

Auctions

advertising auctions

VCG

Advertising – how it used to be



Pay-per-impression

Price depends on how many people your advertisement is shown to.

(whether or not they look at it, or care about it)

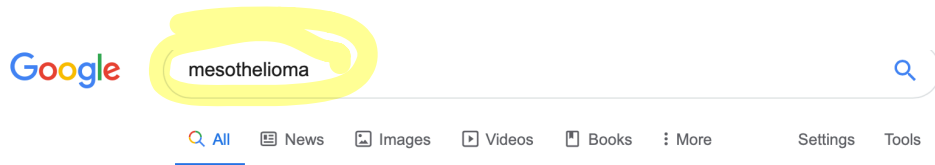
“Half the money I spend on advertising is wasted;
the trouble is, I don't know which half”

Andrew Wanamaker, advertising pioneer



Sponsored Search Ads

Pay per click.



About 7,290,000 results (0.61 seconds)

Ad · www.nationalmesotheliomaclaims.com/ ▾

Mesothelioma Claim Center | Get Asbestos Cancer Payments

Learn how the \$30B Asbestos Trust Fund may pay for your asbestos-caused cancer.

Mesothelioma Claims

Asbestos Trust Fund Claims
This \$30 Billion Trust Can Help

Free Info Package

Covers what you need to know
about medical and financial options

Ad · www.mesotheliomabook.com/ ▾ (888) 637-6234

Family Hurt by Mesothelioma? | We're Here to Help You

We are the largest firm devoted only to Mesothelioma in the US, visit us today. We come to...

Ad · www.mesothelioma-survivor.com/mesothelioma/treatment ▾ (800) 336-0086

I Survived Mesothelioma Cancer - Learn How She Beat the Odds

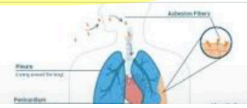
Learn About Heather's Successful Surgery from Preparation to the Procedure & Recovery.

Ad · www.navy-veterans-mesothelioma.org/Financial/Benefits ▾

Mesothelioma & Asbestos Risk | Navy Vets Asbestos Claims

Important info for Navy Vets. Learn About Mesothelioma Claims

Mesothelioma is a type of cancer that develops from the thin layer of tissue that covers many of the internal organs (known as the mesothelium). The most



Mesothelioma

ABOUT SYMPTOMS TREATMENTS

Mesothelial lining

Cough and chest pain

Cancer

Source

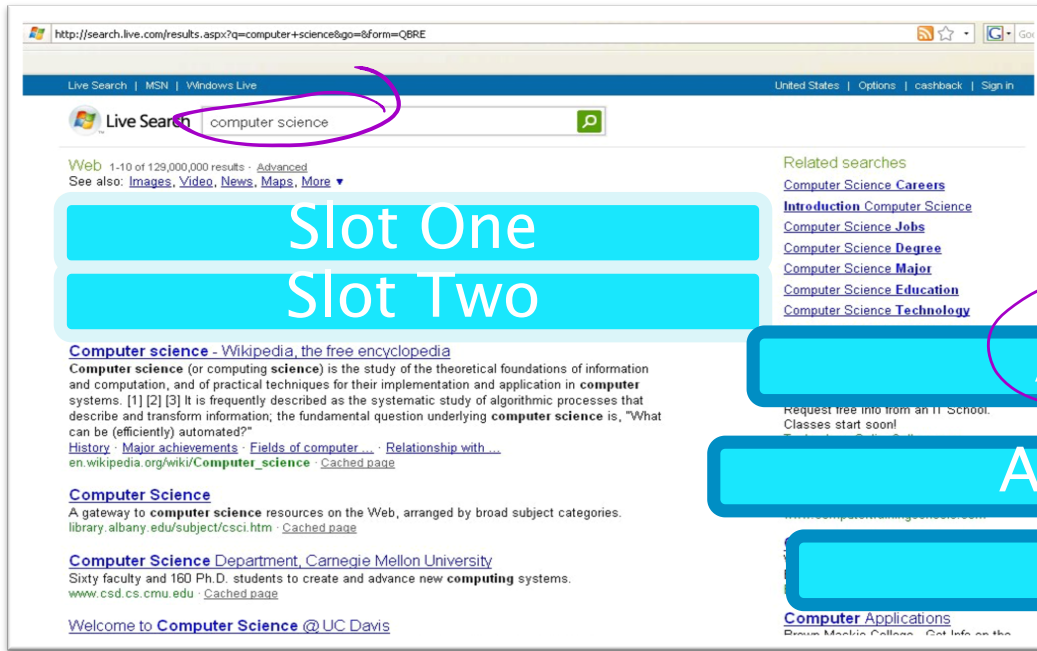
A tumor of the tissue that lines the lungs, stomach, heart, and other organs.

Very rare

Fewer than 20,000 US cases per year


- + Treatment can help, but this condition can't be cured
- 📅 Requires a medical diagnosis
- 🏠 Lab tests or imaging always required

Price determined by auction – per keyword



“Most people don’t realize that all that money comes in pennies at a time”
Hal Varian, Google Chief Economist

Switch to auctions/PPC watershed

- Allows advertiser to much more finely target customers.
 - Customer did a search on related topic
 - Customer then bothered to click on the ad.
 - Reduces risk to advertisers.
 - Provides advertisers with very accurate metrics.
 - Was the ad clicked on?
 - Did the click result in a purchase?
 - Nobody is getting locked into long-term contracts.
 - Users are happier because the ads they are being shown are higher quality and better tailored to their needs.
- 

Model

- goods for sale are k slots for sponsored links on searchpage
- bidders advertisers currently have standing bid on relevant keywords.
- slots not identical
- quantify difference using "click thru rates" CTR.
- $c_1 > c_2 > \dots > c_k$
 $\frac{1}{c_i}$ Prob of a click on an ad in slot i .
 put ad j in slot i
 Prob of a click $c_i q_j$
- q_j - "quality" of advertiser j .
 for now $q_j = 1$
- each advertiser j has expected value v_j for a click on their ad.
- expected value adv j gets if his ad \rightarrow slot i
 $c_i v_j$
- if payment p_j / click.
 exp payment $\underline{c_i v_j - c_i p_j}$

Generalized 2nd Price Auction (GSP)

Collect bids from advertisers
 allocate highest bidder \rightarrow slot 1
 2nd \rightarrow slot 2
 ...

$$b_1 > b_2 > \dots > b_k > b_{k+1}$$

\uparrow \uparrow \uparrow \uparrow
 slot 1 slot 2 slot 3 slot 4

$$p_i = b_{i+1}$$

PPC

Prior to intro GSP, were using pay your bid.

- automated bidding agents
- constant bid & dating.

Example - GSP not truthful

Click-through rates

$c_1 = 1$ (1)

$c_2 = 0.9$ (2)

0 No slot

Slots

Value per click

(A) $10 = v_1$

(B) $9 = v_2$

(C) $0.01 = v_3$

Advertisers

exp utility

$$c_1 v_1 - c_1 p_1 = 1$$

$$c_2 v_2 - c_2 p_2 = 0.9 \cdot 9 - 0.9 \cdot 0.01$$

$$c_2 v_1 - c_2 \cdot 0.01 = 0.9 \cdot 10 - 0.9 \cdot 0.01 = 9 - \dots$$

Benefit to overbidding?

$$c_i v_j - c_i b_{\text{next } b}$$

Fix other bids.

suppose agent j bids

$$b_j > v_j$$

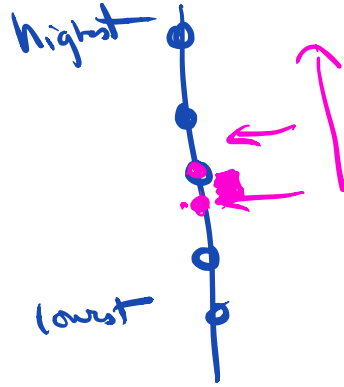
slot s

$$c_i v_j - c_i b_{\text{next highest below } b_j}$$

next highest below b_j

$$b \geq v_j$$

$$b < v_j$$



In this model, is there an auction that

- maximizes Social welfare
- individually rational
- truthful.

$v_1 > v_2 > \dots > v_n$ $c_1 > c_2 > c_3 \dots > c_k$ $c_{k+1} = 0.$

What allocation maximizes SW?

get SW

$$\sum_{i=1}^k v_i c_i$$

\sum_i utilities of bidders
 + utility of auctioneer
 = \sum_i exp value to i of alloc received.

change each bidder their "externality"
 externality: cost or benefit to others due to my behavior/existence

$j \rightarrow$ sub $s(j)$

$$c_{s(j)} v_j$$

Fix bidder 1:
 with bidder 1 present, SW of others is

$$\sum_{i=2}^k v_i c_i$$

with bidder 1 absent, SW of others is

$$\sum_{i=2}^{k+1} v_i c_{i-1}$$

Bidder 1's externality = $\sum_{i=2}^{k+1} (c_{i-1} - c_i) v_i$

this is what we will charge bidder 1 (exp payment)

Bidder 3's externality (exp payment) = $\sum_{i=4}^{k+1} (c_{i-1} - c_i) v_i$

VCG auction for sponsored search

Ask each bidder for bid.
assign bidder $i \rightarrow$ slot i

$b_1 > b_2 > \dots > b_n$

VCG payment per click

PPC $P_i = \frac{1}{c_i} \sum_{j=i+1}^{k+1} b_j (c_{j-1} - c_j)$

GSP

$p := b_{i+1}$

$b_3 \leftarrow$

$i=2$

$b_3 \left(\frac{c_2 - c_3}{c_2} \right) + b_4 \left(\frac{c_3 - c_4}{c_2} \right) + b_5 \left(\frac{c_4 - c_5}{c_2} \right)$

convex comb.

$w = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_n v_n$
 $\sum \alpha_i = 1$
 $\alpha_i \geq 0$

Same example with VCG

Click-through rates

1



0.9



0 No slot

Slots

Value per click

A 10

B 9

C 0.01

Advertisers

GSP
wpp

9

0.01

VCG
wtp

$9(1-0.9) + 0.01(0.9)$

0.01 - 0.9



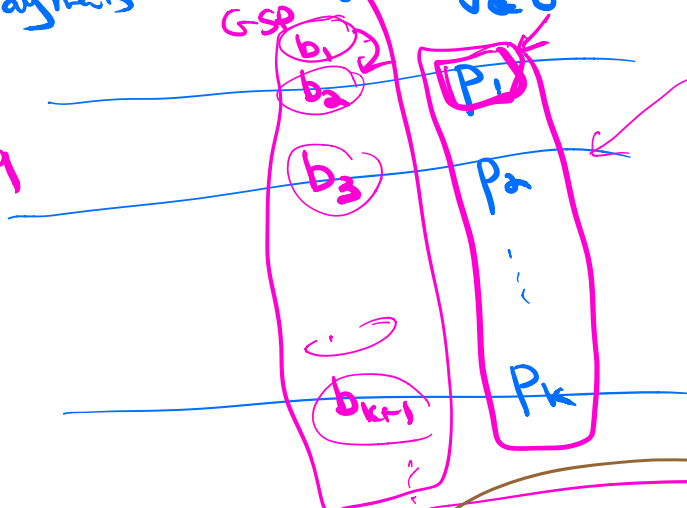
GSP has been the dominant paradigm in sponsored search auctions.

possible that revenue from GSP > rev from VCG.

No direct revenue comparison that you can make.

\exists Nash eq in GSP that results in same allocation & payments as in (truthful) VCG

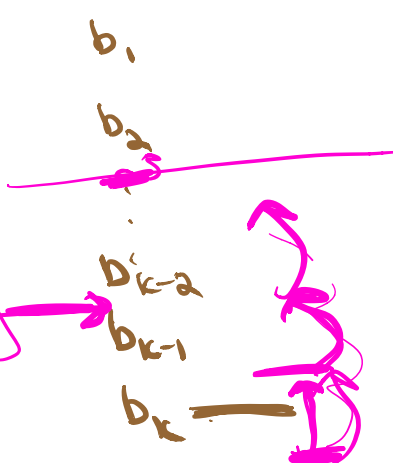
many other eq too.



$$c_i p_i = \sum_{j=i+1}^{k+1} (c_{j-1} - c_j) b_j$$

Thm VCG is truthful.

Pf Fix bidder w/ value v , Fix bids of all other bidders.



$$(c_{k-1} - c_k)v$$

$$+ c_k b_k$$

$$\Delta \text{value} = (c_{k-1} - c_k)v$$

$$\Delta \text{pay} = (c_{k-1} - c_k) b_{k-1}$$

$$\Delta \text{value} = c_k v$$

$$\Delta \text{pay} = c_k b_k$$

Google
relevant ads
determined by
query

"fixed" ordering of slots
"fixed" # of slots

advertisers bidding
on clicks

Facebook

your friends
activity
inf in your newsfeed.

ads different sizes
& formats.

clicks
likes
downloads of app.

many more possible outcomes to auction
- dynamic resizing.

very difficult for advertisers to estimate
CTRs.

in VCG, advertisers don't need to know CTRs
(seller has a lot more info about CTRs)

FB

advertisers can bid on "events"
clicks, likes, app downloads.

much more general model than sponsored search
auction model.

VCG: n bidders

Ω - finite set of outcomes.

each agent i has value

$v_i(w)$

$\forall w \in \Omega$

VCG is truthful mech for choosing outcome $w^* \in \Omega$

such that $\sum_{i=1}^n v_i(w^*)$ is maximized.

$$p_i = \max_w \sum_{j \neq i} b_j(w) - \sum_{j \neq i} b_j(w^*)$$

\uparrow
 $v_j(b_j)$