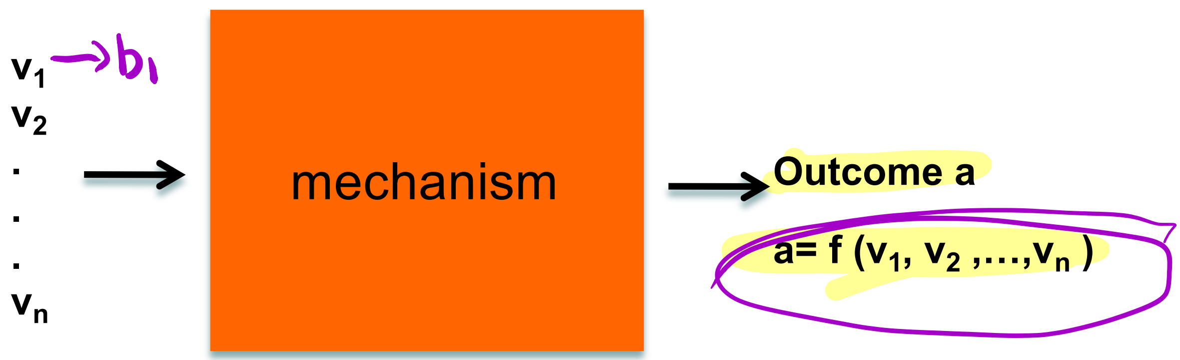
# *Mechanism design*

- How to design algorithms that take inputs from strategic agents, but are still guaranteed to produce the outcome that we as designers want?

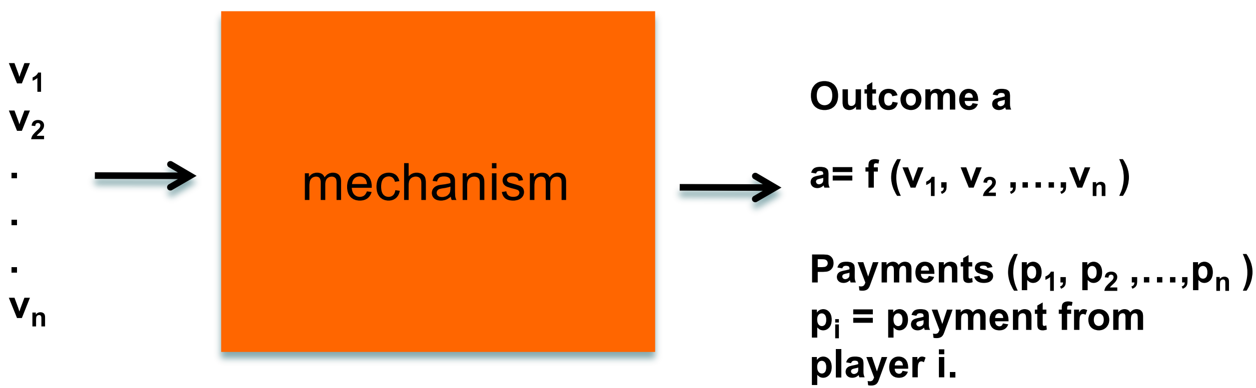
# *Implementing a function*

- n players
- A: set of possible outcomes
- $v_i: A \rightarrow R$, where $v_i(a)$ is the value to player i of outcome a in A. *private information of self-interested participant*
- One of the common goals:
  - implement a function f **i.e., ensure that the outcome selected is $f(v_1, v_2, \ldots, v_n)$**



# *Setup*

- To ensure correct outcome, must incentivize the players to "bid" truthfully => for this, need payments.
- **Utility** of agent i for outcome a: **$v_i(a) - p_i$**
- **Challenge:** choose payments so that mechanism is **truthful** – a player cannot gain by misreporting $v_i$ no matter what others do.



# *A fundamental question*

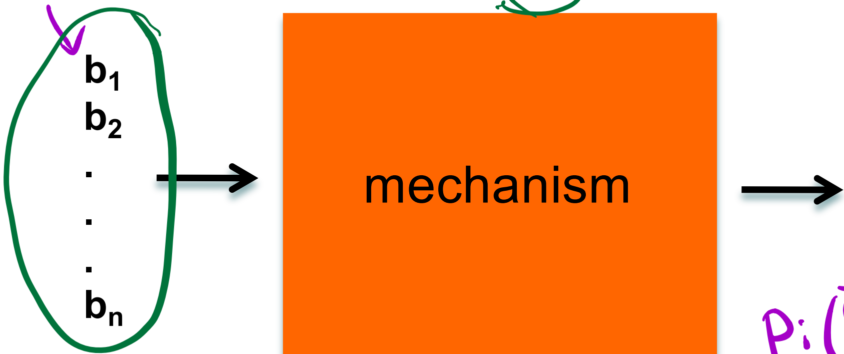
- For what functions f does there exist a payment rule that guarantees that it is in each players' best interest to tell the truth, no matter what the other players do?
- When such a payment rule exists, we say f is "implementable".

- One important case: when the goal is to maximize **social welfare**, i.e.

$$f(v_1, v_2, \ldots, v_n) = argmax_{a \in A} \sum_j v_j(a)$$

# Vickrey Clarke Groves (VCG) mech

$a \in A$




Outcome is:

$a^* = \text{argmax}_a \, \Sigma_j b_j(a)$

payment of i:

$$p_i(\vec{b}) = -\sum_{j \neq i} b_j(a^*) - \text{something independent of player i's report}$$

utility of player i:

$$v_i(a^*) - p_i = v_i(a^*) + \sum_{j \neq i} b_j(a^*) + \text{indep of } b_i$$

incentivized to report

$$b_i(a) = v_i(a) \quad \forall a$$

mechanism chooses outcome $a$ that maximizes

$$b_i(a) + \sum b_j(a)$$

implementable

$j \neq i$

$v_i(a)$

n agents: solve n+1 SW maximizations.

$$p_i(\vec{b}) = \max_{a} \sum_{j \neq i} b_j(a) - \sum_{j \neq i} b_j(a^*)$$

$$P_a = 5 - 5 = 0$$

$$P_b = 4 - 3 = 1$$

EXAMPLE 16.2.5 (**Employee housing**). A university owns a number of homes and plans to lease them to employees. They choose the allocation and pricing by running a VCG auction. A set of $n$ employees, each interested in leasing at most one house, participates in the auction. The $i^{\text{th}}$ employee has value $v_{ij}$ for a yearly lease of the $j^{\text{th}}$ house. Figure 16.3 shows an example.

outcomes A: set of possible matchings

∀ agent i: $v_{ik}$: value of i for $k^{th}$ house

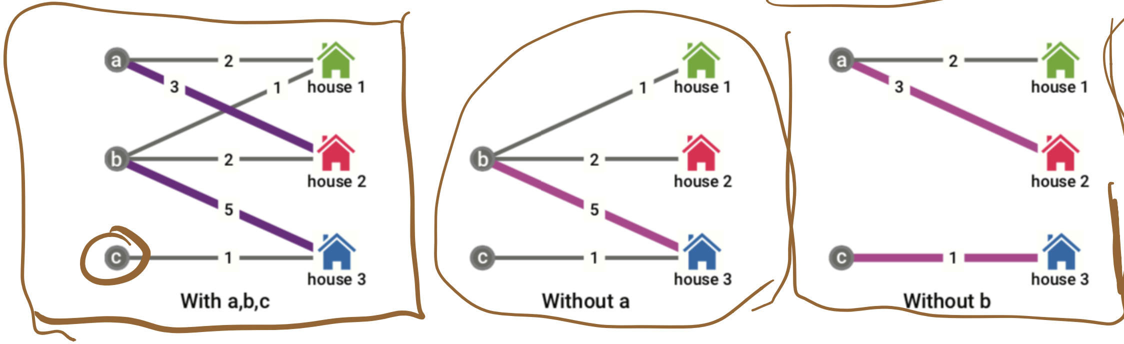



FIGURE 16.3. The label on the edge from $i$ on the left to $j$ on the right is the value $v_{ij}$ that employee $i$ has for a yearly lease of house $j$ (say in thousands of dollars). The VCG mechanism allocates according to purple shaded edges. The payment of bidder $a$ is 0 since in his absence house 3 is still allocated to bidder $b$. The payment of bidder $b$ is 1 since in his absence the allocation is as follows: house 2 to bidder $a$ and house 3 to bidder $c$, and therefore the externality he imposes is 1.

# Combinatorial Auctions

- m items for sale
- n bidders competing for a subset of these items
- Each bidder i has a valuation $v_i(S)$ for each subset S of items

$2^m$ #'s per agent

- **Objective:** Find a partition of the items $(S_1, \ldots S_n)$ that maximizes social welfare $\Sigma_i v_i(S_i)$

$S_i$ set of items → i

government selling licenses for use of spectrum

Applications:

- Spectrum auctions.
- Abstraction of complex resource allocation problems such as routing, scheduling, load balancing, etc.

FCC runs auctions.

VCG gives us a way to find the most efficient outcome.

# *Back to online advertising*

- Since VCG is so general, convenient for complex scenarios such as those Facebook deals with.
  - Outcomes are page layouts, which include a mix of organic and sponsored content.
  - Dynamic resizing
  - Bidders bid on events (click/like/app download)

Practical issue (e.g. for Facebook)

- Design of user interface for bidding.
  - Huge number of possible outcomes, impossible to elicit bid for each.

# *VCG for Facebook*

- Outcomes are page layouts, which include a mix of organic and sponsored content.
- Bidders bid on events (click/like/app download)
- Their bid specifies their value for each such event.
- $v_i$ (w) = value of event x Pr (event occurs in outcome w)
- Facebook devotes enormous effort to learning accurate estimates of these probabilities from data/history.
- Advertisers don't need to know these probabilities.

# *Another practical issue*

- Computational requirements
  - Auction run every time user access news feed.
  - Complexity of implementation.

# *Bigger picture*

- Online advertising ecosystem complex and enormous
- The process for how an ad gets shown to you when you go to a website involves real-time bidding/auctions and a number of intermediaries such as ad exchanges.
- See course web page.
- One interesting thing that has happened recently is that there seems to be a switch from second price auctions to first price auctions (in the display advertising market).
- As far as I can tell, the main reason is transparency.

# *Problem with second price*

" In a second-price auction, raising the price floors after the bids come in allows [online auctioneers] to make extra cash off unsuspecting buyers…. The practice persists because neither the publisher nor the advertiser has complete access to all the data involved in the transaction, so unless they get together and compare their data, publishers and buyers won't know for sure who their vendor is ripping off"

# *Other significant issues*

Repeated auctions.

Interaction between bidding and budgets.

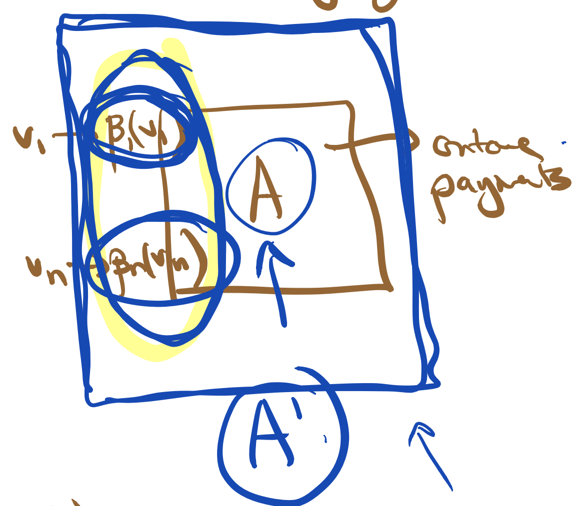
# Revenue Maximization.

1 seller  1 item  n bidders.

$V_1, V_2, \ldots, V_n$ r.v. $\sim F$

**Objective:** Find the "optimal" auction among all truthful auctions, has the highest expected revenue.

Truthfulness is without loss of generality




1 bidder $V \sim F$  VCG $\Rightarrow$ payment 0

set a price. $p$ $\Rightarrow$ choose $p$ to max $p(1-F(p))$

monopoly price.  $U(0,1)$  $p(1-p)$

2 bidders $V_1, V_2$ $U(0,1)$  $E(\text{rev}) = \frac{1}{3}$ $\Rightarrow E(\text{rev}) = \frac{5}{12}$

Run Vickrey auction with reserve price of $r$, run a Vickrey auction ⟹ winner, payment $p$

offer winner item at price $= \max(p, r)$

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A valuation distn fn $F$ with density $f$ is called **regular** if

$$x - \frac{1-F(x)}{f(x)}$$

is a nondecreasing fn of $x$.

unif
normal
exponential
heavy-tailed.

**Thm** If bidders valuations are drawn iid. from a regular distn $F$, then the auction that maximizes exp auctioneer revenue is

Vickrey auction with reserve price

$= $ monopoly price

$= \arg\max_p \, p(1-F(p))$