Suppose we knew that the game has a joint Nash equilibrium strategy \( \{ \vec{p}, \vec{q} \} \) that assigns probabilities \( p_i > 0 \) to a subset \( I_R \) of row player \( R \)'s pure strategies and \( q_i > 0 \) to a subset \( I_C \) of column player \( C \)'s pure strategies. Such a strategy would need to satisfy the following conditions:

- No matter what pure strategy \( r \in I_R \) \( R \) plays, the expected payoff to \( R \) given \( C \)'s mixed strategy \( \vec{q} \) should be the same value; denote it as \( V_R \). (If this value is not the same for all \( r \in I_R \), \( R \) has an incentive to deviate by choosing a pure strategy that brings the biggest payoff, i.e. \( \{ \vec{p}, \vec{q} \} \) isn’t a Nash equilibrium.)

- If \( R \) decides to play a pure strategy \( r \notin I_R \), the expected payoff to \( R \) shouldn’t be more than \( V_R \). (Otherwise, \( \vec{p} \) should assign a non-zero probability to that pure strategy, i.e. the strategy should be in \( I_R \).)

- Similarly, no matter what pure strategy \( c \in I_C \) \( C \) plays, the expected payoff to \( C \) should be the same; denote this value as \( V_C \).

- If \( C \) plays a pure strategy \( c \notin I_C \), \( C \)'s expected payoff should be no more than \( V_C \).

- Under \( \{ \vec{p}, \vec{q} \} \), the probabilities of pure strategies in \( I_R \) and in \( I_C \) respectively must sum to one and be no less than zero.

- Under \( \{ \vec{p}, \vec{q} \} \), the probabilities of pure strategies not in \( I_R \) and not in \( I_C \) respectively must be zero.

These requirements translate into the following system of linear inequalities:
To find a Nash equilibrium for given $I_R$ and $I_C$, we need to find any $\{\vec{p}, \vec{q}\}$ satisfying the above system of inequalities. In other words, we need to solve the corresponding LP feasibility problem. LP feasibility problems are just a type of LP problems — in the latter one is interested in an optimal solution to a system of inequalities, while in the former one is interested in any solution to it. Therefore, if a solution exists it can be found with an LP solver in finite time.

Of course, we don’t actually know $I_R$ and $I_C$ for which a Nash equilibrium exists. Therefore, we need to iterate over all possible pairs of action sets and for each to try to find a Nash equilibrium strategy as described above. Once such a strategy is found, the algorithm will halt. There are a total of $2^{n+m}$ possible action set pairs, so the whole algorithm runs in a finite, although large, amount of time.