Probabilistic Models

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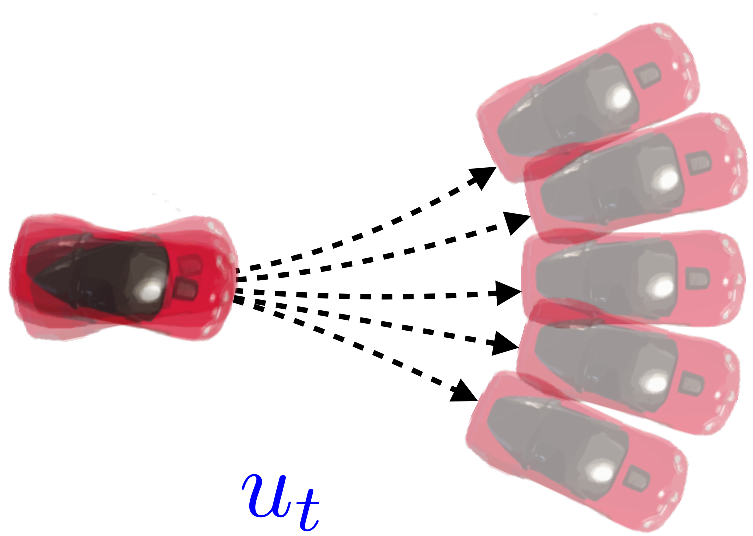
TAs: Matthew Rockett, Gilwoo Lee, Matt Schmittle

Probabilistic models in localization

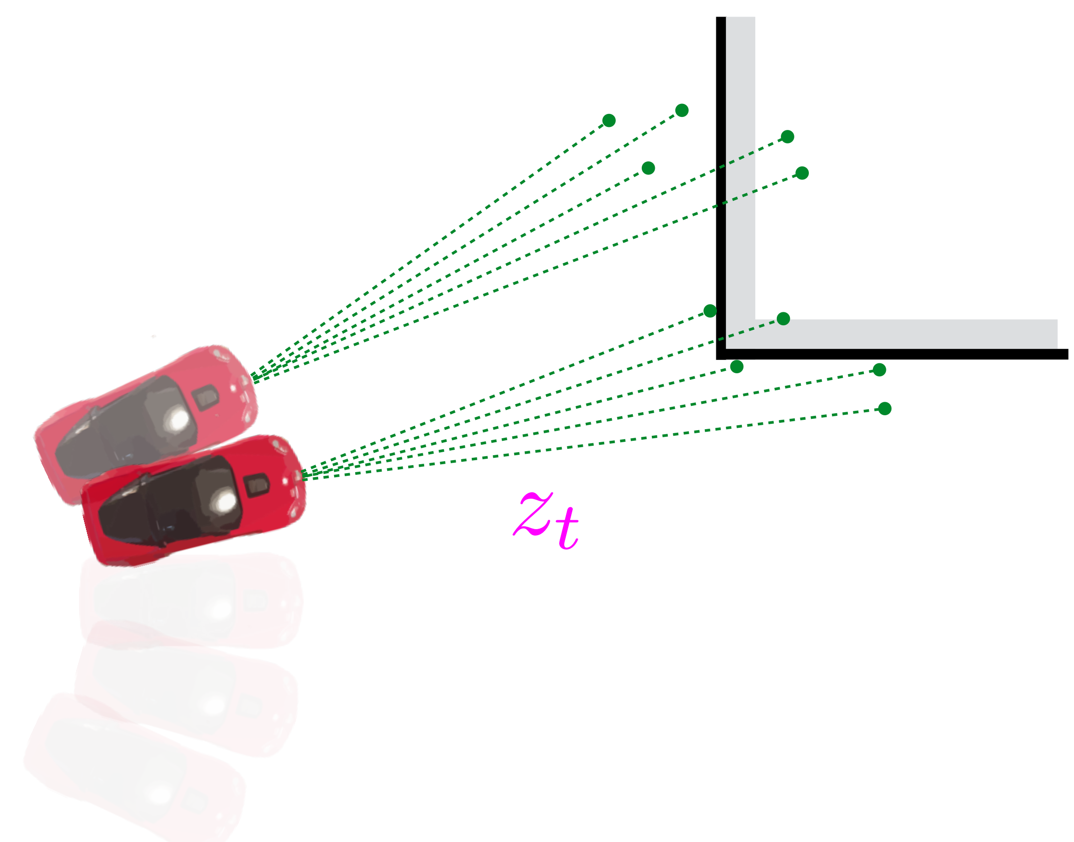
Motion model

$$P(x_t | u_t, x_{t-1})$$

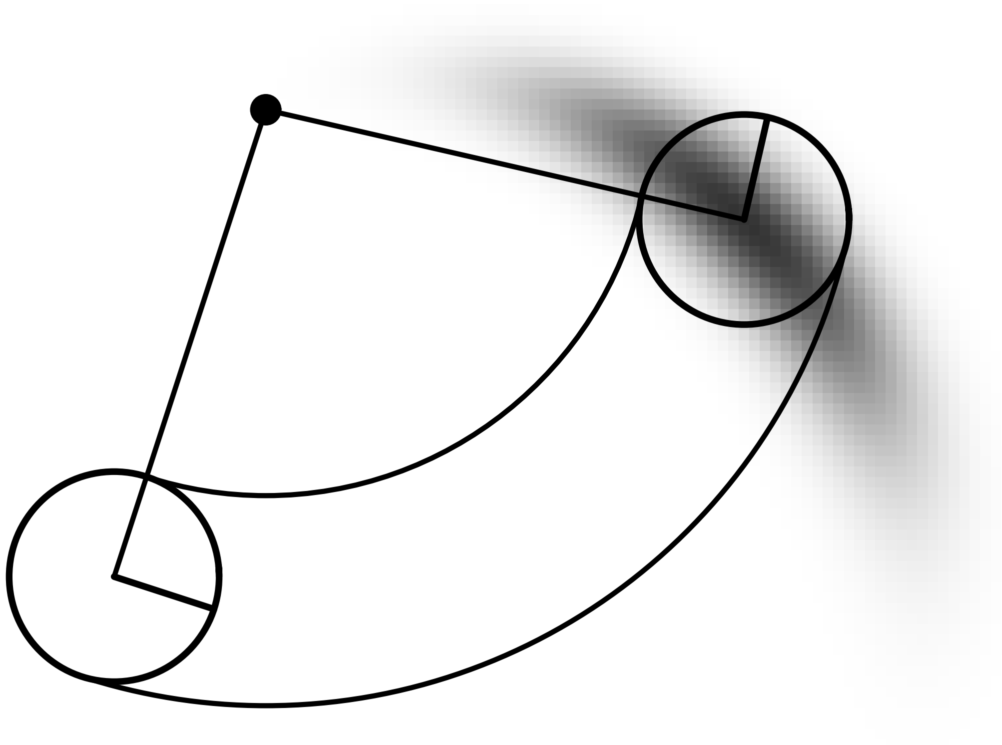


Measurement model

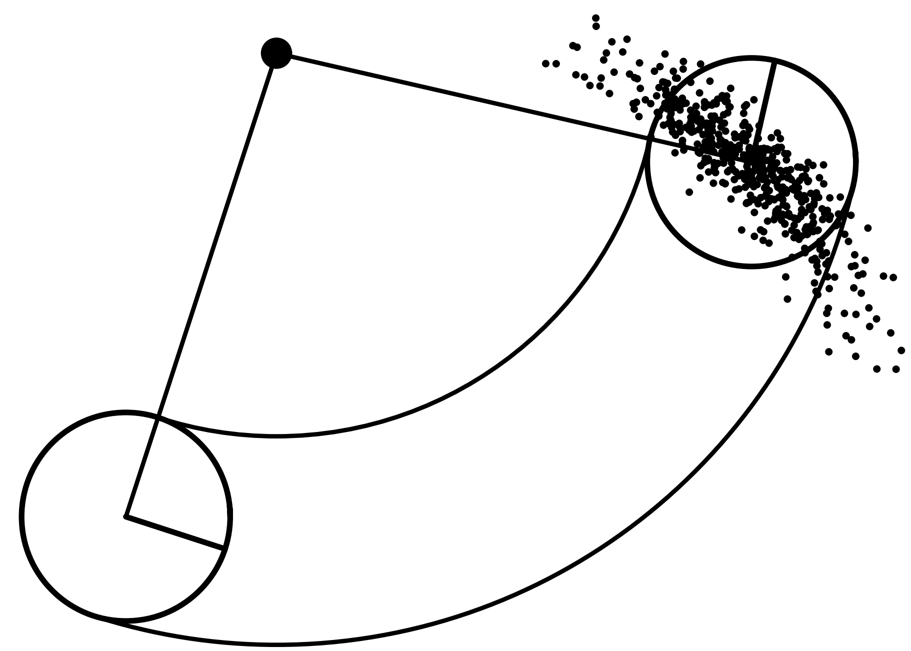
$$P(z_t | x_t)$$



Example of a motion model



Probability density function



Samples from the pdf

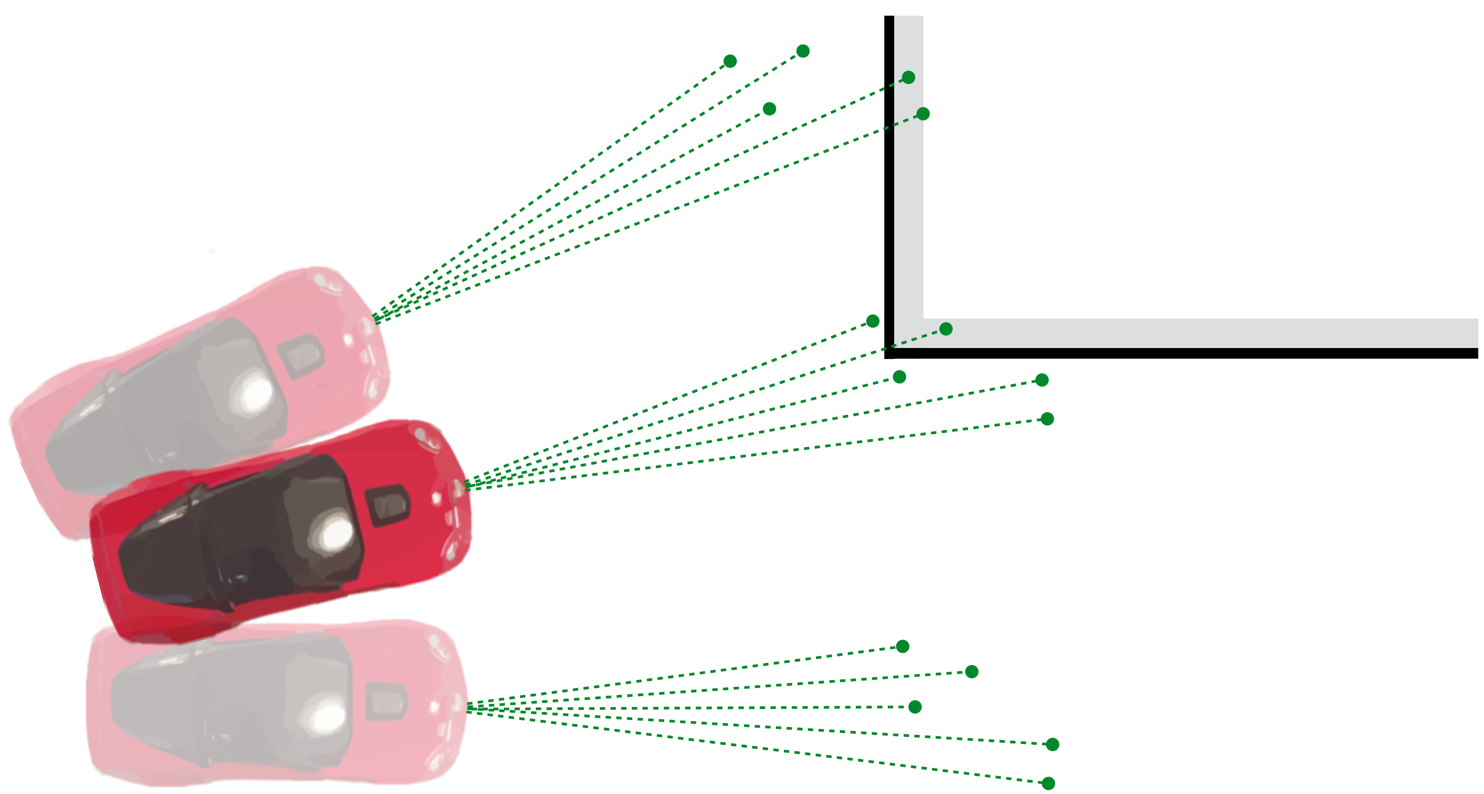
Measurement Model

$$P(z_t | x_t, m)$$

sensor reading

state

map



How does a LiDAR work?

https://www.youtube.com/watch?v=NZKvf1cXe8s&t=67s

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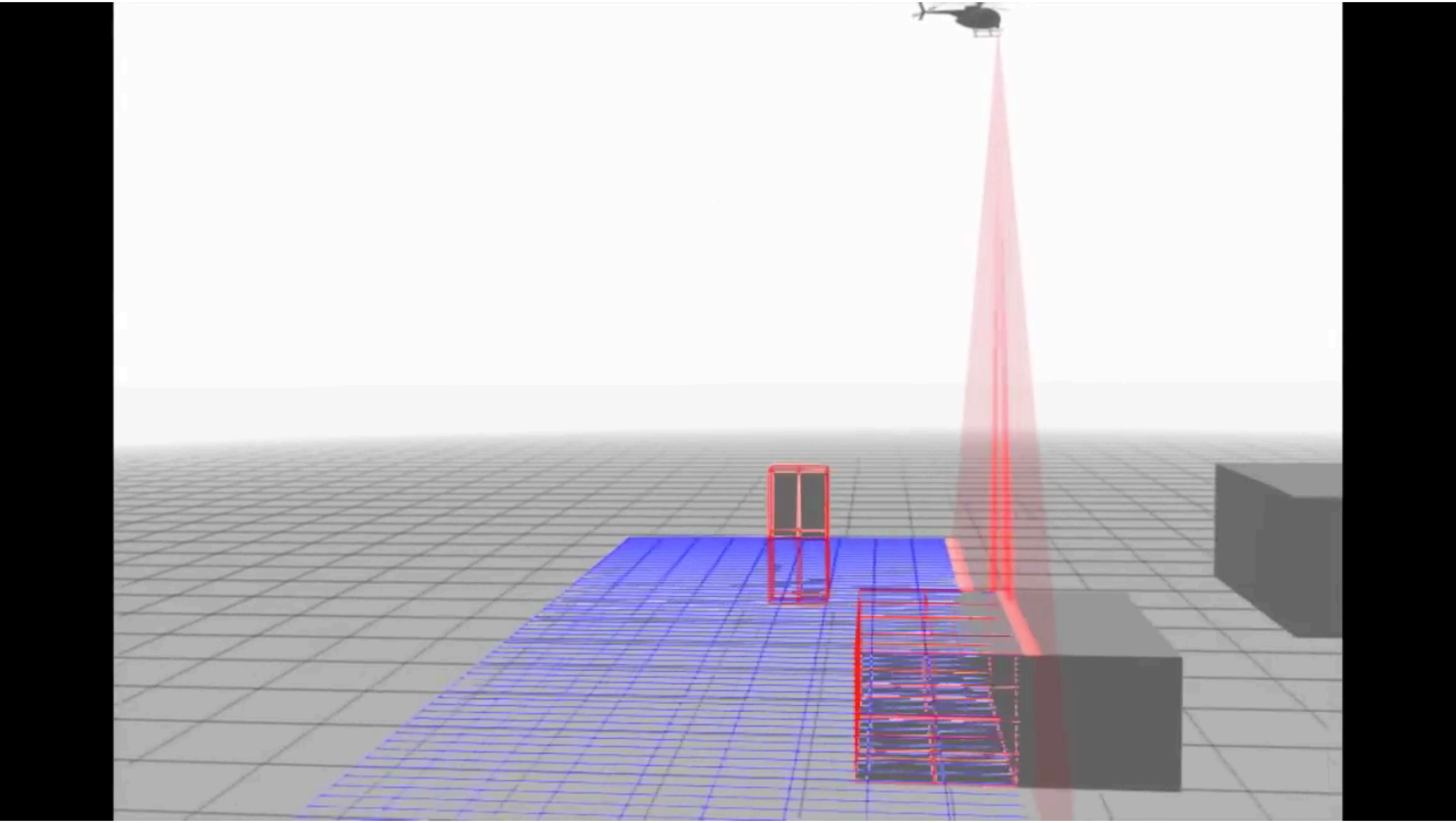
Working with lasers in the real world



(courtesy Lyle Chamberlain)

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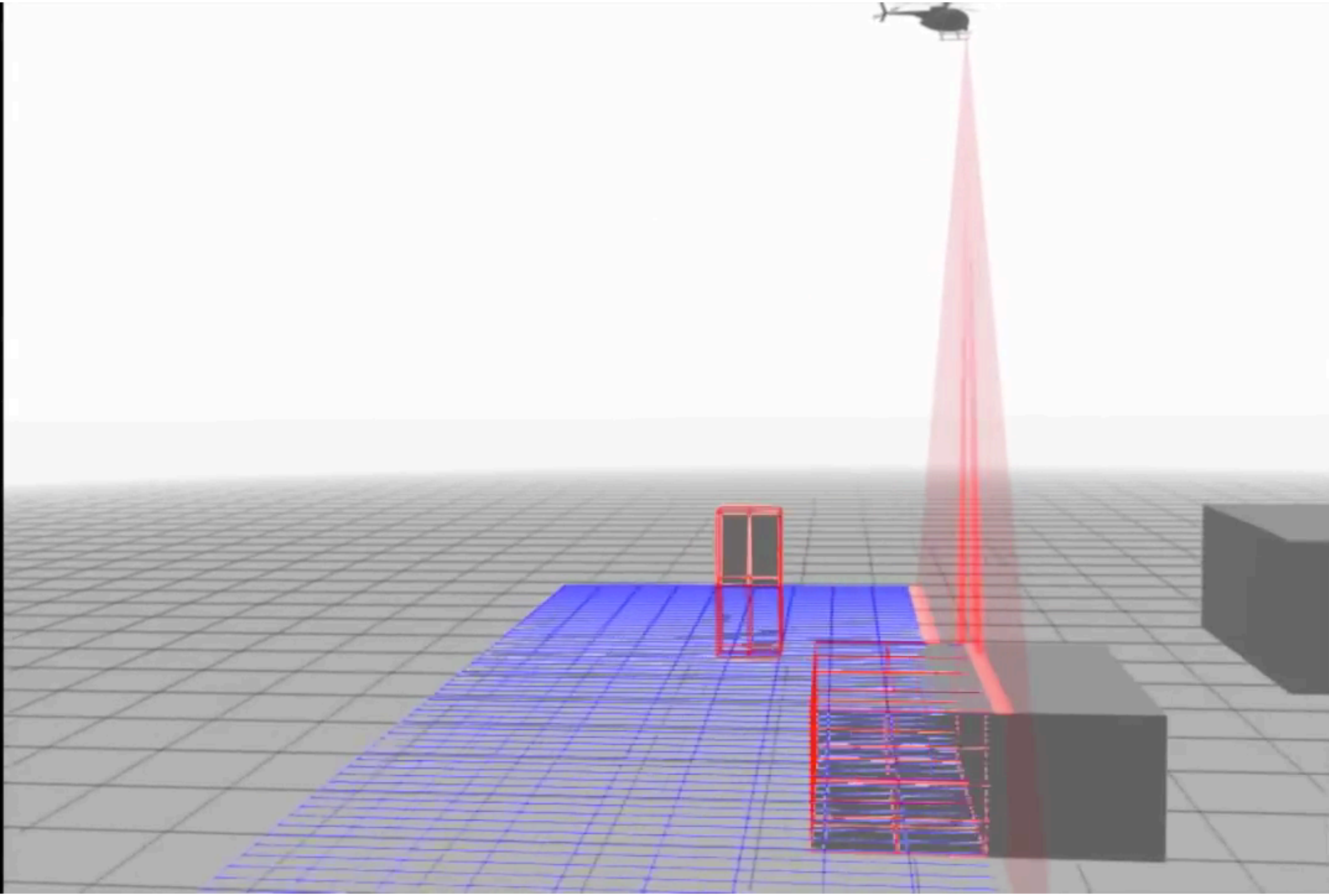

(courtesy Lyle Chamberlain)

Three questions you should ask

1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

Why is the measurement model probabilistic?

Several sources of stochasticity

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Category	**Example**

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Incomplete / Incorrect map	Pedestrians, objects moving around

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Incomplete / Incorrect map	Pedestrians, objects moving around
Unmodelled physics	Lasers goes through glass
Sensing assumptions	Multiple laser returns

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Solution: Anticipate specific types of failures and add stochasticity accordingly.

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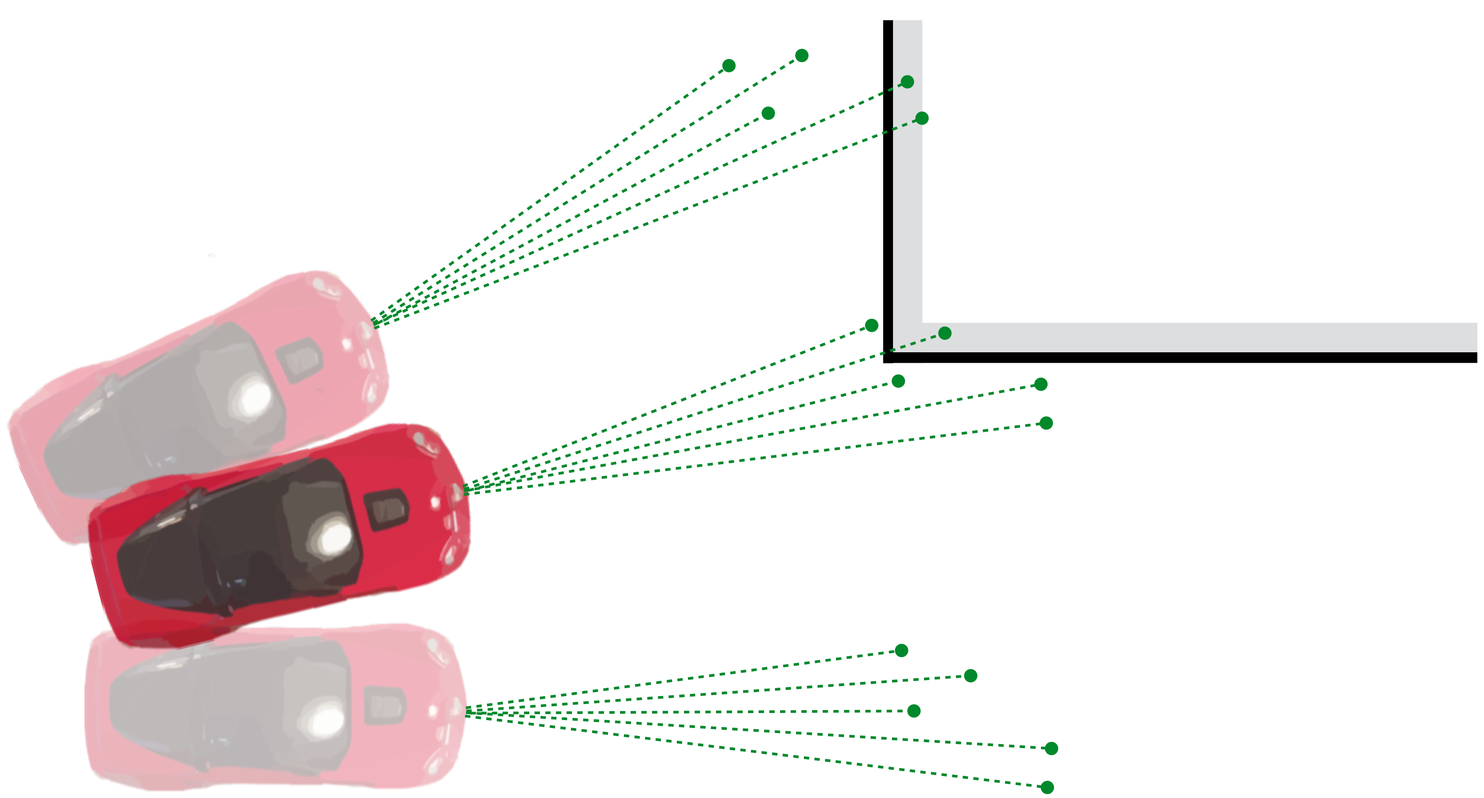
Measurement model for LiDAR

$$P(z_t | x_t, m)$$

laser scan　　state　　map



Measurement model for LiDAR

Assume individual beams are conditionally independent given map

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laser scan

individual beams

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Assume individual beams are conditionally independent given map

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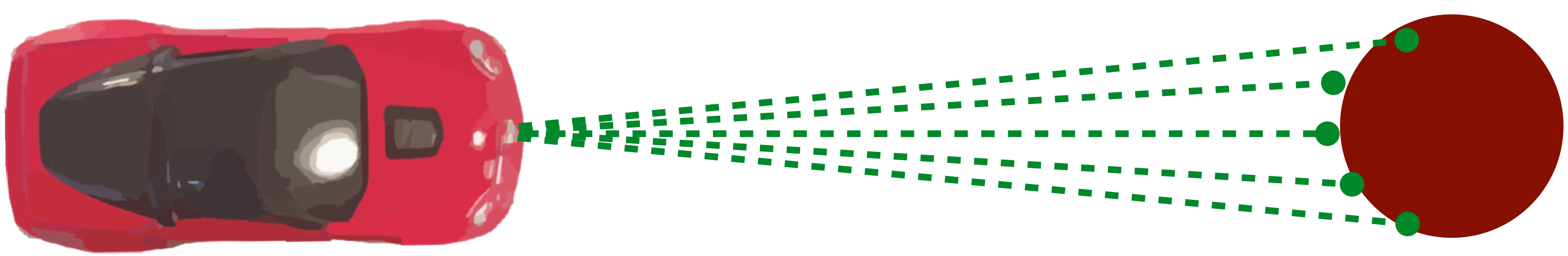
laser scan — individual beams

When is this assumption invalid?



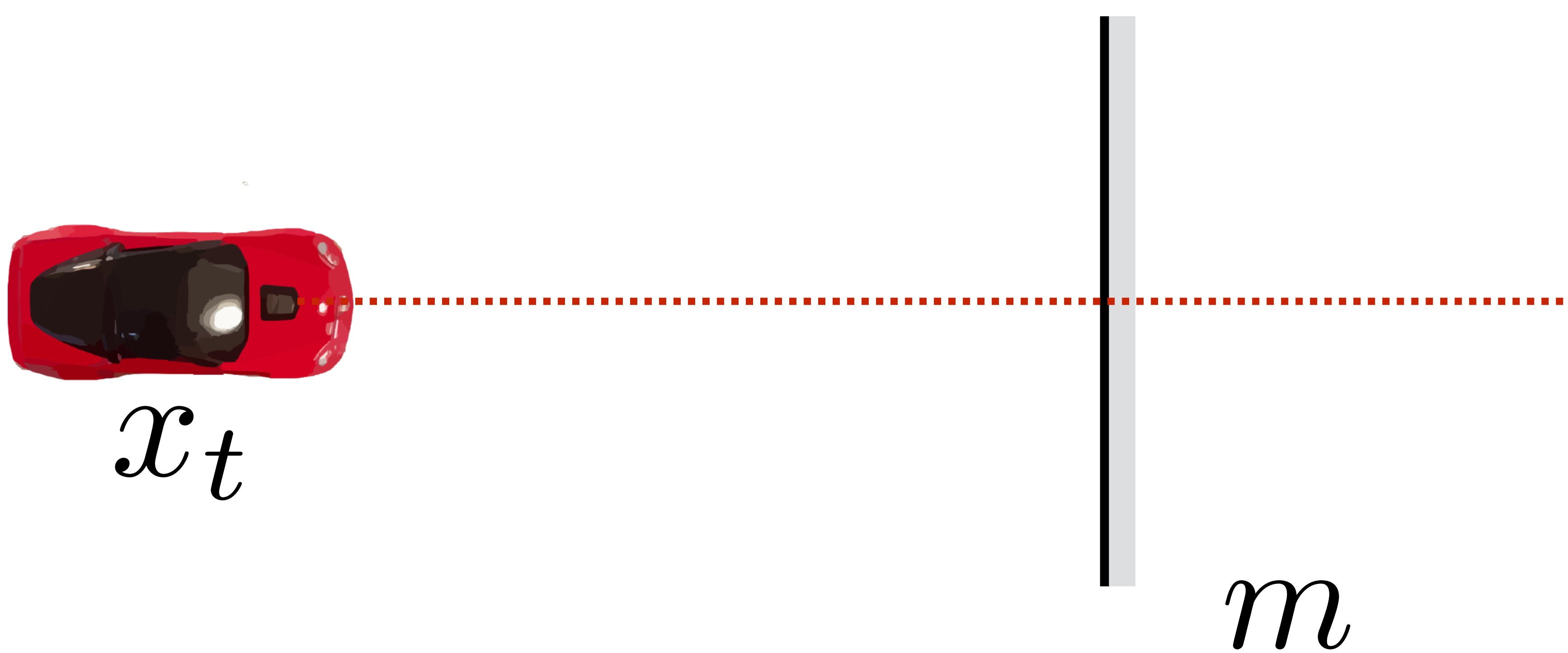
Obstacle not in map

All beams are correlated!

Measurement model for single beam

$$P(z_t^k \mid x_t, m)$$

distance value | state | map



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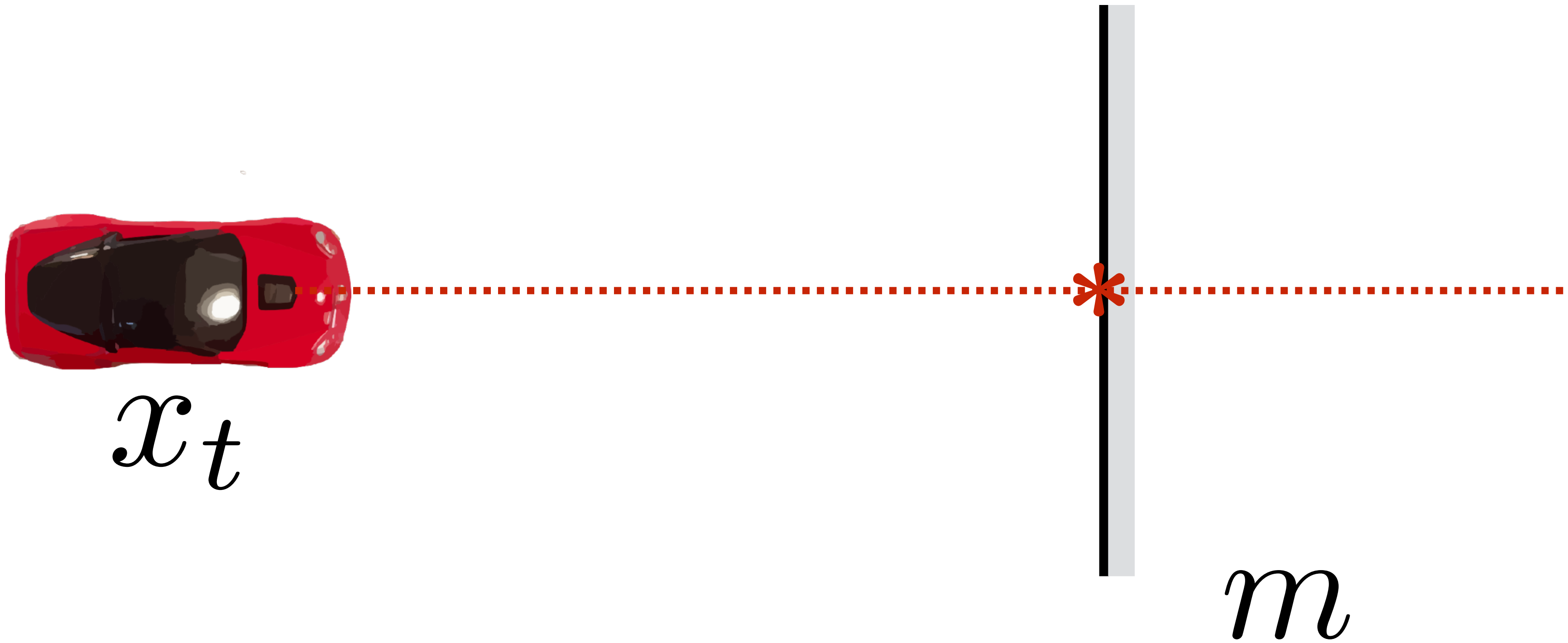

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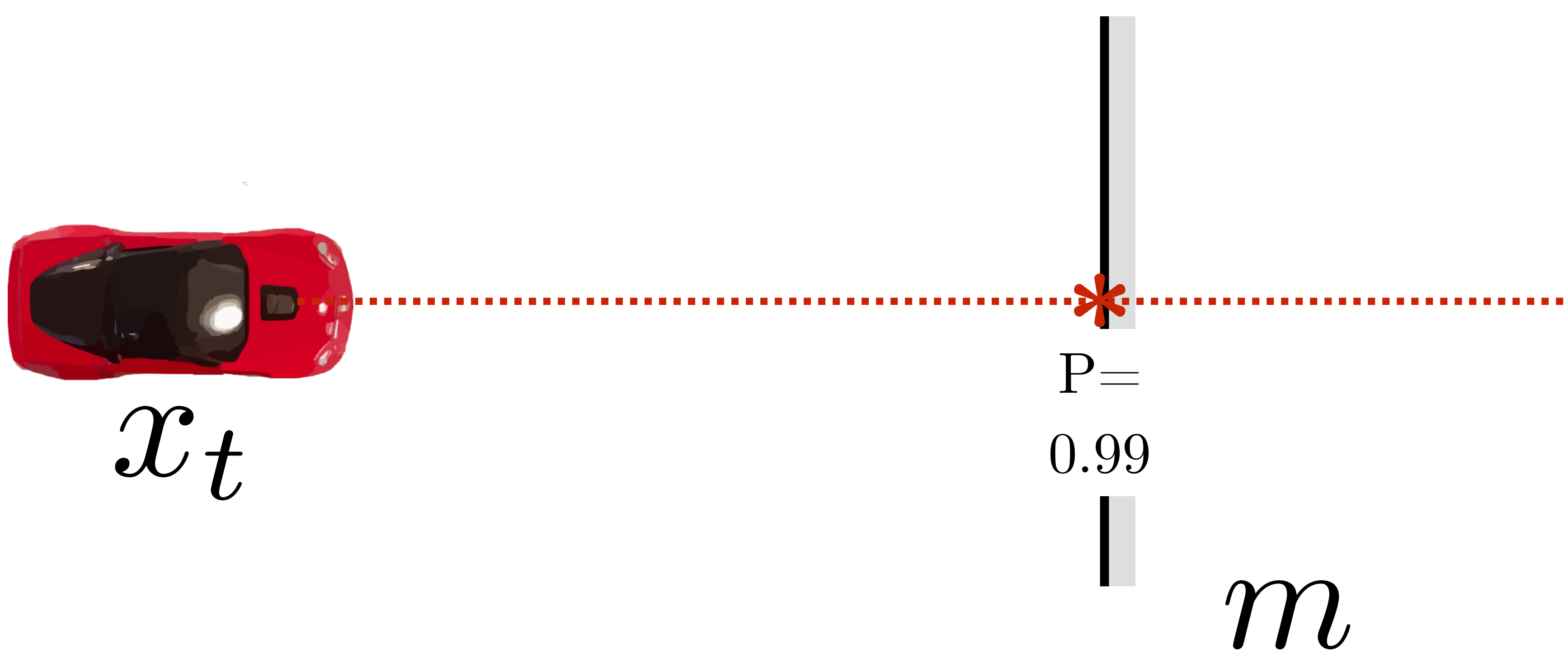

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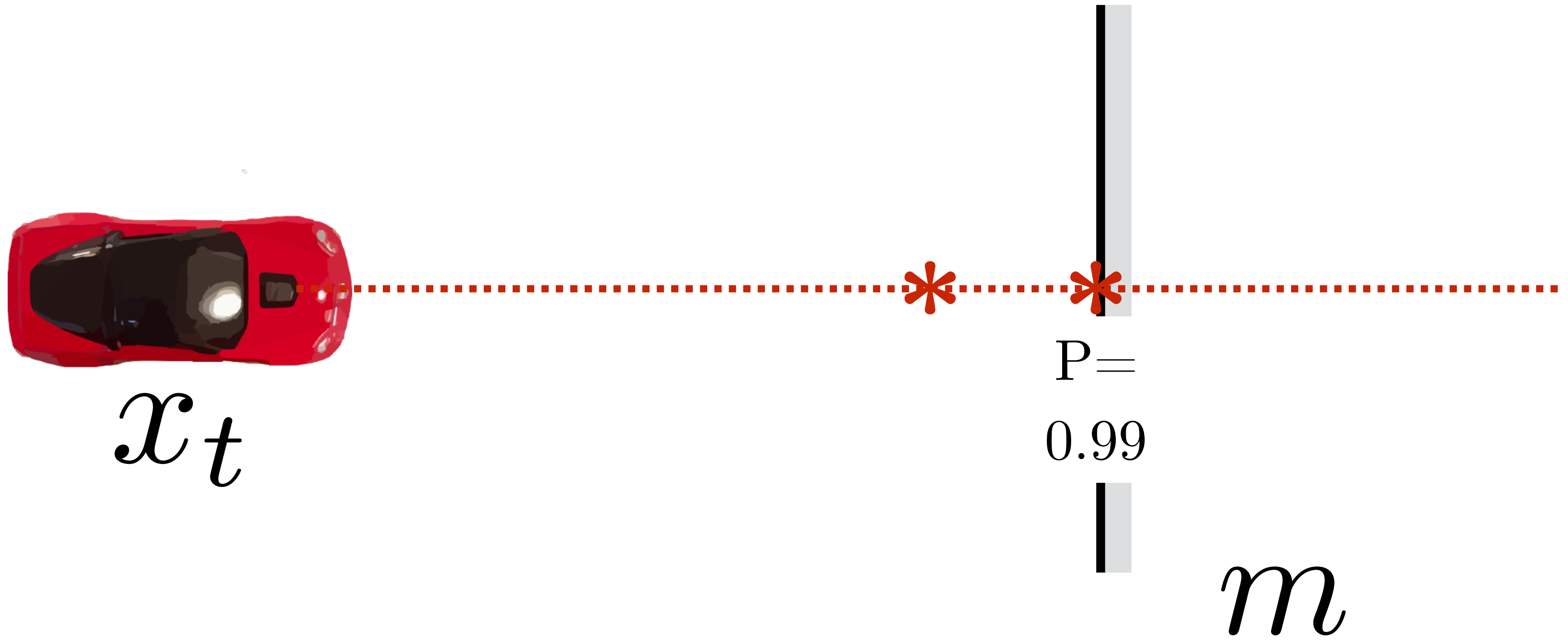

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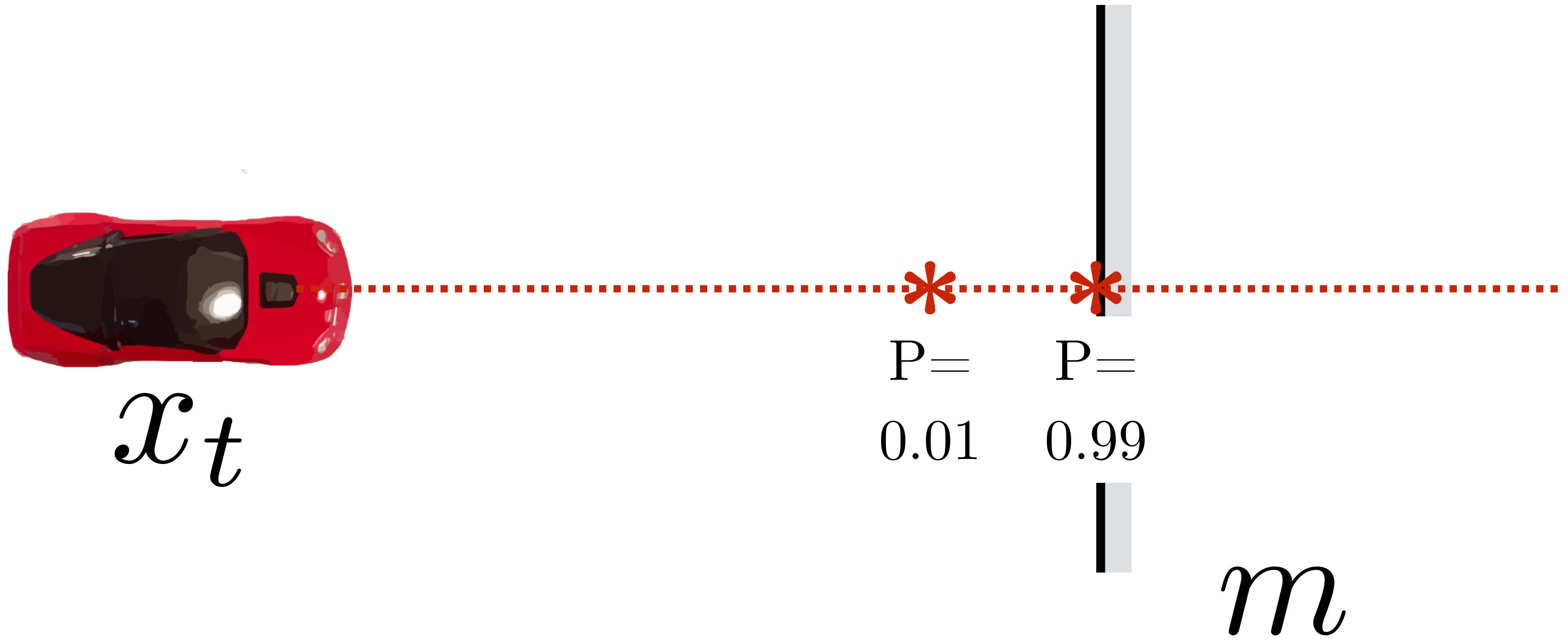

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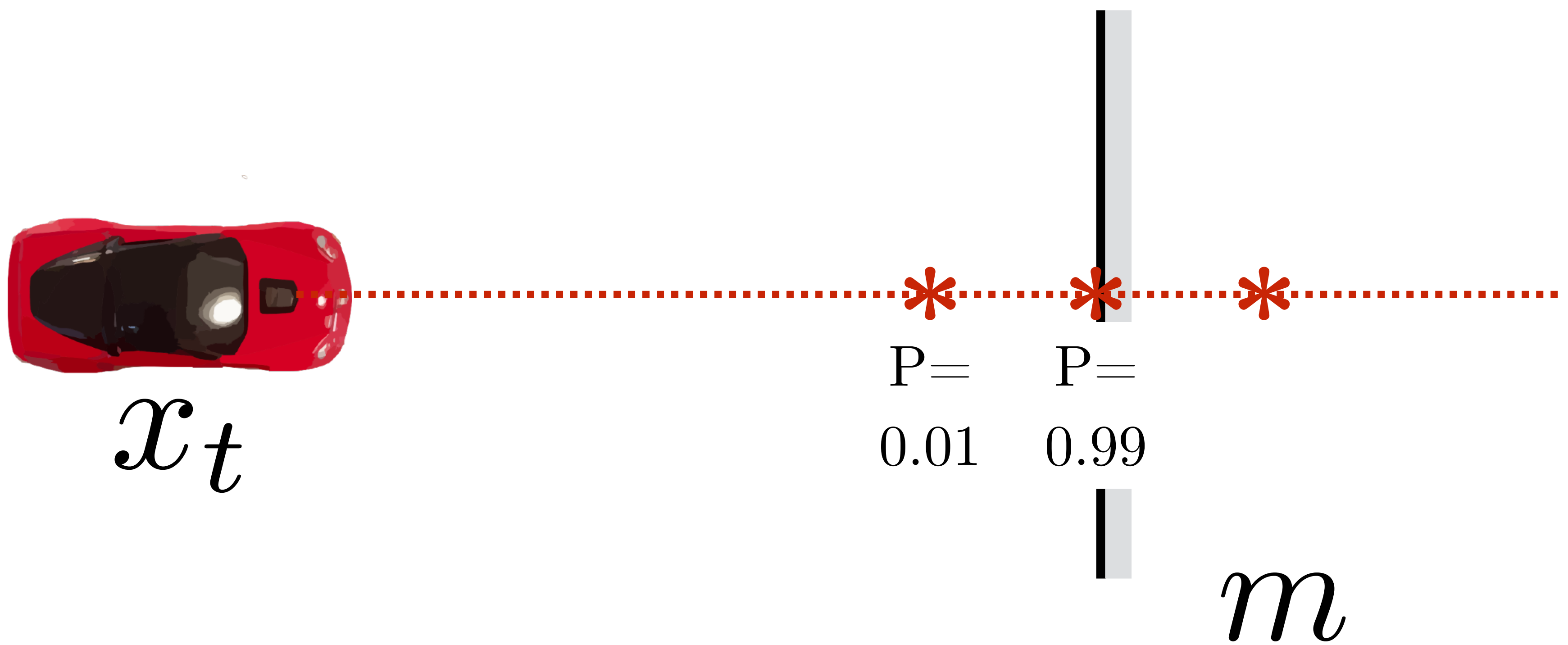

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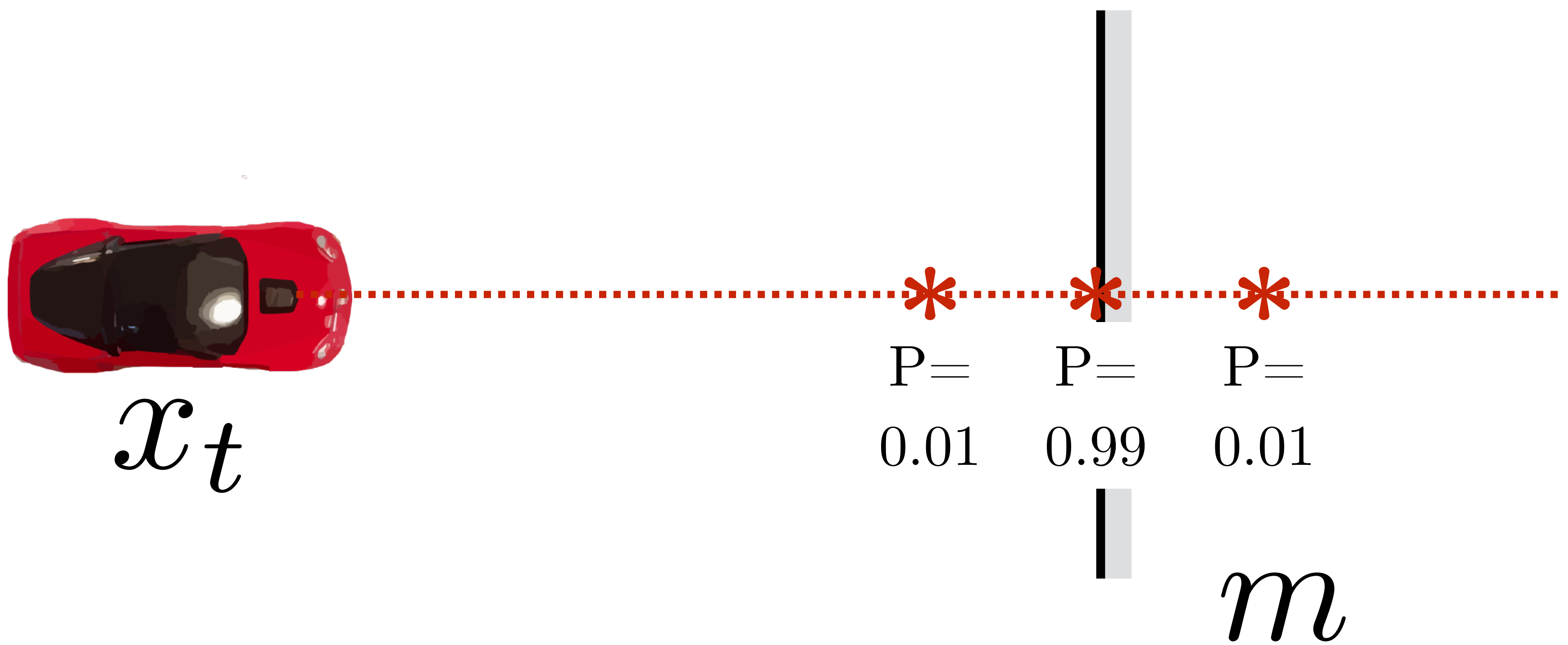

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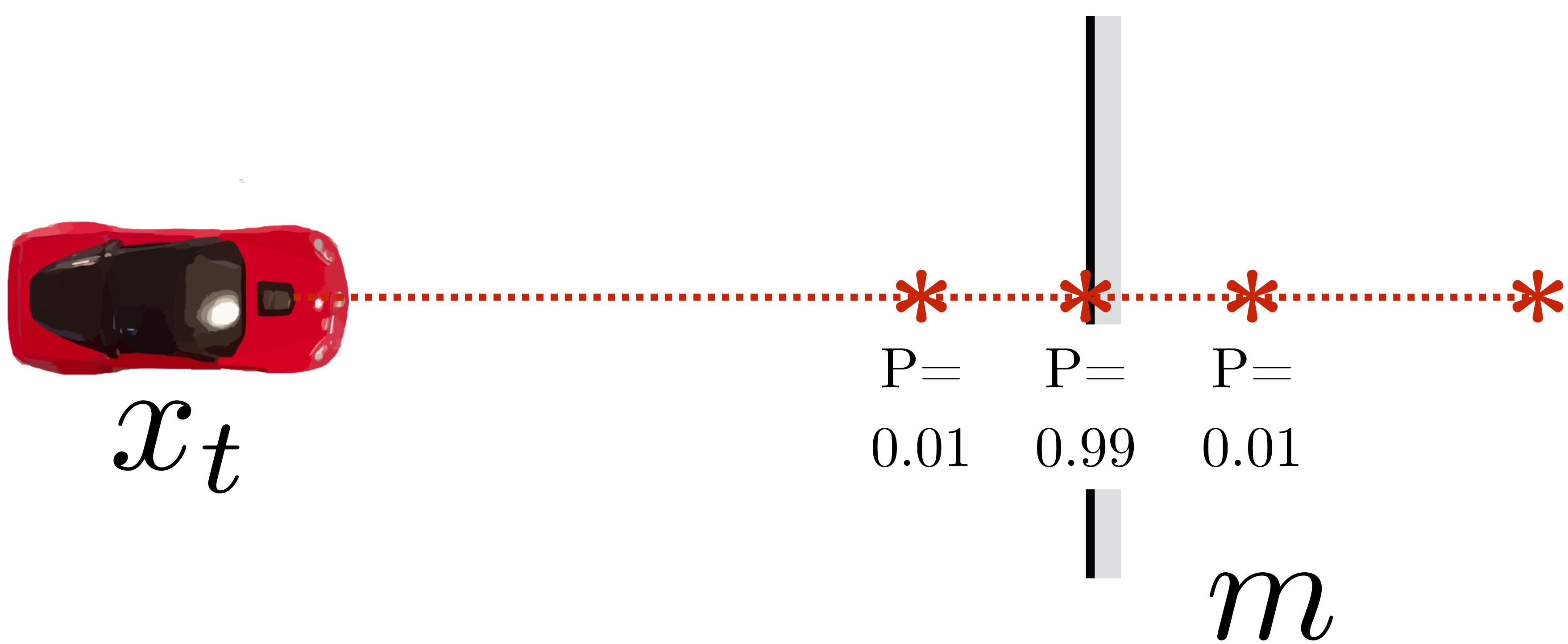

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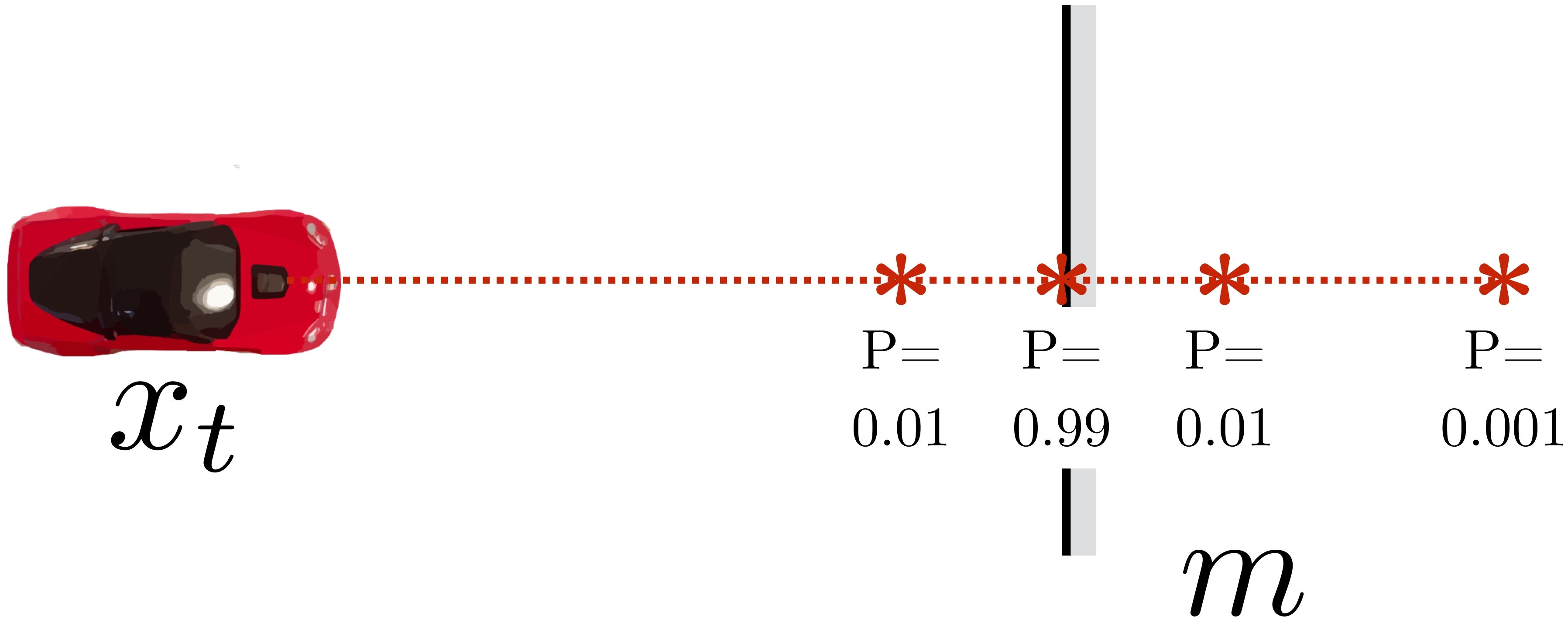

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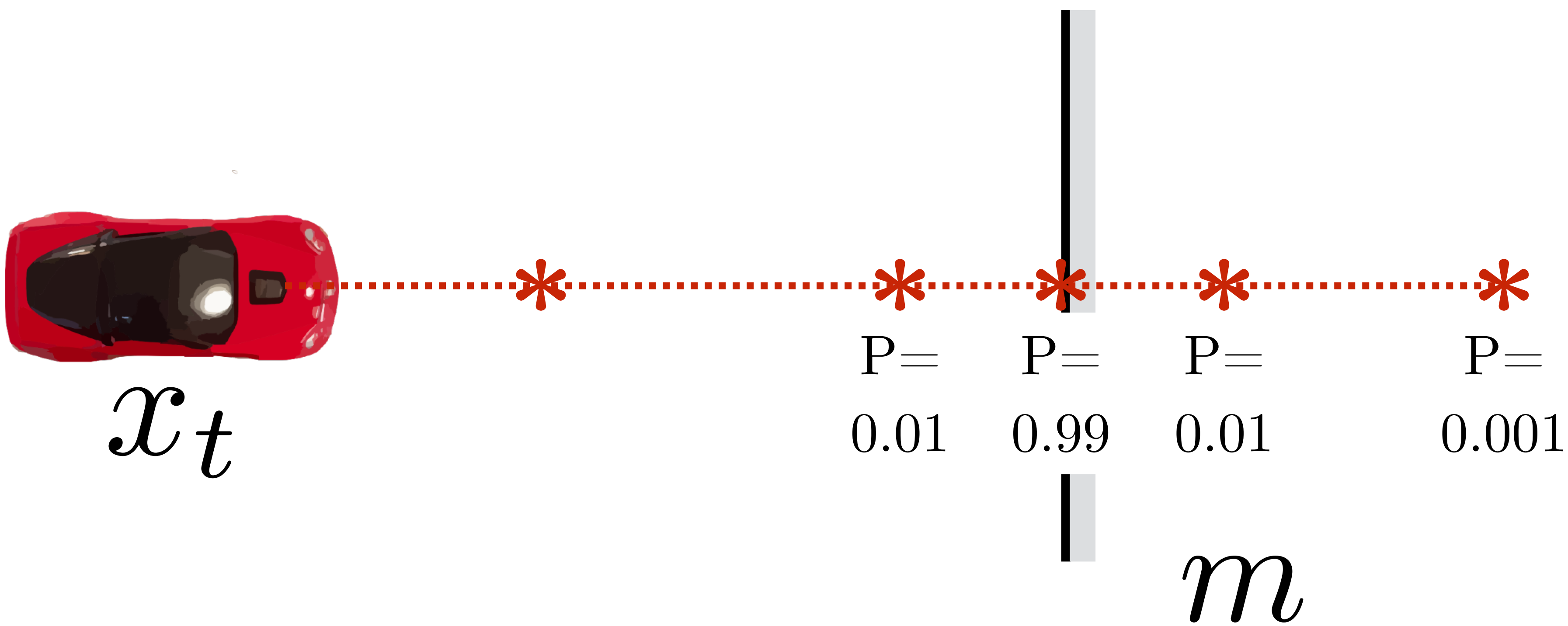

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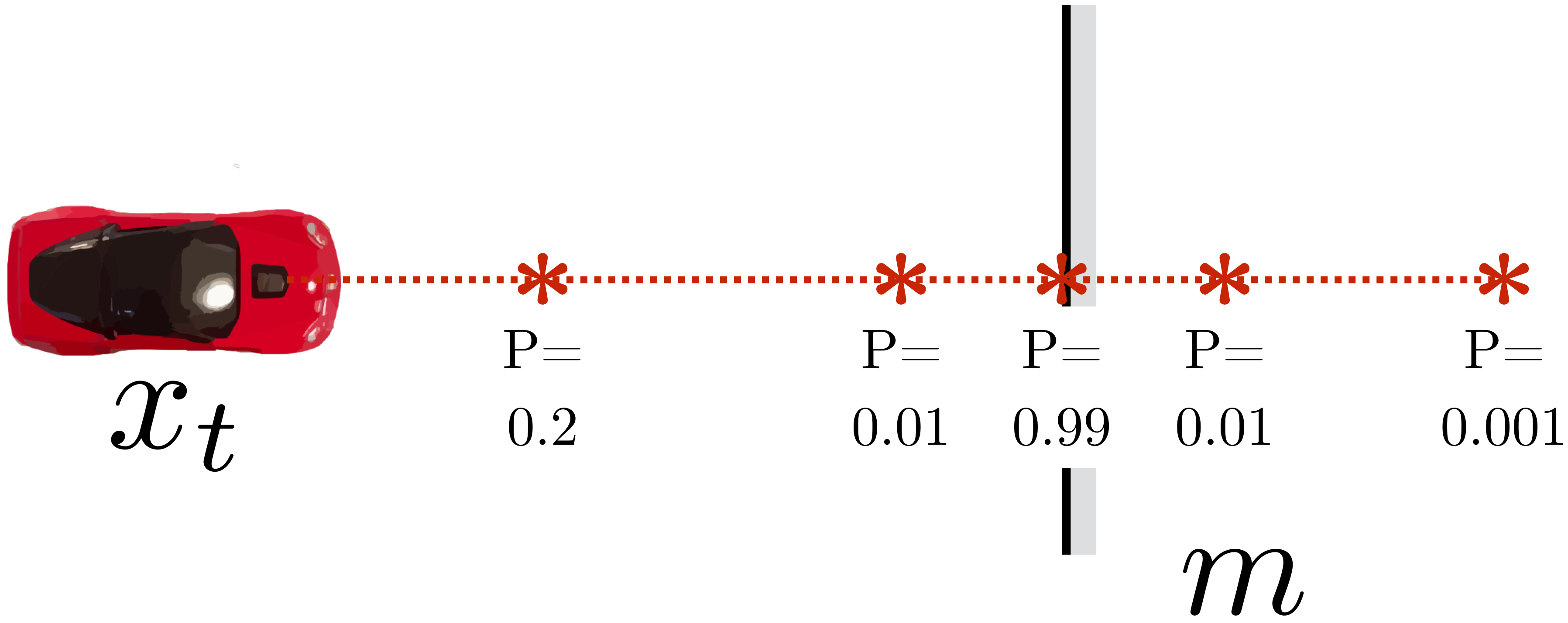

Pseudo-algorithm for sensor model

Input: State of the robot x, Map m, True laser scan z

Output: Probability p

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5. Multiply all probabilities to get p

What kind of stochasticity should we consider?

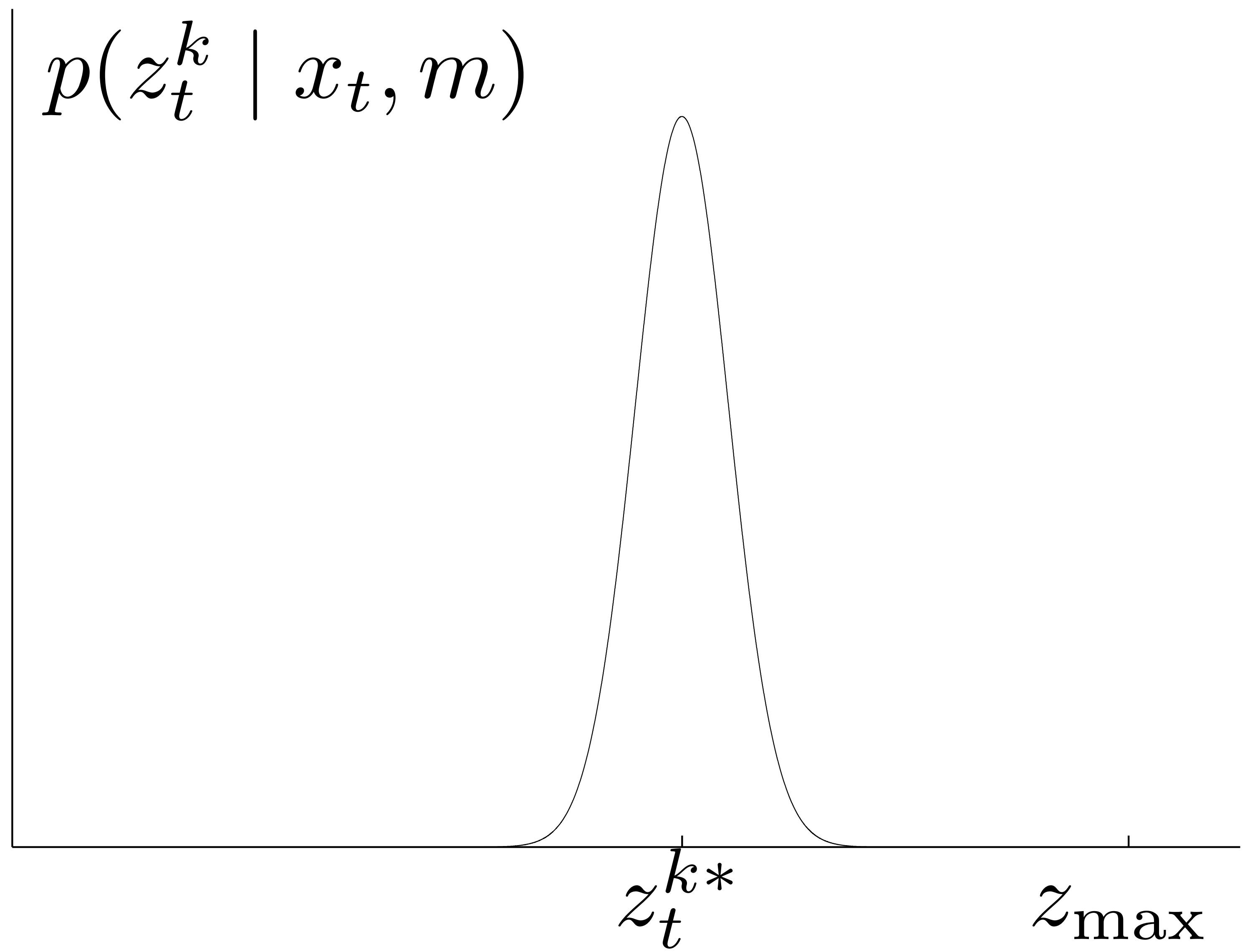
1. Simple measurement noise in distance value

2. Presence of unexpected objects

3. Laser returns max range when no objects

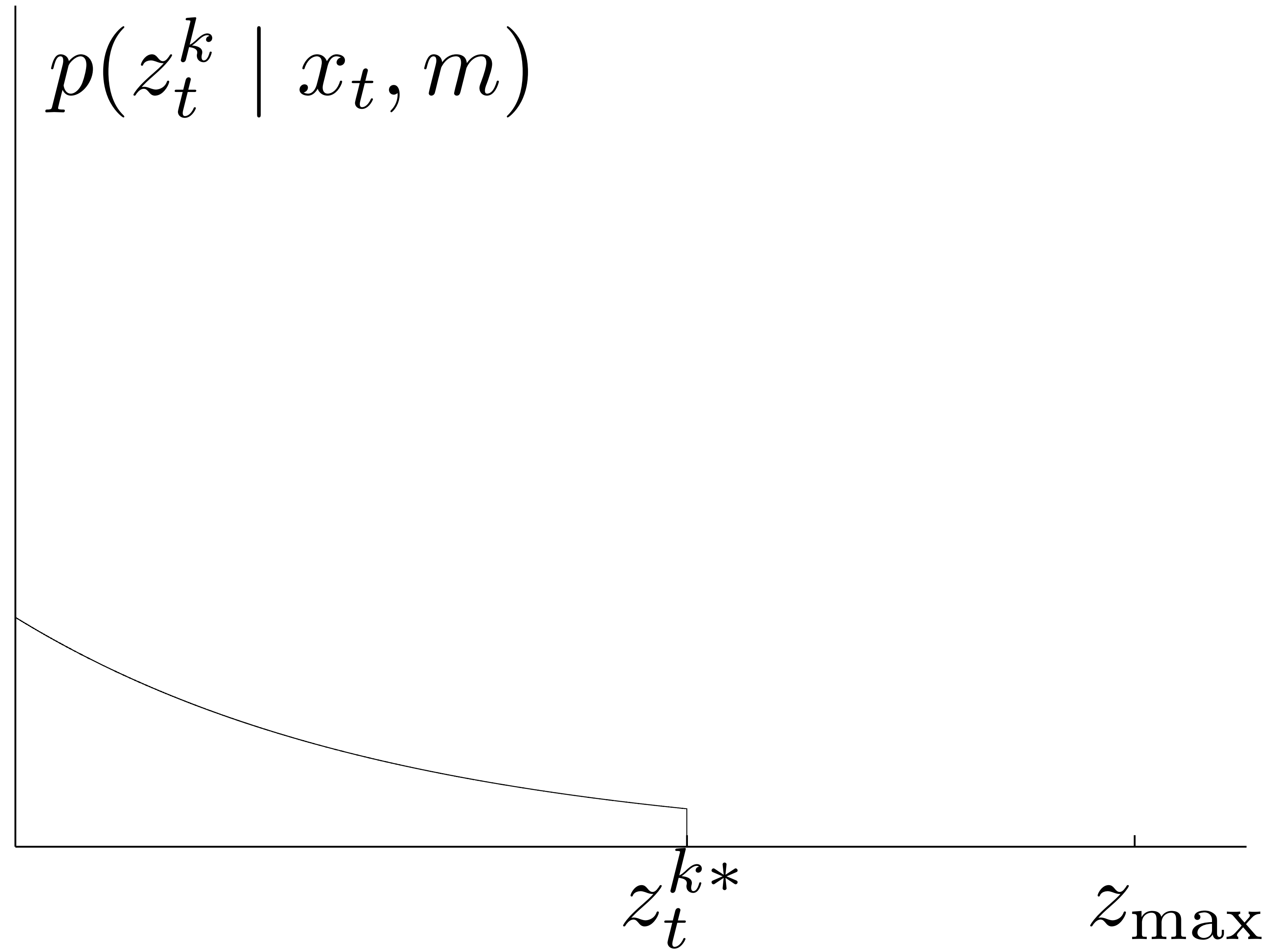
4. Failures in sensing

Factor 1: Simple measurement noise



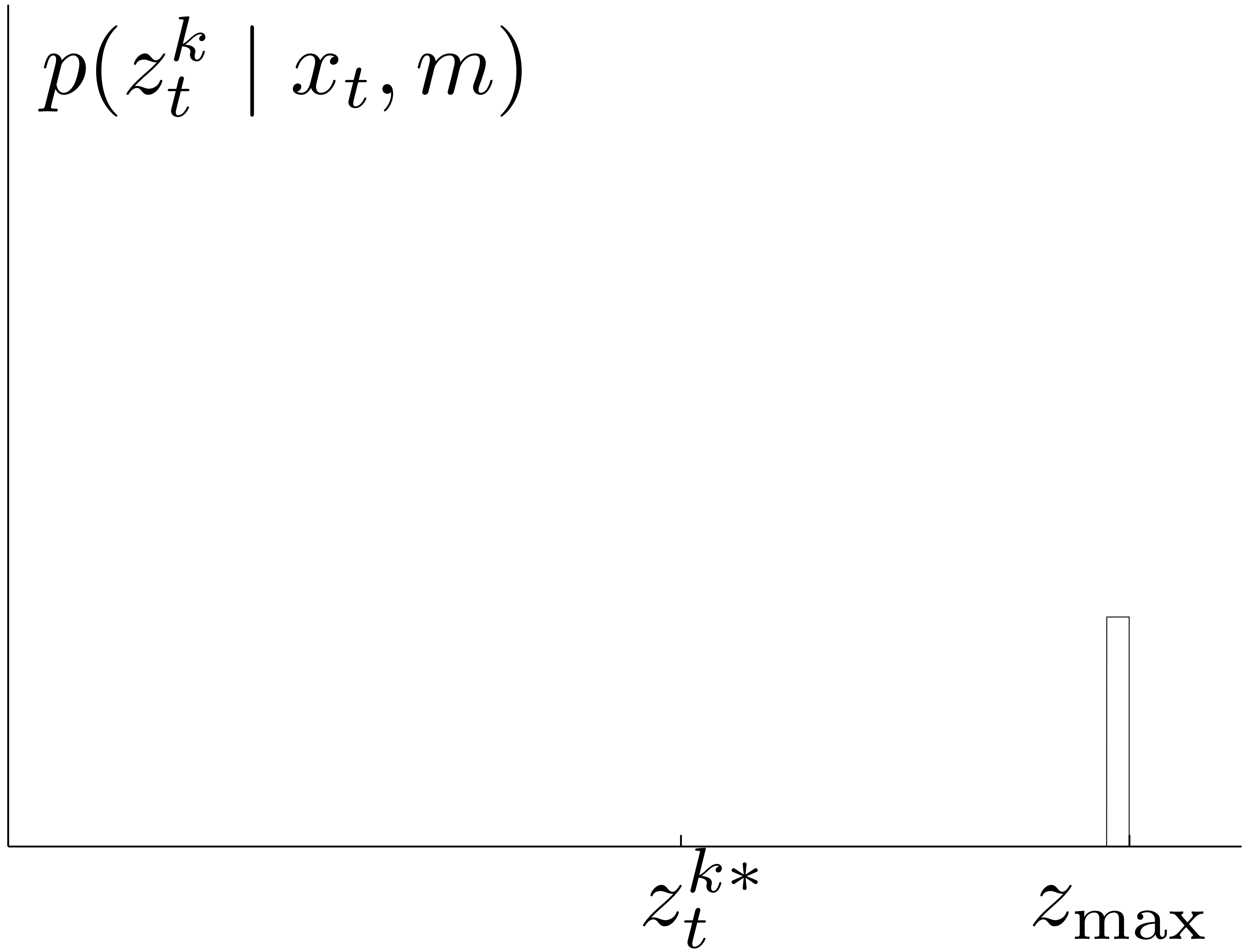
$$p_{\text{hit}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) & \text{if } 0 \leq z_t^k \leq z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Factor 2: Unexpected objects



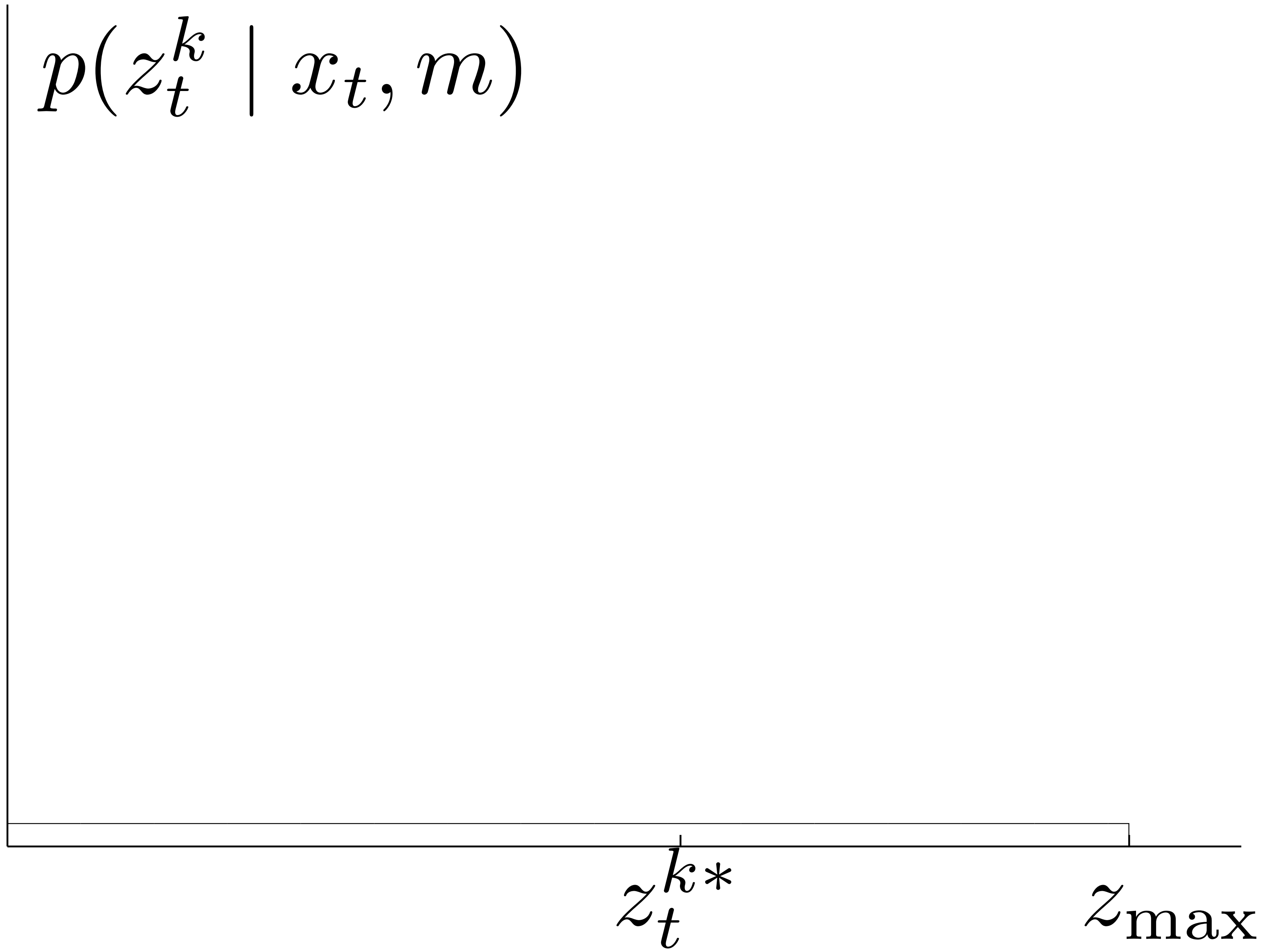
$$p_{\text{short}}(z_t^k \mid x_t, m) \quad = \quad \begin{cases} \eta \, \lambda_{\text{short}} \, e^{-\lambda_{\text{short}} z_t^k} & \text{if } 0 \leq z_t^k \leq z_t^{k*} \\ 0 & \text{otherwise} \end{cases}$$

Factor 3: Maximum range



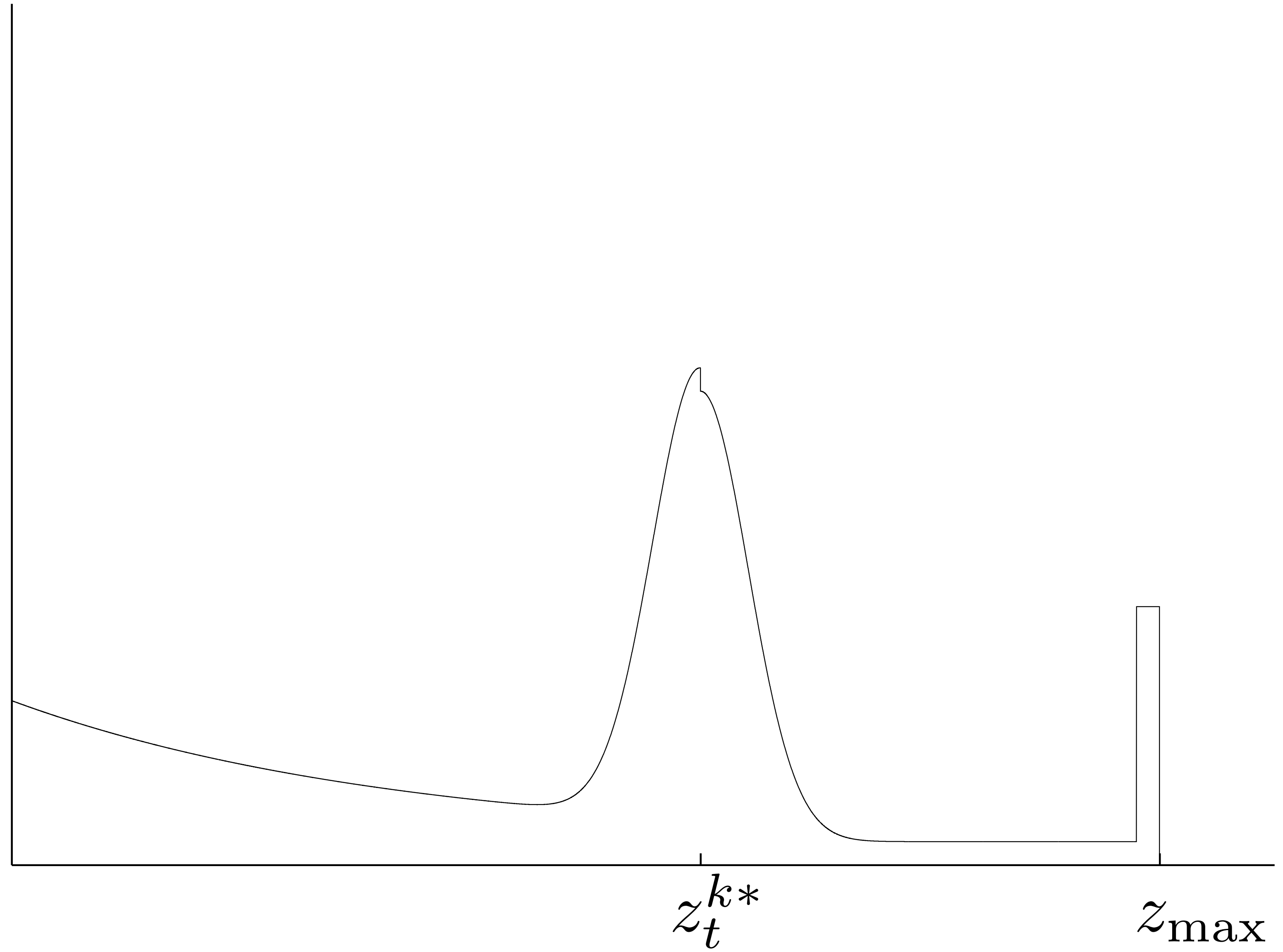
$$p_{\max}(z_t^k \mid x_t, m) \quad = \quad I(z = z_{\max}) \quad = \quad \begin{cases} 1 & \text{if } z = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Factor 4: Failures in sensing



$$p_{\text{rand}}(z_t^k \mid x_t, m) \quad = \quad \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \leq z_t^k < z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

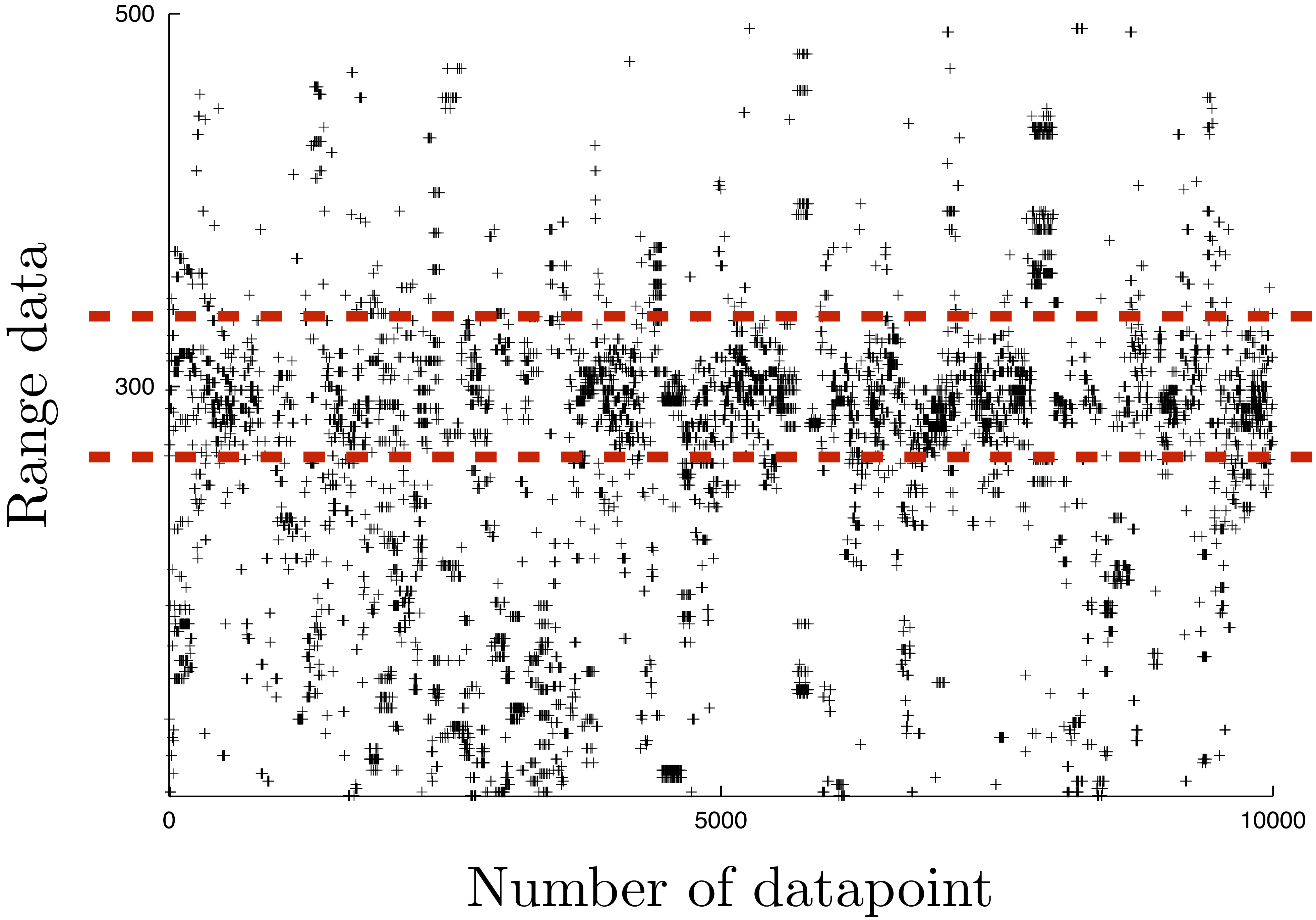
Combined probabilistic model



$$p(z_t^k \mid x_t, m) \quad = \quad \begin{pmatrix} z_{\text{hit}} \\ z_{\text{short}} \\ z_{\text{max}} \\ z_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z_t^k \mid x_t, m) \\ p_{\text{short}}(z_t^k \mid x_t, m) \\ p_{\text{max}}(z_t^k \mid x_t, m) \\ p_{\text{rand}}(z_t^k \mid x_t, m) \end{pmatrix}$$

Question: How do we tune parameters?

In theory: Collect lots of data and optimize parameters to maximize data likelihood

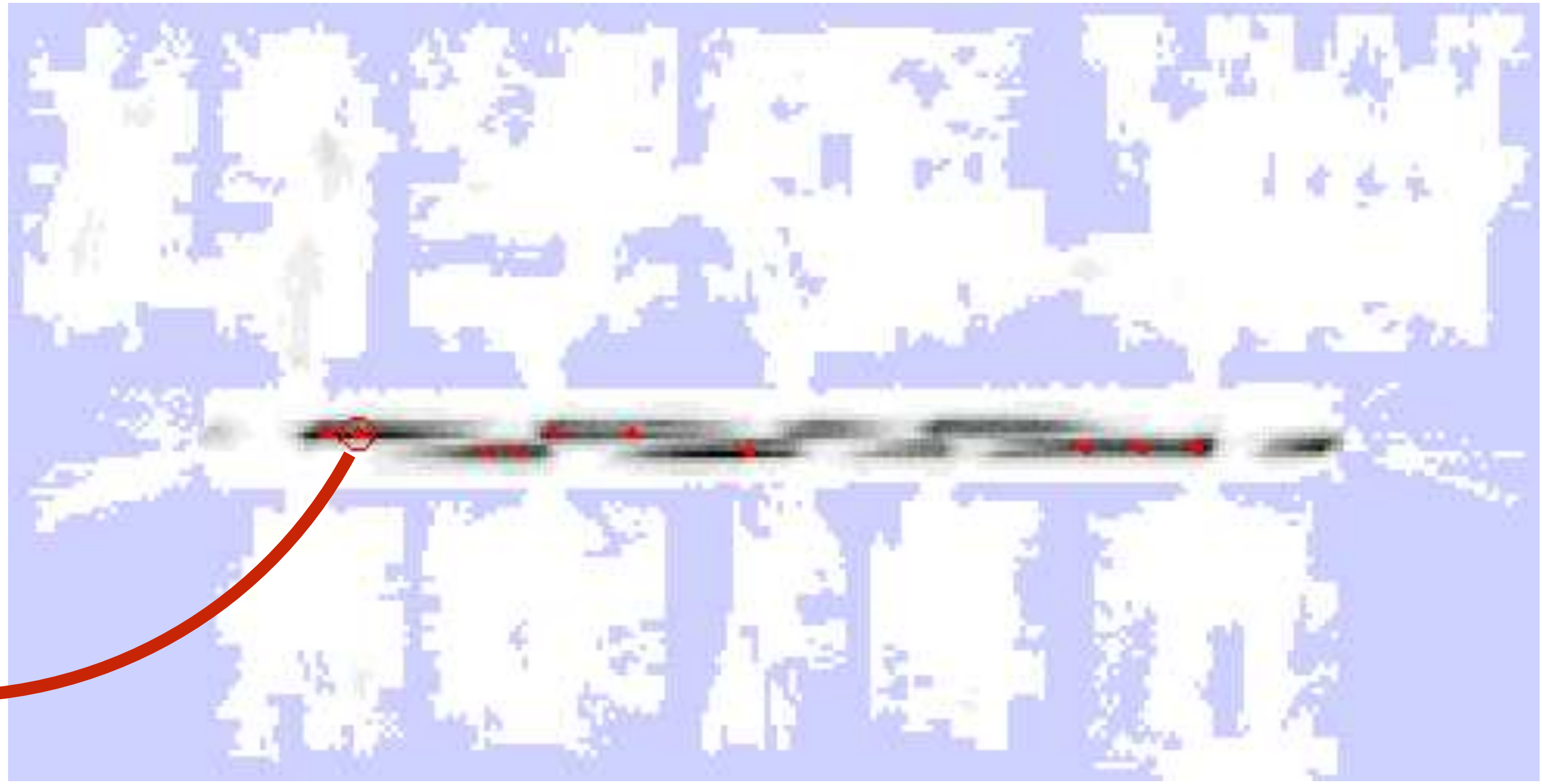


Example: Place a robot 300 cm from a wall and collect lots of data

Question: How do we tune parameters?

In practice: Simulate a scan and plot the likelihood from different positions

Actual scan



Likelihood at various locations

Problem: Overconfidence

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Independence assumption may result in repetition of mistakes

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Solution

1. Subsample laser scans: Convert 180 beams to 18 beams

2. "Smooth" out the probability model

$$P(z_t^k|x_t, m) \longrightarrow P(z_t^k|x_t, m)^{\frac{1}{N}}$$