Mechanisms for profit maximization

- Research divided into three strands:
  - Bayesian:
    - Agents values assumed to come from publicly known prior distributions.
    - Goal: to do well in expectation
  - Prior-independent
    - There is a prior, but auctioneer doesn't know it.
    - Goal: to do well in expectation.
  - Prior-free
    - What if we don't want to assume a prior?
    - Want to do well in worst case

Prior-free

- Key questions:
  - How do we design mechanisms for profit maximization that work well without priors?
  - How do we evaluate these mechanisms?

Example: Digital Goods Auction

- Given
  - Unlimited number of copies of identical items for sale
  - n bidders, bidder i has private value v_i for obtaining one item (and no additional value for more than one)
- Goal: Design truthful auction to maximize profit

Maximizing Profit: A Competitive Analysis Framework

- Goal: truthful profit maximizing basic auction
- There is no auction that is best on every input.
  - How do we evaluate auctions?

Maximizing Profit: A Competitive Analysis Framework

- Goal: truthful profit maximizing basic auction

- There is no auction that is best on every input.
  - How do we evaluate auctions?

- Competitive analysis
  - Compare auction profit to “profit benchmark” OPT(v).

Profit Benchmark for Digital Goods Auction

Definition:

A truthful auction is c-competitive if for all v its profit is at least OPT(v)/c

- Define OPT(v) = optimal fixed price revenue
  - Example: v= (3, 2, 2, 1, 1) OPT(v)= 6

Profit Benchmark for Digital Goods Auction

- Define OPT(v) = optimal fixed price revenue
  - Example: v= (3, 2, 2, 1, 1) OPT(v)= 6

- How do we design competitive truthful auction? (An auction is c-competitive if for all v its profit is at least OPT(v)/c.)

- Key observation: for an auction to be truthful has to be bid independent – price offered to bidder is independent of that player's bid.

Generic Truthful Auction: BI_f

On input $\mathbf{b}$, for each bidder i:

1. $p \leftarrow f(\mathbf{b}_{-i})$.
2. If $p \leq b_i$, sell to bidder i at price p.
3. Otherwise, reject bidder i.

Profit Benchmark for Digital Goods Auction

- Define OPT(v) = optimal fixed price revenue
  - Example: v= (3, 2, 2, 1, 1) OPT(v)= 6

- How do we design competitive truthful auction?

- Key observation: for an auction to be truthful has to be bid independent – price offered to bidder is independent of that player's bid.

- Most natural choice for f:

  Optimal fixed price for b_{-i}

Generic Truthful Auction: BI_f

On input $\mathbf{b}$, for each bidder i:

1. $p \leftarrow f(\mathbf{b}_{-i})$.
2. If $p \leq b_i$, sell to bidder i at price p.
3. Otherwise, reject bidder i.

Digital Goods Auction

- [GHW] no deterministic symmetric auction achieves any constant competitive ratio.

Digital Goods Auction

- [GHW] no deterministic symmetric auction achieves any constant competitive ratio.
- Must turn to randomized auctions.

Random Sampling Auction for Digital Goods

1. Randomly partition bids into S and S'
2. Offer optimal price for S'' to bidders in S'
3. Offer optimal price for S' to bidders in S''

Theorem [Goldberg,Hartline, Karlin, Saks, Wright**]**

This auction is truthful and achieves a constant competitive ratio.

What's in vogue now…

- Design auctions that obtain revenue close to that of the optimal auction designed for a particular prior distribution, **simultaneously for all** (or a large class of) distributions.