Solutions for Quiz #1

Andrey Kolobov

November 8, 2011

1. **True or False:** Every dominant strategy equilibrium in a normal form game is also a Nash equilibrium.

   ANSWER: True, by the definition of dominant strategy equilibrium.

2. **True or False:** Every zero-sum game has a pure Nash equilibrium.

   ANSWER: False. A counterexample is the Moriarty vs. Holmes game from homework #1.

3. **True or False:** Suppose a particular zero-sum game has value 0. Then no matter how the two players play, the first player's expected payoff is at least 0.

   ANSWER: False. If the first player plays suboptimally, his payoff can be less than the value of the game.

4. **True or False:** All Nash equilibria in a normal form (not necessarily zero-sum) game give each player the same expected payoff.

   ANSWER: False. A counterexample is the game of chicken.

5. **True or False:** Every normal form game has a pure Nash equilibrium (i.e. a Nash equilibrium where the players play pure strategies).

   ANSWER: False. A counterexample is the Moriarty vs. Holmes game from homework #1.

6. **True or False:** It's advantageous in a zero-sum game to know the other player's mixed strategy before deciding on your own. (You can assumer the other player plays "optimally".)

   ANSWER: False. Such knowledge gives no advantage, since if the other player plays "optimally", your best strategy is maximin, which can be computed independently of whether the other player tells you his strategy or not.

7. Consider a normal form game, where the row player has 3 strategies (let's call them 1,2,3) and the column player has 3 strategies (let's also call them 1,2,3) and let $A_{i,j}$ be the payoff to the row player, when the row player plays strategy i and the column player plays strategy j, and let $B_{i,j}$ be the payoff to the column player, when the row player plays

strategy i and the column player plays strategy j. (i and j here both range from 1 to 3.) Suppose I told you that the mixed row strategy $(\frac{1}{3}, \frac{2}{3}, 0)$ and the mixed column strategy $(0, \frac{1}{5}, \frac{4}{5})$ are a Nash equilibrium in this game. Write down all the conditions you would need to check to be sure that I am right.

ANSWER: The conditions to check are the following:

- Each player's strategy equalizes expected payoffs to the opponent from the opponent's positive-probability pure strategies, i.e.

$$\frac{B_{1,2}}{3} + \frac{2B_{2,2}}{3} = \frac{B_{1,3}}{3} + \frac{2B_{2,3}}{3}$$

  and

$$\frac{A_{1,2}}{5} + \frac{4A_{1,3}}{5} = \frac{A_{2,2}}{5} + \frac{4A_{2,3}}{5}$$

- Given the opponent's mixed strategy, he expected payoffs to each player from his zero-probability pure strategies are no bigger than from his positive-probability pure strategies, i.e.

$$\frac{B_{1,1}}{3} + \frac{2B_{2,1}}{3} \leq \frac{B_{1,2}}{3} + \frac{2B_{2,2}}{3}$$

  and

$$\frac{A_{3,2}}{5} + \frac{4A_{3,3}}{5} \leq \frac{A_{1,2}}{5} + \frac{4A_{1,3}}{5}$$

- (Optional.) The pure strategy probabilities under each player's mixed strategy should sum to 1 and be no less than 0. Since the probabilities were given explicitly, these checks are trivial so you wouldn't lose any points if you didn't mention them. Remember, though, that when you are formulating a game as a linear program, these checks are crucial.

COMMON MISTAKES AND EVALUATION.

- Many people wrote that each player's mixed strategy should equalize the payoffs *to that player across his opponent's pure strategies*, i.e. something like

$$\frac{A_{1,2}}{3} + \frac{2A_{2,2}}{3} = \frac{A_{1,3}}{3} + \frac{2A_{2,3}}{3}$$

  and

$$\frac{B_{1,2}}{5} + \frac{4B_{1,3}}{5} = \frac{B_{2,2}}{5} + \frac{4B_{2,3}}{5}$$

  This is a rather serious mistake since it indicates a lack of understanding of how Nash equilibria are computed. If you made it you would have lost 1.5 points out of 4. Some people gave the correct explanation that each player's mixed strategy should equalize the payoffs to the opposing player, but nonetheless got the equations wrong. They lost 0.5 or 1 point, depending on the clarity of explanation.

- Many people stated that payoffs to each player from *all* of their pure strategies should be equal. This excludes valid N.E.s in which the payoff for the zero-probability pure strategy is less than the payoff from non-zero-probability pure strategies. This point is perhaps subtle and wasn't emphasized much in class, so you would only lose 1 point if you made this mistake.

Several people presented solutions in a way that I could not understand very well. If you believe I misinterpreted your solution and you should have gotten more points, let me know!

8. Consider two players playing the following game repeatedly.

	C	D
C	(2,2)	(-1, 3)
D	(3, -1)	(0,0)

(a) Player 1's minimax value is

ANSWER: 0, achieved when both players deterministically play their second pure strategies.

(b) Write down strategies for the 2 players that are a Nash equilibrium in the infinitely repeated game and achieve average payoffs for the 2 players of $(0, 2\frac{2}{3})$. Note that $\frac{1}{3}(2,2) + \frac{2}{3}(-1,3) = (0, 2\frac{2}{3})$.

ANSWER: Player 1 always plays C. Player 2 plays C once and D twice in every three rounds. If one of the players deviates (this can be checked at the end of any three-round period), the other plays D forever.

COMMON MISTAKES AND EVALUATION.

Several people gave the strategy under which in every round Player 1 plays C, and Player 2 plays C with probability $\frac{1}{3}$ and D with probability $\frac{2}{3}$. Although this strategy achieves the desired payoffs, it is not a N.E. strategy, since, for instance, Player 2 always wants to deviate and play D if Player 1 plays C. Again, this is a serious mistake, and a few people made it on the last homework. Unfortunately, when correcting the homeworks I didn't explicitly say what the correct answer was — my fault. To compensate for it, if you made this mistake again on the quiz you would only lose 1 point, even though this mistake makes the solution almost entirely wrong.