Non-linear Synthesis: Beyond Modulation

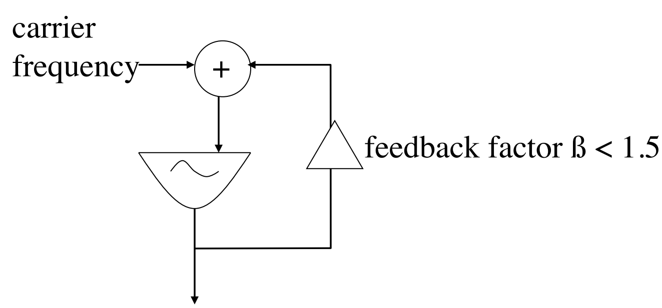
Feedback FM

Invented and implemented by Yamaha

Solves the problem of the rough changes in the harmonic amplitudes caused by Chowning FM.

Single Osc Feedback FM

Feedback factor acts similarly to modulation index and controls the width of the spectrum.



Single Osc Feedback FM

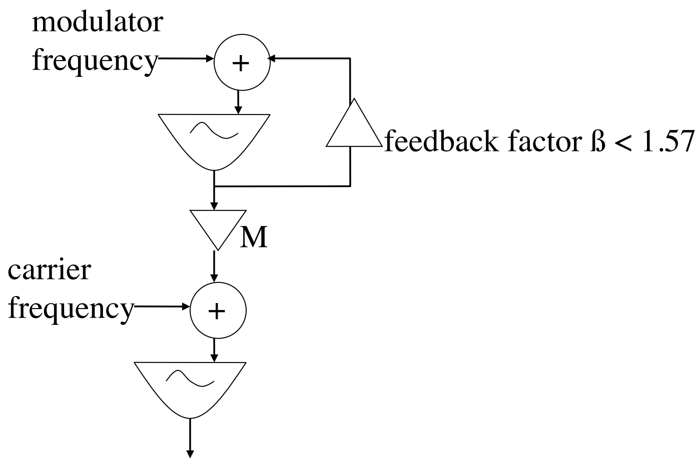
Amplitude of the nth harmonic is proportional to:

$$\frac{2}{(n * \beta)} * J_n(n * \beta)$$

Each harmonic has a unique scaling factor and a unique Index.

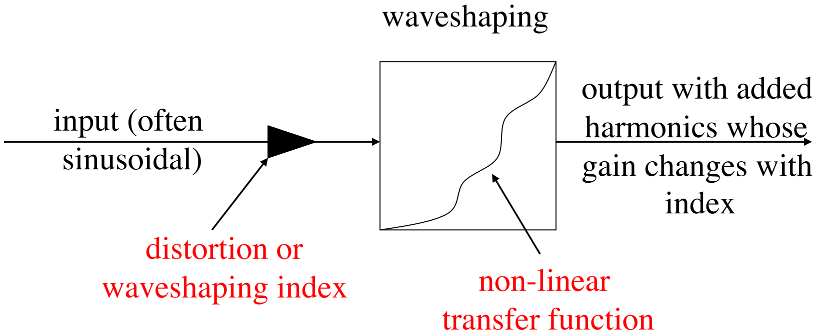
Two Osc Feedback FM

Output of the one-oscillator feedback FM modulates another oscillator.



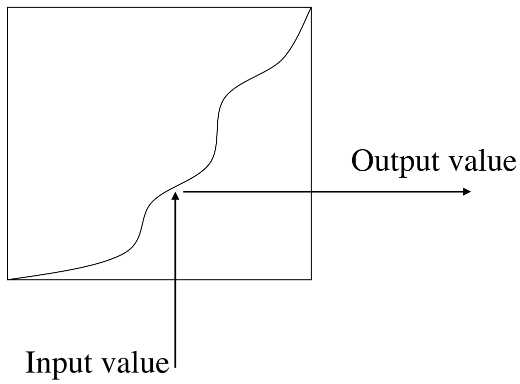
Non-linear Waveshaping

Waveshaping is a type of *distortion synthesis* that can create dynamic spectra.

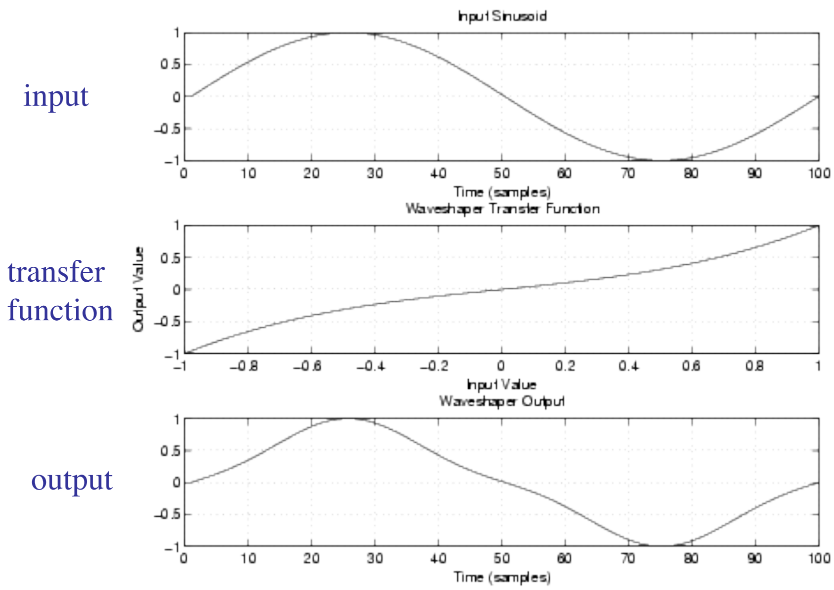


Waveshaping

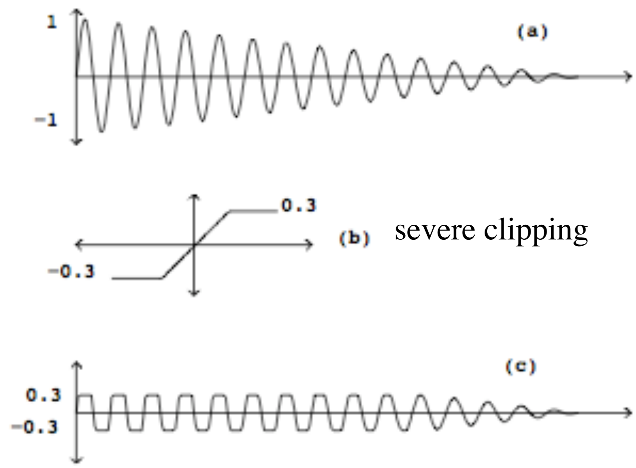
Waveshaper and transfer function



Waveshaping

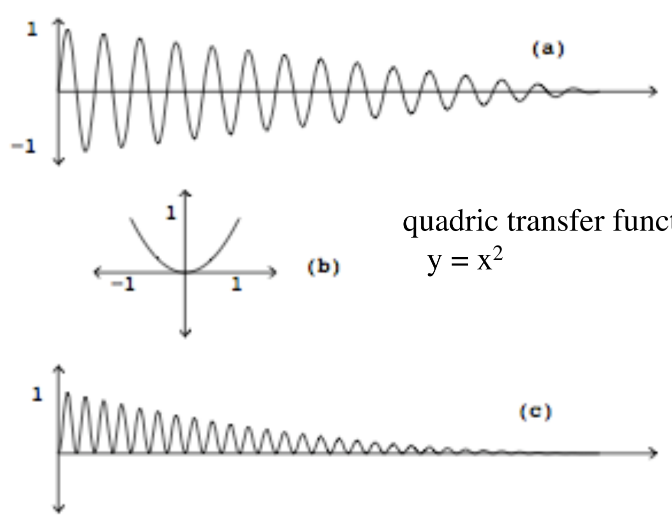


Dynamic Non-linear Distortion



severe clipping

Dynamic Non-linear Distortion



quadric transfer function
$y = x^2$

Intermodulation

- When more than one sinusoid is applied to the waveshaper, additional frequencies are generated called intermoduation products.
- Intermodulation becomes more and more dominant as the number of components in the input increases.
- If there are k sinusoids in the input, there are only k 'regular' sinusoids in the product, but there are $(k^2 - k)/2$ additional sinusoids by intermodulation.

Look at SuperCollider examples.

How input levels below 1.0 affect output

The waveshaping function can be written as a power series:

$$f(x) = f_0 + f_1 x + f_2 x^2 + f_3 x^3 + \cdots$$

If the input is a sinusoid, then the effect of each term can be examined separately:

$$f(x[n]) = f_0 + a f_1 \cos(\omega n) + a^2 f_2 \cos^2(\omega n) + a^3 f_3 \cos^3(\omega n) + \cdots$$

Low amplitudes emphasize low harmonics and the level of the higher harmonics increases as amplitude approaches 1.0.

Using Chebyshev Polynomials

When a sinusoid of unity amplitude is applied to a *Chebyshev polynomial* of order *k*, the output contains energy only at the *kth* harmonic. This property makes Chebyshev polynomials potentially useful for building more complex waveshaping functions in terms of a specific desired harmonic content.

cheby(n) = cos(n * acos(x))

Using Chebyshev Polynomials

In order to create an output that has specific gains for each harmonic, use the target gains to scale the individual Chebyshev polynomials in the transfer function.

transfer function = 0.5 *$Cheby_1$ + 0.3 * $Cheby_2$ + 0.2 $Cheby_3$

When a sinusoid with a peak amplitude of 1.0 is applied to this transfer function, the output will contain the first three harmonics at gains of 0.5, 0.3 and 0.2.

Chebyshev Polynomials

$Cheby_0 = 1$

$Cheby_1 = x$

$Cheby_2 = 2x^2 - 1$

$Cheby_3 = 4x^3 - 3x$

$Cheby_4 = 8x^4 - 8x^2 + 1$

$Cheby_5 = 16x^5 - 20x^3 + 5x$

Etc.

Low amplitudes will produce greater output gain with the low-order Chebyshev polynomials and the output will approach the target as the input amplitude approaches 1.0.

Look at SuperCollider examples.