

Revenue Maximization

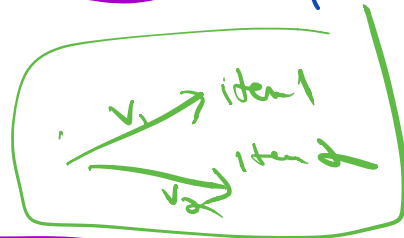
1 seller n bidders
bidder's value $V_i \sim F$

Revenue maximizing truthful auction:

Thm If all $V_i \sim F$ independently (regular)
then rev maximizing auction is Vickrey VCG
auction with monopoly reserve price r^*

$$r^* = \underset{p}{\operatorname{argmax}} \underbrace{p(1-F(p))}_{\Pr(V \geq p)}$$

to any "single parameter setting"



Auction selling 2 items, single bidder

$$V_1 \sim F_1, \quad V_2 \sim F_2$$

$r_1 \quad r_2$

Example:

$$V_i = \begin{cases} 1 & \frac{1}{2} \\ 2 & \frac{1}{2} \end{cases}$$

$i=1,2$

set price of 1 $[2, 1-\epsilon]$

set $p_1 = 2-\epsilon$ $p_2 = 2-\epsilon$

$$E(u_i) = 2 - 2\epsilon$$

$$u_i = (V_i - p_i)^+ + (V_2 - p_2)^+$$

$$E(u_i) = \left(\frac{2-\epsilon}{2}\right) \cdot 2$$

set price of $3-\epsilon$ for both items

$$Pr(\text{buys}) = 1 - Pr(V_1=1=V_2) = \frac{3}{4}$$

$$E(\text{rev}) = 3 \cdot \frac{3}{4} = \frac{9}{4} >$$

$p_{\text{bundle}} = \arg \max P Pr(V_1+V_2 \geq p)$

$$V_1, V_2 = \begin{cases} 0 \\ 1 \\ 2 \end{cases} \quad \begin{matrix} \text{out} \\ \text{in} \\ \text{out} \end{matrix}$$

		V_2		
		0	1	2
V_1	0	X	$1-\epsilon$	$1-\epsilon$
	1	$1-\epsilon$	$2(1-\epsilon)$	
	2	$1-\epsilon$	"	"

P_1, P_2

$$P_1 = P_2 = 1 - \epsilon$$

customers. revenue.

$$P_1 = 1 - \epsilon \quad P_2 = 1 - \epsilon$$

$$E(\text{rev}) = \frac{4}{9} \cdot 1 + \frac{4}{9} \cdot 2 = \frac{12}{9}$$

		V_2		
		0	1	2
V_1	0	X	X	$2-\epsilon$
	1	X	X	$2-\epsilon$
	2	$2-\epsilon$	$2-\epsilon$	$4-\epsilon$

$$P_1 = 2 - \epsilon \quad P_2 = 2 - \epsilon$$

$$E(\text{rev}) = \frac{4}{9} \cdot 2 + \frac{4}{9} \cdot 4 = \frac{16}{9}$$

		V_2		
		0	1	2
V_1	0	X	X	$2-\epsilon$
	1	X	$2-\epsilon$	$2-\epsilon$
	2	$2-\epsilon$	$2-\epsilon$	$2-\epsilon$

$$P_{\text{bundle}} = 2 - \epsilon$$

$$E(\text{rev}) = (2-\epsilon) \cdot \frac{2}{3} = \frac{4-\epsilon}{3}$$

$$P_{\text{bundle}} = 3 - \epsilon$$

	0	1	2
0	X	X	X
1	X	X	$3-\epsilon$
2	X	$3-\epsilon$	$3-\epsilon$

$$E(\text{rev}) = 3 \cdot \frac{1}{3} = 1$$

$$P_{\text{bundle}} = 4$$

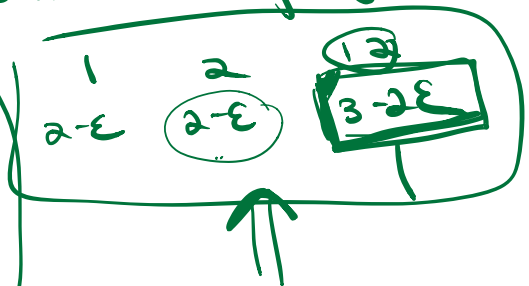
$$E(\text{rev}) = 4 \cdot \frac{1}{9}$$

			$4-\epsilon$

Better

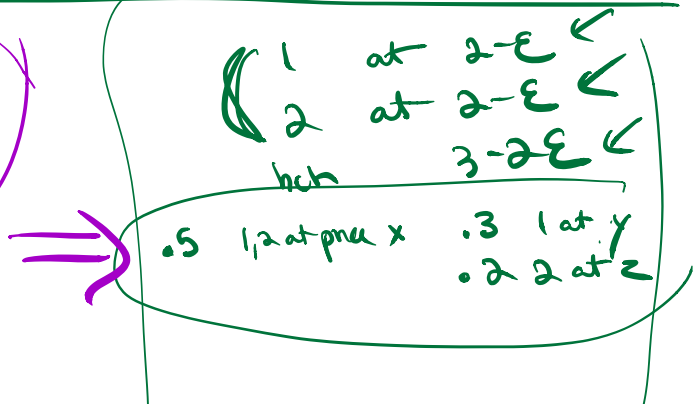
Offer buyer choice between any single item at price of $2-\epsilon$ or bundle of both items at price $3-2\epsilon$

	0	1	2
0	X	X	$2-\epsilon$
1	X	X	$3-2\epsilon$
2	$2-\epsilon$	$3-2\epsilon$	$3-2\epsilon$



$$E(\text{rev}) = 3 \cdot \frac{3}{9} + 2 \cdot \frac{2}{9} = \frac{13}{9}$$

F \Rightarrow menu size of optimal auction is infinite.



When values are correlated,
possible to design an auction
that extracts "full value"

Single item auction:

best possible case

$$E(\text{rev}) = E(\max(V_1, \dots, V_n))$$

2 item
single item $U[0,1]$ $\frac{1}{2}$ ~~1/2~~ $\frac{1}{2}$

Prior indep auctions.

How do we know F ?

Thm Suppose F is regular. ^{single item auction}
Then expected revenue of Vickrey auction with $n+1$ bidders
 \geq exp rev of opt auction w/ n bidders

competition is more important than reserve price.

Corollary If $n \geq 2$, exp revenue of Vickrey auction (n)

assuming all values $\sim F$

$\geq \frac{n-1}{n}$ of exp rev of opt auction (n)

Pf

A_n^* opt auction n bidder

$\Downarrow \Rightarrow A_{n-1}$

$\frac{n-1}{n}$ of rev of A_n^*

\hat{A}_{n-1}

(b_1, \dots, b_{n-1}) inputs
 b_n Auctioneer.

$\hat{b}_n \sim F$

Run $A_n^q(b_1, \dots, b_{n-1}, \hat{b}_n)$

if \hat{b}_n sells to i_{n-1}

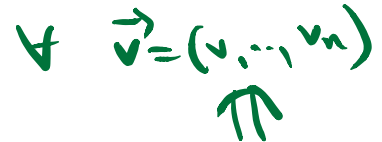
if it sells to bidder n do nothing.

$$\text{rev}(\hat{A}_{n-1}) \geq \frac{n-1}{n} \text{rev}(A_n^q)$$

$$\text{rev Vicking (n bidders)} \geq \text{rev OPT}(n-1) \geq \text{rev}(\hat{A}_{n-1}) \geq \frac{n-1}{n} \text{rev}(A_n^q)$$

Mechanisms for profit maximization

- Research divided into three strands:
 - Bayesian:
 - Agents values assumed to come from publicly known prior distributions.
 - Goal: to do well in expectation
 - Prior-independent
 - There is a prior, but auctioneer doesn't know it.
 - Goal: to do well in expectation.
 - Prior-free
 - What if we don't want to assume a prior?
 - Want to do well in worst case

$$v \quad \vec{v} = (v_1, \dots, v_n)$$


Prior-free

- Key questions:
 - How do we design mechanisms for profit maximization that work well without priors?
 - How do we evaluate these mechanisms?

Example: Digital Goods Auction

What does VCG do for digital goods?

- Given
 - Unlimited number of copies of identical items for sale
 - n bidders, bidder i has private value v_i for obtaining one item (and no additional value for more than one)
- Goal: Design truthful auction to maximize profit

Maximizing Profit: A Competitive Analysis Framework

- Goal: truthful profit maximizing basic auction
- There is no auction that is best on every input.
 - How do we evaluate auctions?

truthfulness \Rightarrow imposes constraints on what we can achieve. \$100

absolute optimality \Rightarrow relative optimality. \$100

Maximizing Profit: A Competitive Analysis Framework

- Goal: truthful profit maximizing basic auction
- There is no auction that is best on every input.
 - How do we evaluate auctions?
- **Competitive analysis**
 - Compare auction profit to “profit benchmark”

$\vec{v} = (v_1, \dots, v_n)$

$\text{OPT}(\vec{v}) = \sum_{i=1}^n v_i$

Profit Benchmark for Digital Goods Auction

Definition:

$$\vec{v} = (v_1, \dots, v_n)$$

A truthful auction is c-competitive if for all v its profit is at least

$$\text{OPT}(\vec{v})/c$$

\geq

- Define $\text{OPT}(v)$ = optimal fixed price revenue

- Example: $v = (3, 2, 2, 1, 1)$

$$\text{OPT}(\vec{v}) = 6$$

3
2

1

3
6

c

$$\text{OPT}(\vec{v}) = \max_p P$$

(# bidders with value $\geq p$)