

1. Suppose that 3 players are competing in an auction and their values are each drawn independently from a uniform distribution on $[0, 1]$. Use the revenue equivalence theorem to derive a Bayes-Nash equilibrium strategy for the players in this auction if it is an all-pay auction. Prove that the strategies you derive are indeed a Bayes-Nash Equilibrium.
2. Consider a single-parameter allocation problem where the agents are divided into 2 markets, and the auctioneer is only allowed to sell in one of them. Suppose also that the players in the first market have values drawn from an exponential distribution with parameter $\lambda = 1$ and the players in the second market have values drawn from an exponential distribution with parameter $\lambda = 2$. (If you need a reminder look at the wikipedia article on the exponential probability distribution.)
 - Describe precisely the allocation rule and the payment rule in the auction the auctioneer should run to maximize his expected profit.
 - Describe precisely the allocation rule and the payment rule the auctioneer should use to maximize social welfare.
 - Describe the outcome and payments when there are 3 agents in the first market and their values are 0.2, 1.5 and 6, and there are 3 agents in the second market and their values are 0.3, 1 and 5.6, in both of the previous 2 scenarios.
3. **Extra Credit:** Consider a single-parameter allocation problem where the auctioneer is either allowed to sell to no-one or to everyone.
 - Describe precisely the allocation rule and the payment rule the auctioneer should run to maximize his expected profit when there are n agents whose values are each drawn independently from the uniform $[0,1]$ distribution.
 - Give an asymptotic (as a function of n , the number of agents) analysis of the expected revenue the auctioneer will collect.