Auctions

advertising  auctions

VCG
Advertising – how it used to be
Pay-per-impression

Price depends on how many people your advertisement is shown to.

(whether or not they look at it, or care about it)

“Half the money I spend on advertising is wasted; the trouble is, I don’t know which half”

Andrew Wanamaker, advertising pioneer
Sponsored Search Ads

Google search results for mesothelioma

1. **Mesothelioma Claim Center | Get Asbestos Cancer Payments**
   - Learn how the $30B Asbestos Trust Fund may pay for your asbestos-caused cancer.
   - **Mesothelioma Claims**
     - Asbestos Trust Fund Claims
     - This $30 Billion Trust Can Help
   - **Free Info Package**
     - Covers what you need to know about medical and financial options

2. **Family Hurt by Mesothelioma? | We're Here to Help You**
   - We are the largest firm devoted only to Mesothelioma in the US, visit us today. We come to...

3. **I Survived Mesothelioma Cancer - Learn How She Beat the Odds**
   - Learn About Heather's Successful Surgery from Preparation to the Procedure & Recovery.

4. **Mesothelioma & Asbestos Risk | Navy Vets Asbestos Claims**
   - Important info for Navy Vets. Learn About Mesothelioma Claims

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**Mesothelioma**

- A type of cancer that develops from the thin layer of tissue that covers many of the internal organs (known as the mesothelium). The most

- **Very rare**
  - Fewer than 20,000 US cases per year

- Treatment can help, but this condition can't be cured
- Requires a medical diagnosis
- Lab tests or imaging always required

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Pay per click.
Price determined by auction – per keyword

“Most people don’t realize that all that money comes in pennies at a time”

Hal Varian, Google Chief Economist
Switch to auctions/PPC watershed

- Allows advertiser to much more finely target customers.
  - Customer did a search on related topic
  - Customer then bothered to click on the ad.
- Reduces risk to advertisers.
- Provides advertisers with very accurate metrics.
  - Was the ad clicked on?
  - Did the click result in a purchase?
- Nobody is getting locked into long-term contracts.
- Users are happier because the ads they are being shown are higher quality and better tailored to their needs.
Model
- goods for sale are $k$ slots for sponsored links on search page
- bidders, advertisers currently have standing bid on relevant keywords
- slots not identical
- quantity difference using "clickthrough rates" CTR
- $c_1 > c_2 \ldots \geq c_k$
- $p_i$ - probability a click on an ad in slot $i$
- $q_j$ - "quality" of advertiser $j$
- each advertiser $j$ has expected value $V_j$ for a click on their ad.
- expected value adv $j$ gets of his ad -> slot $i$ if payment $p_j$/click:
  \[
  \text{exp. payment } c_i \cdot V_j - c_i \cdot p_j
  \]

Generalized 2nd Price Auction (GSP)
- Collect bids from advertisers
- Allocate highest bidder -> slot $1$
- Allocate next highest bidder -> slot $2$

$\mathbf{p_i} = b_{i+1}$

PPC

Prior to initiate GSP, we use pay-per-click:
- automated bidding agents
- constant bid per click.
Example – GSP not truthful

Click-through rates
- $c_1 = 1$
- $c_2 = 0.9$

0 No slot

Slots

Value per click
- A $10 - v_1$
- B $9 - v_2$
- C $0.01 - v_3$

Advertisers

Benefit to overbidding?
- $c_j v_j - c_i b_j$
- Next $b$

Fix open bids
- Suppose agent $j$ bids $b_j > v_j$
- Slot $b$

Exp utility
- $\exp \left( \frac{c_j v_j - c_i b_j}{c_i v_i - c_i p_i} \right) = 1$
- $c_2 v_2 - c_2 p_2 - 0.9 \cdot 0.9 - 0.9 \cdot 0.01$

Next highest below $b_j$
- $b = v_j$
- $b < v_j$
In this model, is there an auction that:
- maximizes social welfare
- is individually rational
- is truthful.

$v_1 > v_2 > ... > v_n$
$c_1 > c_2 > c_3 ... > c_k$
$c_{k+1} = 0.$

What algorithm maximizes SW?

$v_i \rightarrow c_i$

$k \sum_{i=1}^k v_i c_i$

change each bidder their externally
externally: cost or benefit to others due to my behavior/existence

Fix bidder:
with bidder 1 present, SW of others is
$k \sum_{i=1}^k v_i c_i$

with bidder 1 absent, SW of others is $\sum_{i=2} v_i c_{i-1}$

Bidder's externality $= \sum_{i=2}^{k+1} (c_{i-1} - c_i) v_i$
this is what we will change hidden 1 (exp payout)

\[
\text{Bidder } 3\text{'s externally } = \sum_{i=4}^{k+1} (c_{i-1} - c_i) v_i
\]

VCG algorithm for sponsored search

Ask each bidder for bid. 
assign bidder \(i\) to slot \(i\)

\[
P_i = \frac{1}{c_i} \sum_{j=i+1}^{k+1} b_j (c_j - c_i)
\]

GSP

\[
p_i = b_i + 1
\]

convex comb,

\[
x = \sum_{i=1}^{k} x_i + \sum_{i=1}^{k} w_i
\]

\[
\sum_{i=1}^{k} x_i = 1
\]

\[
\sum_{i=1}^{k} w_i = 0
\]
Same example with VCG

Click-through rates | Value per click
---|---
1 | A 10
0.9 | B 9
0 | No slot

Slots | Advertisers
---|---
C 0.01

GSP

VCG

9 (1-0.9) + 0.01 (0.9)

0.01 - 0.9
GSP has been the dominant paradigm in sponsored search and was possible that revenue from GSP is greater than that from VCG.

No direct revenue comparison that you can make.

Nash in GSP that results in same allocation & payments as in (truthful) VCG in VCG.

Many other eq too.

Thm VCG is truthful.

Fix bid of value $v_i$ for bidder $i$, Fix bids of all other bidders.

\[ c_i \cdot p_i^* = \frac{k+1}{\text{max}(j \neq i)(c_j - c_i) b_j} \]

\[ \Delta \text{value} = \frac{(c_k - c_{k-1}) \cdot \Delta V}{c_k - c_{k-1}} \]

\[ \Delta \text{pay} = \frac{\Delta V}{c_k b_k} \]

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Google
relevant ads determined by query
"fixed" ordering of slots
"fixed" # of slots
advertisers bidding on clicks

Facebook
your friends activity in your newsfeed.
ads different sizes, formats,
likes, clicks, downloads of app.

Many more possible outcomes to anchor
-dynamic resizing.

Very difficult for advertisers to estimate
CTRs.

In VCG, advertisers don't need to know CTRs
(seller has a lot more info about CTRs).

FB

Advertisers can bid on "events"
chicks, likes, app downloads,
much more general model than sponsored search auction model.

VCG: n bidders
\( \mathcal{N} \) - finite set of outcomes.
Each agent i has value \( v_i(\mathcal{N}) \) if \( w \in \mathcal{N} \).

VCG is truthful mech for choosing outcome \( w \in \mathcal{N} \).
such that \( \sum_{i=1}^{n} v_i(w^*) \) is maximized.

\[
p_i = \max \sum_{w} \sum_{j \neq i} b_{ij}(w) - \sum_{j \neq i} b_{ij}(w^*)
\]