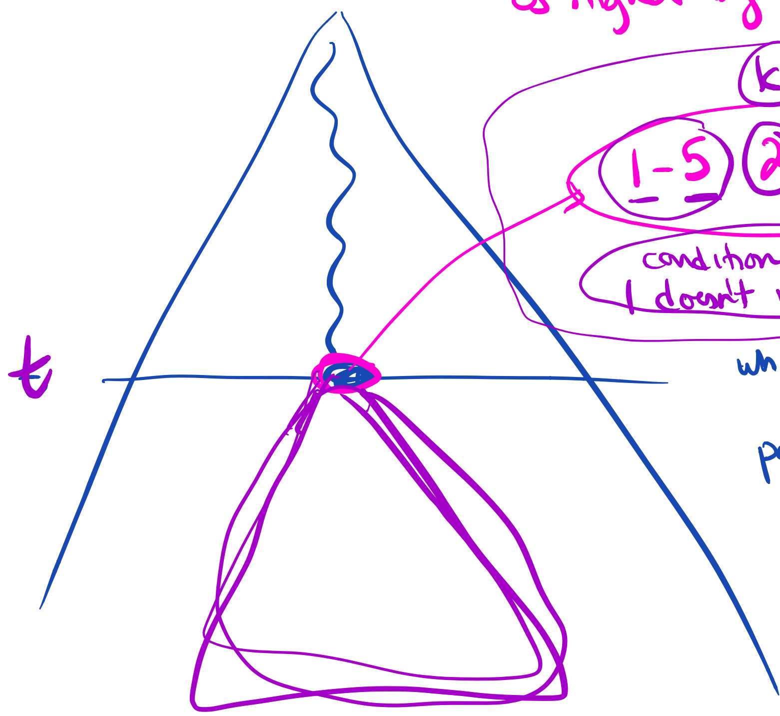


suffices to show that no node at which exp payoff is higher by deviating.

k = # bad agents at this point.

1-5 2-3 4-6

condition of β s.t. I doesn't want to deviate at this node.

which agents have defected in the past

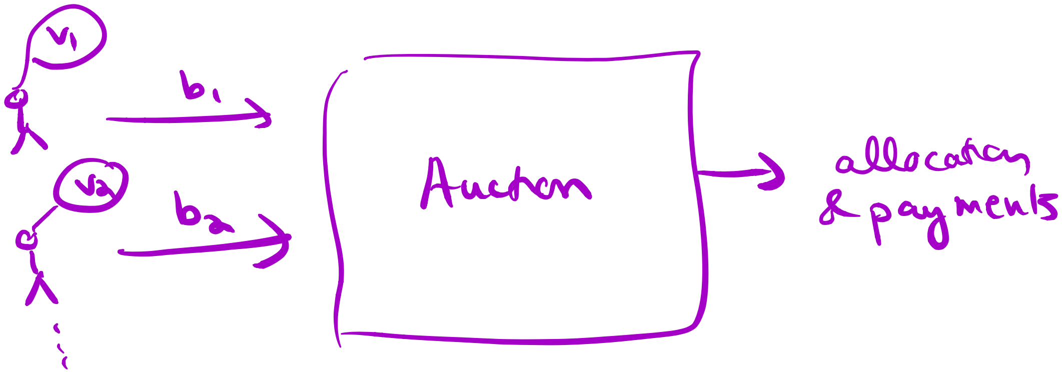
pairing of who each agent is playing.

Auctions

Last time — **Single item auctions**

n agents

$v_1, v_2, \ldots$ are private values



1st price auction: winner is highest bidder
winner pays bid

2nd price (Vickrey) auction: winner is highest bidder
winner pays 2nd highest bid

All-pay auction: winner is highest bidder.
All bidders pay their bid.

utility = value for item if they win it − payment

2^{nd} price auction truthful, DSIC.

$\Rightarrow$ $\boxed{\text{outcome maximizes}}$ social welfare, surplus, efficiency.

$\Rightarrow$ Σ utilities of all players including auctioneer

value of the winning bidder

item goes to player that has highest value.

IR utility of each agent nonnegative.

3 nice properties: truthful, max social welfare, IR

Suppose allocate to 2nd highest bidder at 3rd highest bid — price = 3rd

100
90
80
⋮

allocate to top bidder at 3rd highest bid. — price = 3rd

first price auction

Suppose have k copies of an item.

v_i for single item, $v_1 \geq v_2 \geq \dots \geq v_n$

Auction with all 3 nice properties

to max SW allocate to $1, \dots, k$

$\sum_{i=1}^{k} v_i$

Payment rule? to guarantee truthfulness.

$\forall i \; 1 \leq i \leq k$ charge bidder i the bid of bidder i+1.

Truthful? No

v_1
v_2
v_3
⋮
v_k
v_{k+1}
v_n

Vickrey

Have top k $\underbrace{\text{bidders}}_{\text{(winners)}}$ pay $(k+1)^{st}$ highest bid.

$\underline{B}$ k^{th} highest of other bids

$$\text{utility} \le \max(v - B, 0)$$

truthful bidding attains this

Auctioneer revenue

Assume each player's value $v_i \sim F$ & F is known to all players

A bidding strategy: β: values $\to$ bids

$\beta(100)$

2 players values $\boxed{V_1, V_2 \sim U[0,100]}$

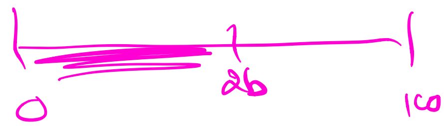
Suppose I know your bidding strategy $\beta(v) = \frac{v}{2}$

Suppose my value is v_1

I choose my bid (b) to maximize $(v_1 - b)\Pr(I \text{ win bidding } b)$

$\Pr\left(\frac{V_2}{2} < b\right)$

$\Pr(V_2 < 2b) = \frac{2b}{100}$



choose b to max

max $(v_1 - b)\cdot\frac{b}{50}$

$$b = \frac{v_1}{2}$$

Bayes-Nash Equilibrium (BNE)

$(\beta_1, \dots, \beta_n)$ [where β_i is the bidding strategy of player i]

is a BNE if $\forall v, \forall i, \forall b$.

$$E\left[u_i\left(\underbrace{\beta_i(v), \beta_{-i}(V_{-i})}_{j \text{ bids } \beta_j(V_j)}\right)\right] \geq E\left[u_i\left(b, \beta_{-i}(V_{-i})\right)\right]$$

2 players $U[0, 100]$ 1st price auction

$\beta(v) = \frac{v}{2}$. (β, β) is a BNE.

2nd price auction $\tilde{\beta}(v) = v$

$(\tilde{\beta}, \tilde{\beta})$ is a BNE

Back to revenue

2 players $U[0,1]$ V_1, V_2

$$\underset{V_1, V_2}{E}[\text{revenue of auctioneer in 1st price auction}]$$

$$= E\left[\max\left(\frac{V_1}{2}, \frac{V_2}{2}\right)\right]$$

$U[0, 100]$

$$= \frac{1}{2} E[\max(V_1, V_2)]$$

$$= \frac{1}{2} \cdot \frac{2}{3} \cdot 100 = \frac{100}{3}$$

$0 \quad \frac{100}{3} \quad \frac{2}{3}100 \quad 100$

2nd price auction

$$E_{V_1, V_2}[\text{auctioneer revenue in 2nd price auction}] = E[\min(V_1, V_2)]$$

$$= \frac{100}{3}$$

$$\int_0^{\omega} p'(v)\,dv = p(\omega) - p(0)$$

Revenue Equivalence Thm

Consider single-item auction where buyers values are drawn from same dist'n F $[0, H]$

Then any auction that allocates to the highest bidder & charges any bidder who bids 0, 0 has the same expected revenue.

$\forall v$ Exp payment of bidder with value v is the same in all of these auctions

Can use this to compute eq bidding strategies.

n bidders, values $\sim F$

$$\underbrace{E[p(v)]}_{\text{payment in } 2^{nd} \text{ price auction}} = \boxed{E\left[\max_{j \neq i} V_j \,\middle|\, V_j < v \;\forall j \neq i\right]} \boxed{Pr\begin{pmatrix} V_j < v \\ \forall j \neq i \end{pmatrix}}$$

E[payment | win] Pr(win)

=

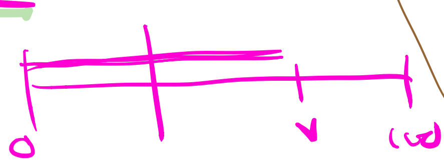
$$\underbrace{E[p(v)]}_{1^{st} \text{ price auction}} = \boxed{\beta(v)} Pr(win)$$

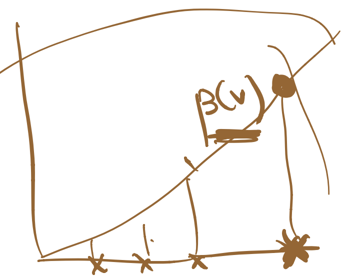
$Pr\left(V_j < v \; \forall j \neq i\right)$

2 player $\beta(v) =$

$$E\left[V_2 \mid V_2 < v_1\right] = \frac{v_1}{2}$$

$U[0,100]$





When $V_i \sim F$

$\beta_i(v) = \beta_j(v) = \beta(v)$ $\forall i, j$

Can use this to compute eq bidding strategies.

n bidders, values ~ F

$$E[p(v)] = E\left[\max_{j\neq i} V_j \,\middle|\, V_j < v \ \forall j \neq i\right] Pr\left(V_j < v \ \forall j \neq i\right)$$

payment in 2nd price auction

$\boxed{\beta_{AB}(v)}$

$$E[p(v)] = \beta_{AP}(v)$$

2 players $U[0,1]$

$$= \frac{v}{2} \underbrace{Pr(V_2 < v)}_{U[0,1] \;=\; v}$$

$\beta_{2nd}(v) = v$

$\beta_{1st}(v) = \frac{v}{2}$

$\beta_{AP}(v) = \frac{v^2}{2}$