

# Map Representations

Sanjiban Choudhury

# Announcements

Deadline for lab1 extended to **Wednesday 4/30 at 11:59 p.m**

This is the due date for the writeup

The lab evaluation is still on **Thursday 4/25**  
from **9.00 a.m - 12:00 p.m**

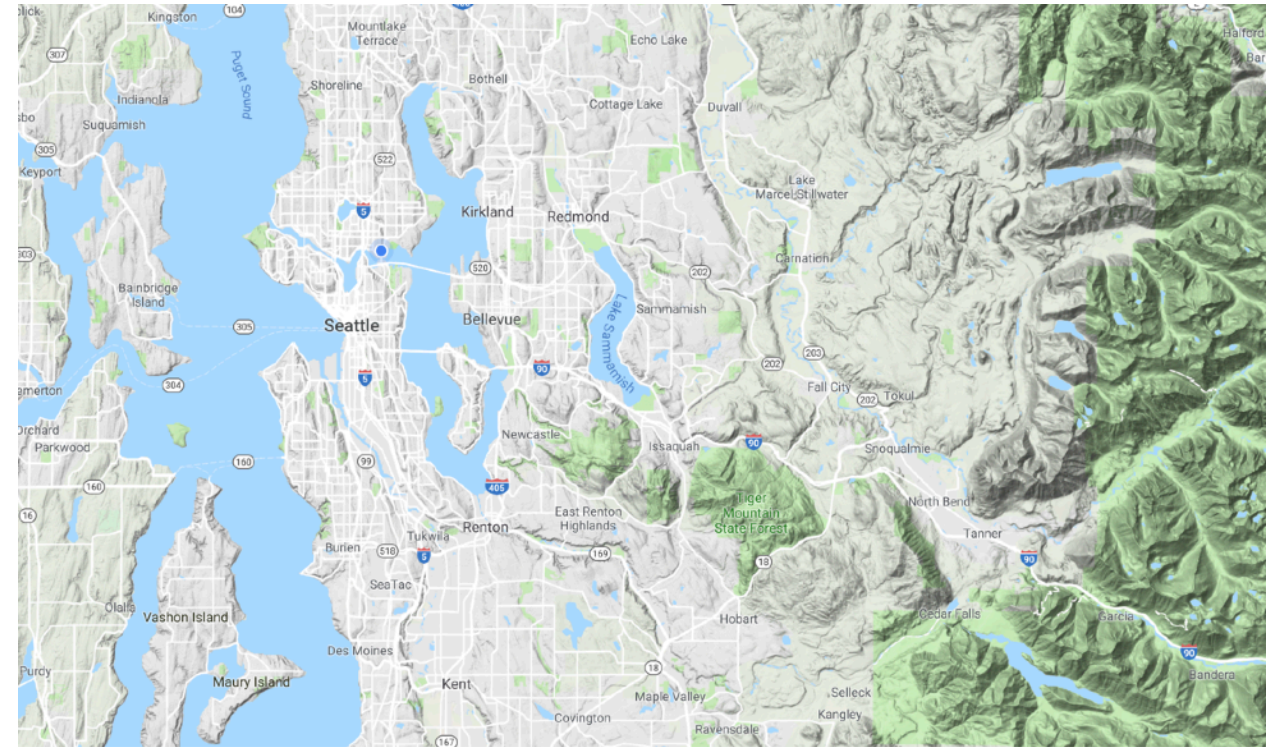
Please continue to update blogs by Friday of **each week**

What is a map?

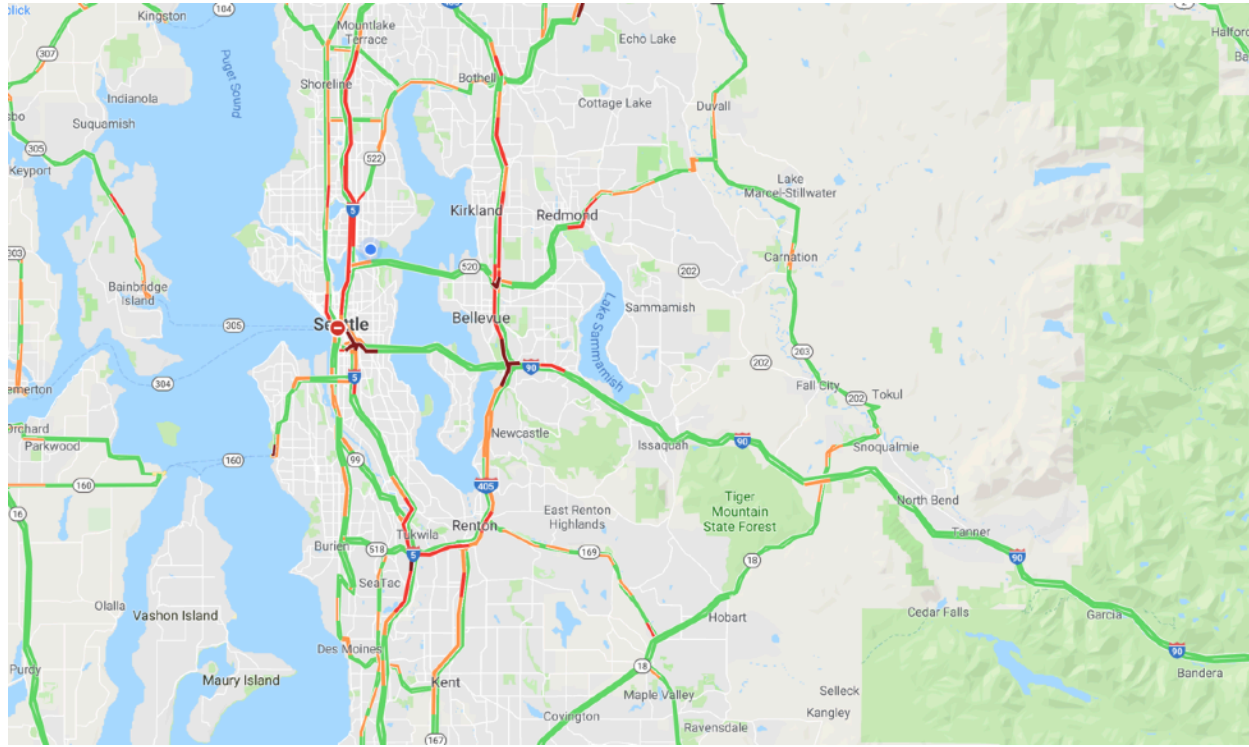
# Do all maps convey the same information?



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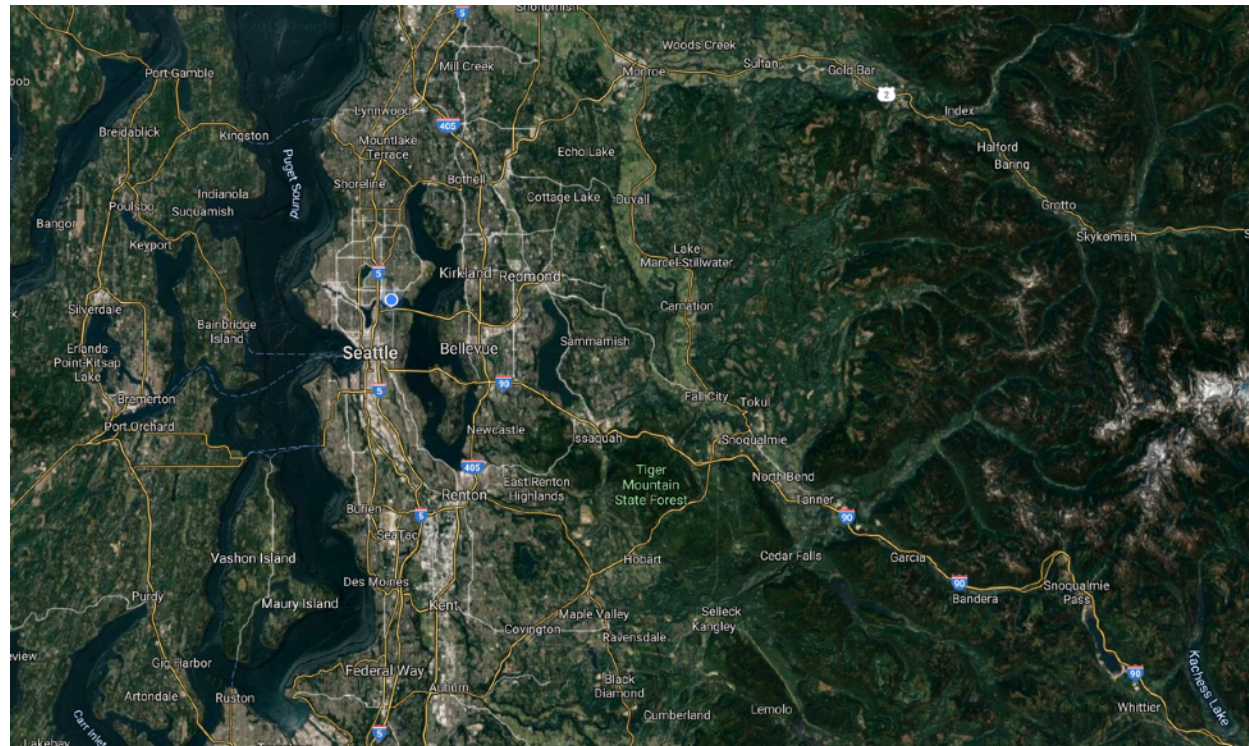
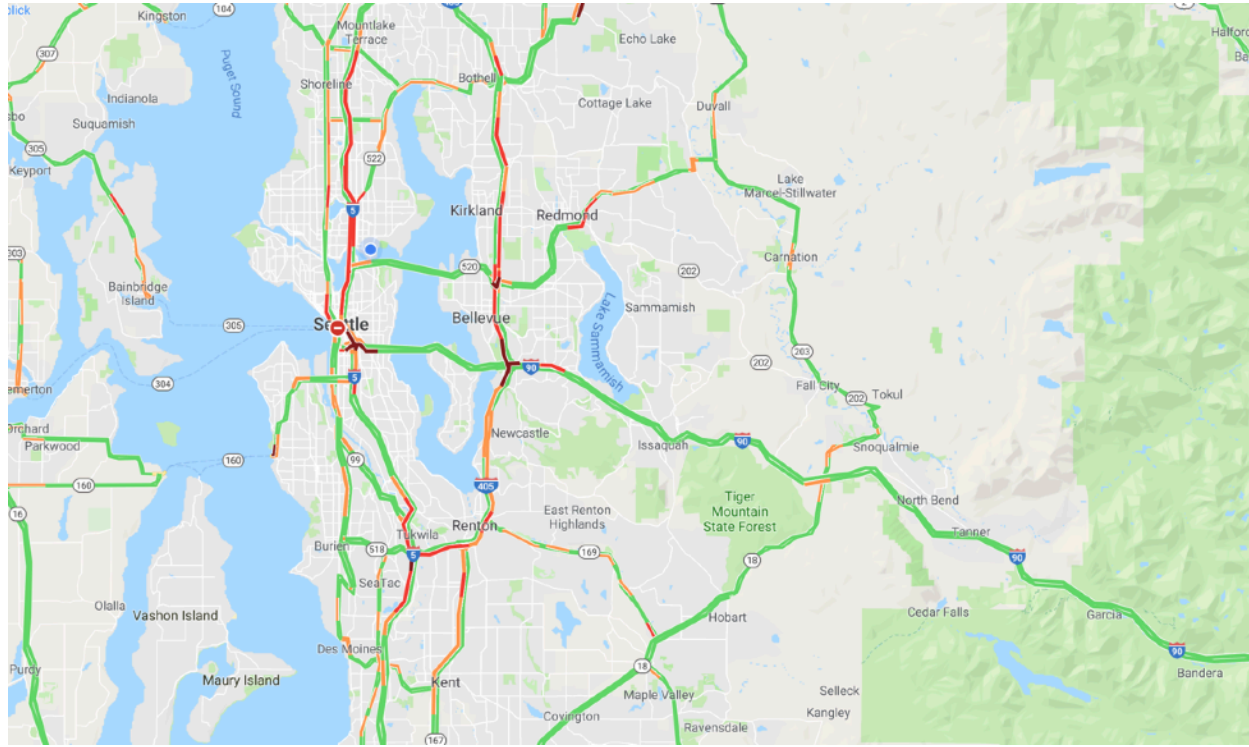


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Maps are a **summary of information** about the world

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What sort of information? Depends on the **task**

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What sort of information? Depends on the **task**

Task also determines how we  
query, update, store maps

# Today's objective

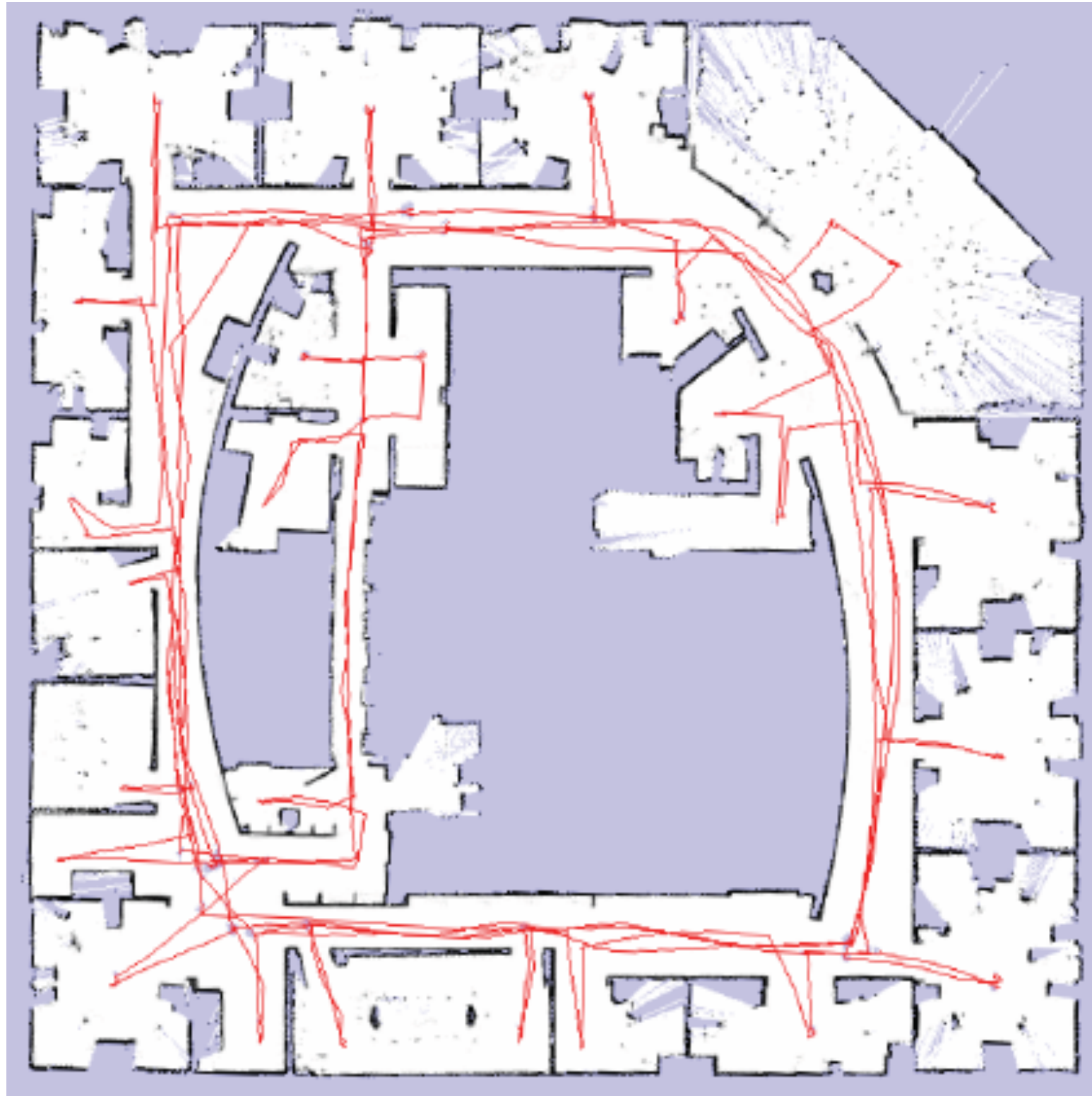
1. Framework / taxonomy to think about maps
2. Look at various maps and the underlying tasks they serve
3. Distance map



# What do we want from maps?

1. **Information** - What task does it help me solve?  
(Help me localize, help me navigate, help humans navigate / plan their lives etc)
2. **Query** - Can we query it online? How often?
3. **Updatable** - Can we update it online? Can it deal with noisy measurements?
4. **Memory** - How much storage does it need? Is it transportable? How does it scale with time? Scale with amount of stuff we see ?

# Example 1: Occupancy grids



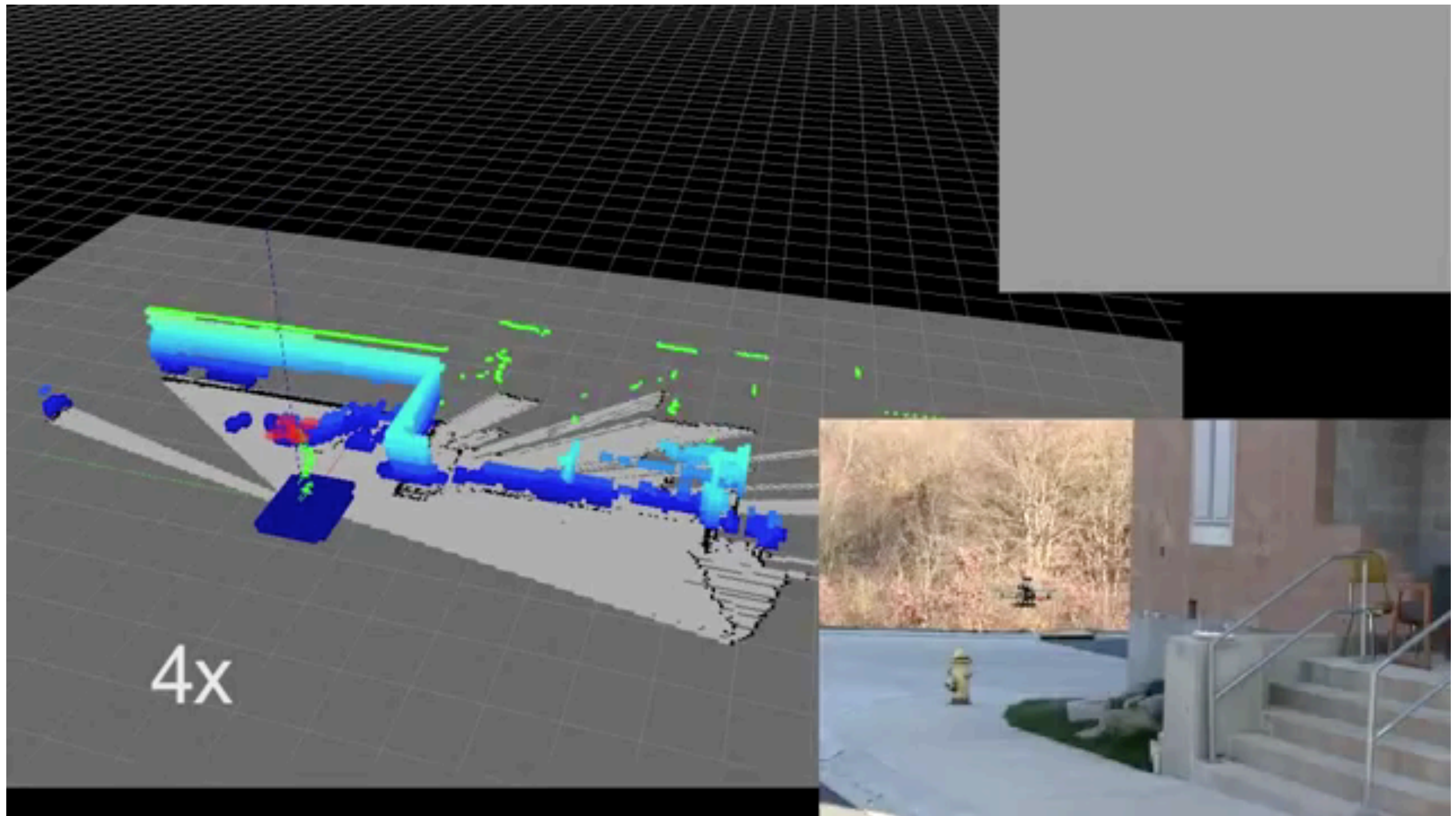
# Example 1: Occupancy grids

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Category	Details
Information	
Query	
Update	
Memory	

---

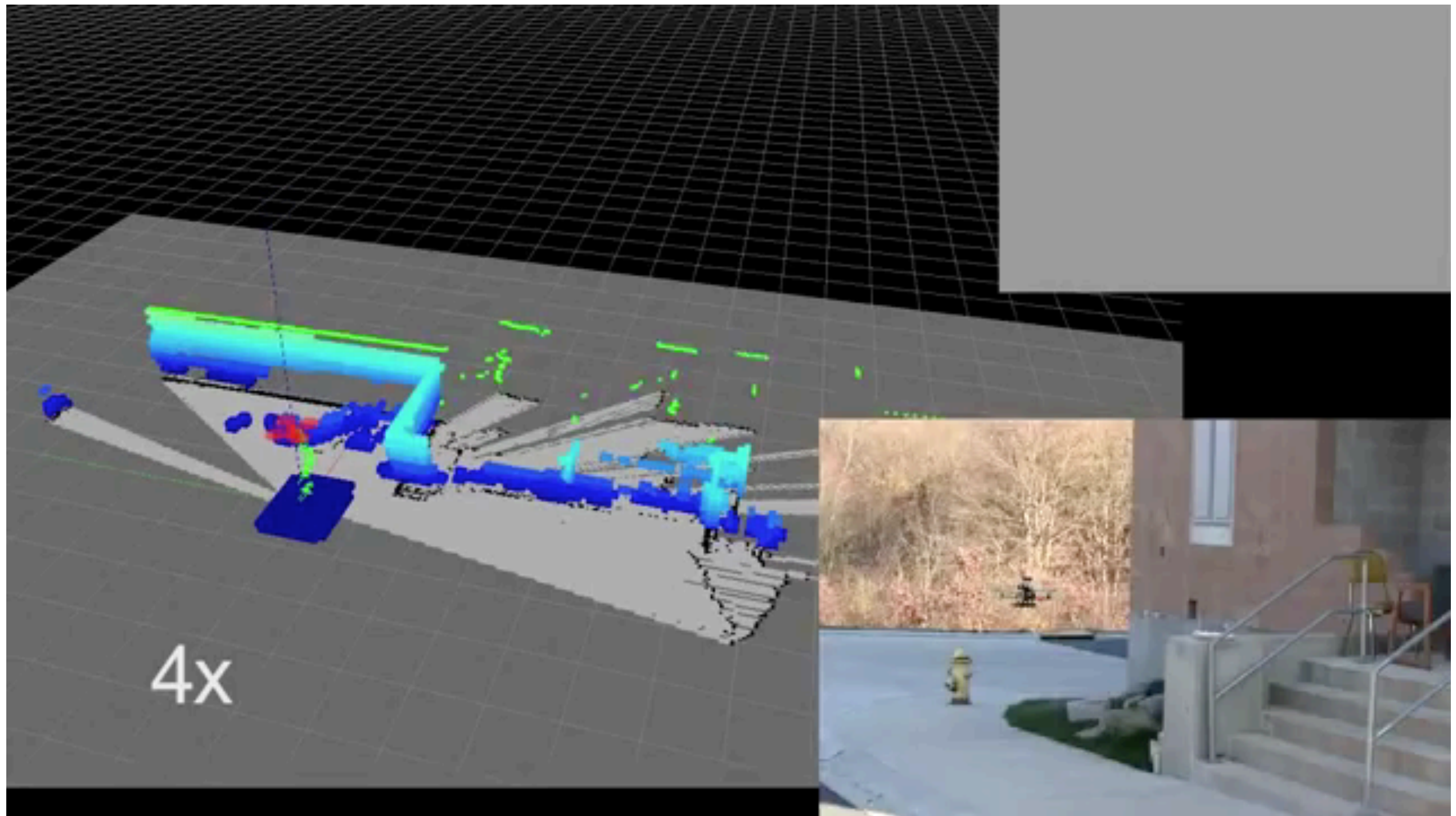
# Occupancy grids in action



“Autonomous Multi-Floor Indoor Navigation with a Computationally Constrained MAV”, S. Shen, N. Michael, V.Kumar, 2010



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Update	Can deal with <b>noisy sensors</b> (log likelihood update) Updates equal ray-casting ( $O(l)$ where $l$ is length of ray)
Memory	<b>Bounded</b> Can still be large if we want really fine resolution Need to allocate all the memory upfront

# Problems with occupancy grids

# Problems with occupancy grids

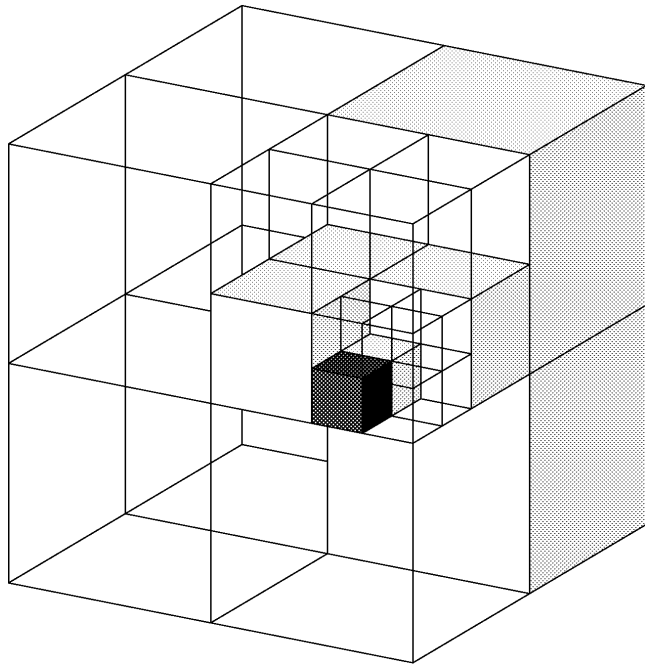
1. Memory scales with distance travelled in any one direction

# Problems with occupancy grids

1. Memory **scales with distance** travelled in any one direction
2. Do I need high resolution information **everywhere?**

# Example 2: Occupancy Trees (OctoMap)

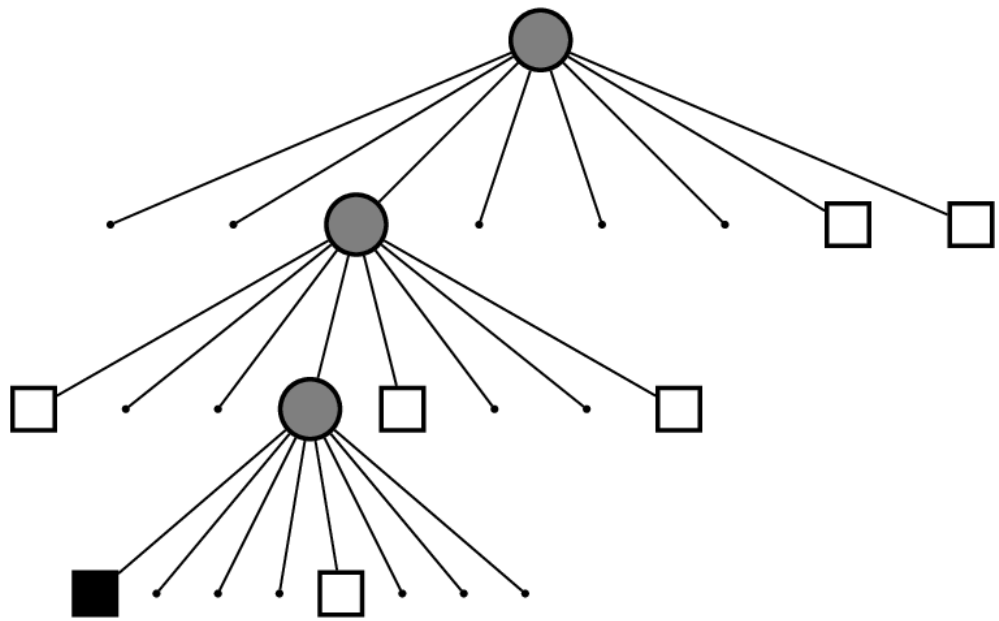
Hornung et al. 2013



Tree-based data structure

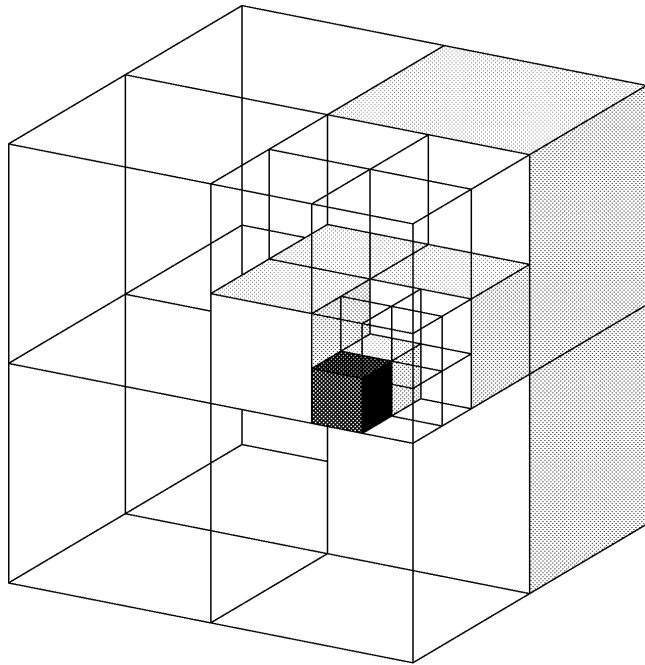
Recursive sub-division of space

Query maps at multiple-resolutions!



## Example 2: Occupancy Trees (OctoMap)

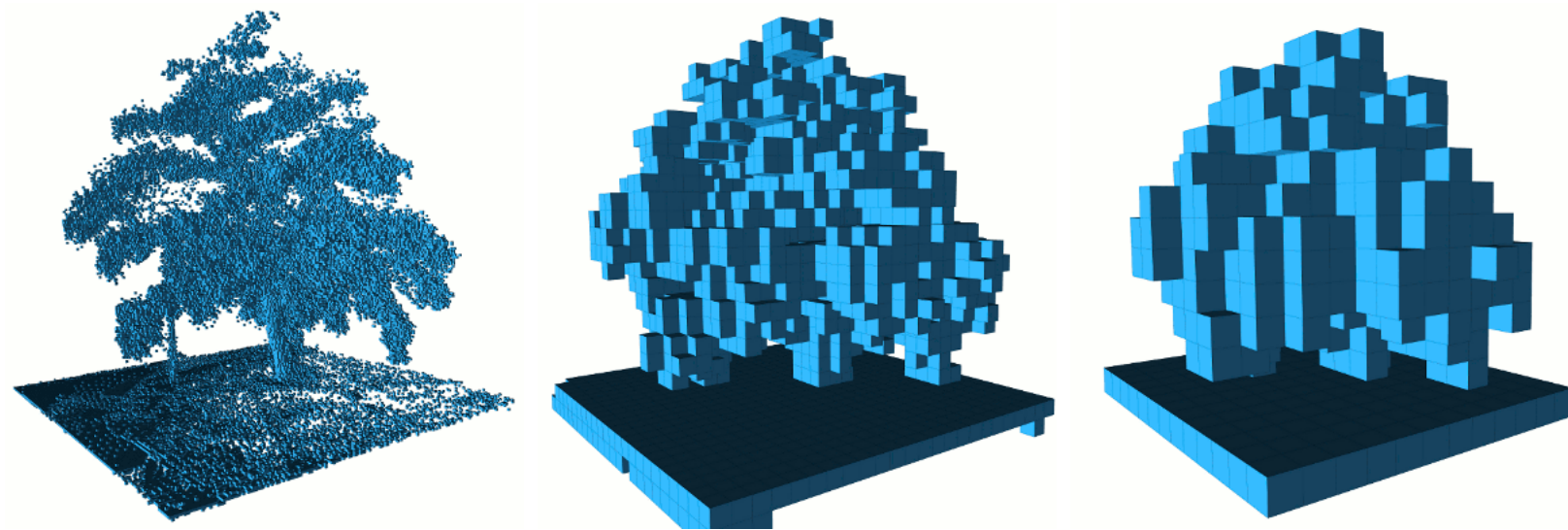
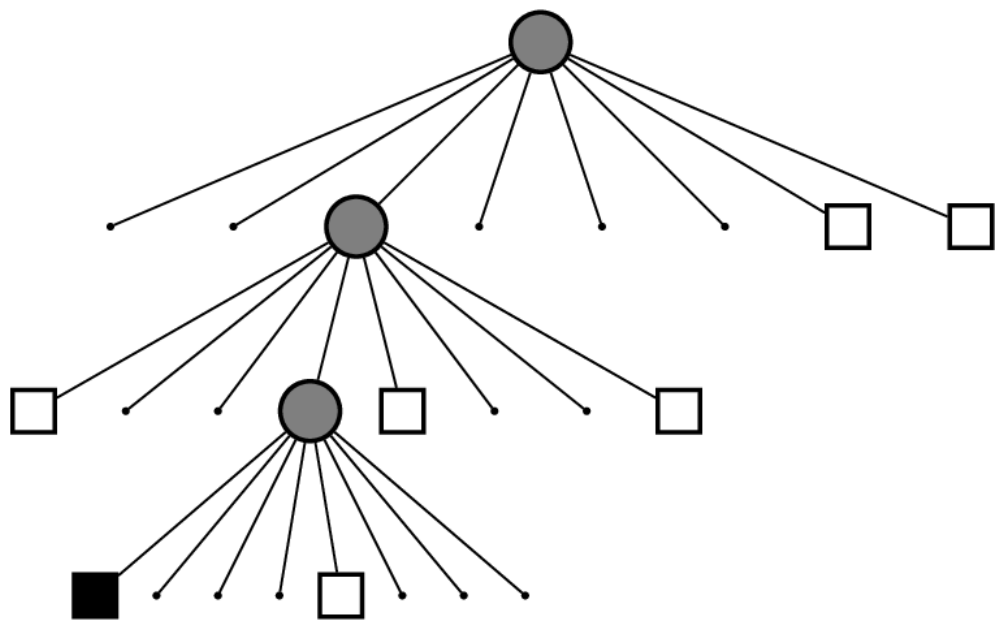
Hörnung et al. 2013



## Tree-based data structure

## Recursive sub-division of space

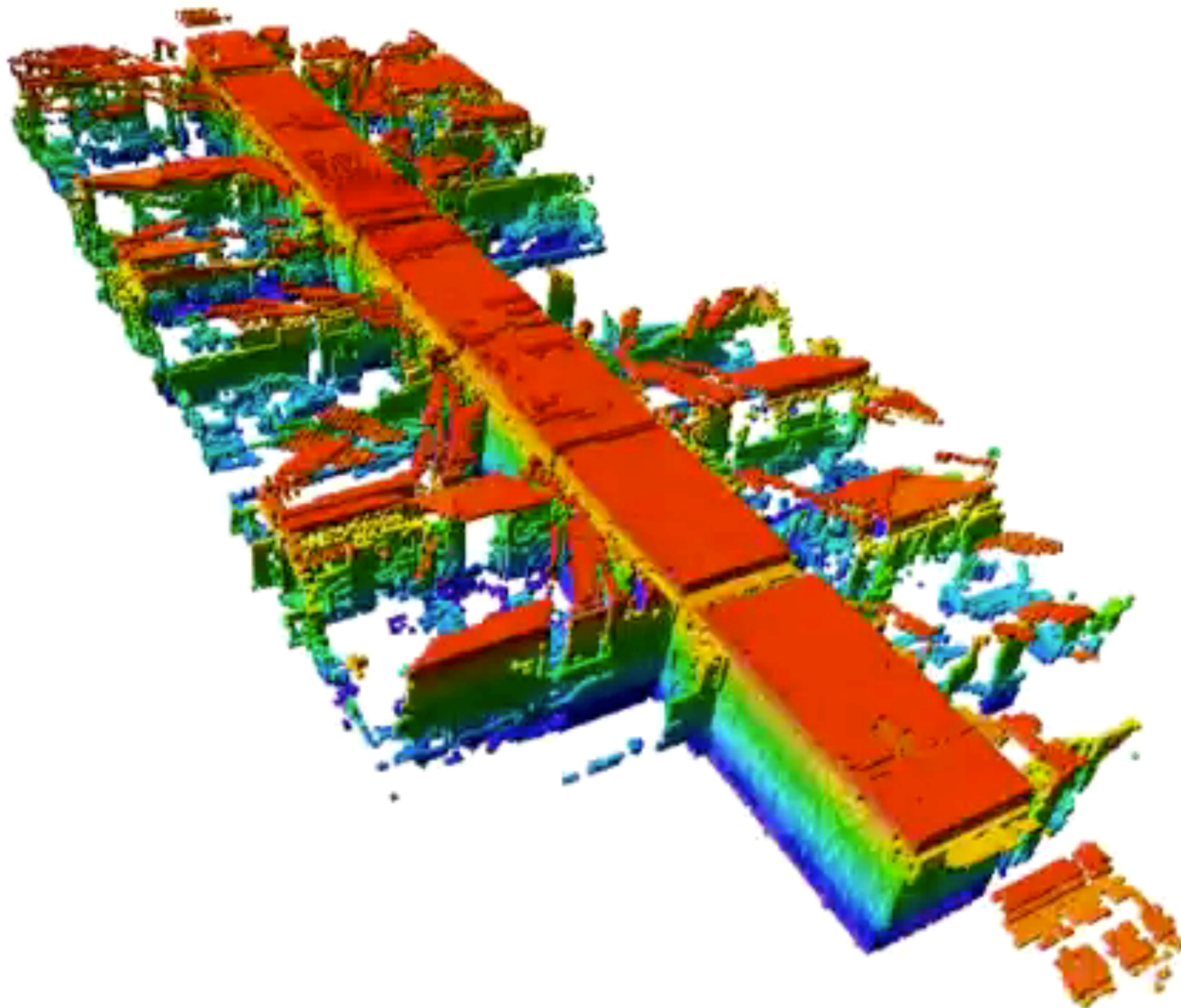
## Query maps at multiple-resolutions!



<https://octomap.github.io/>

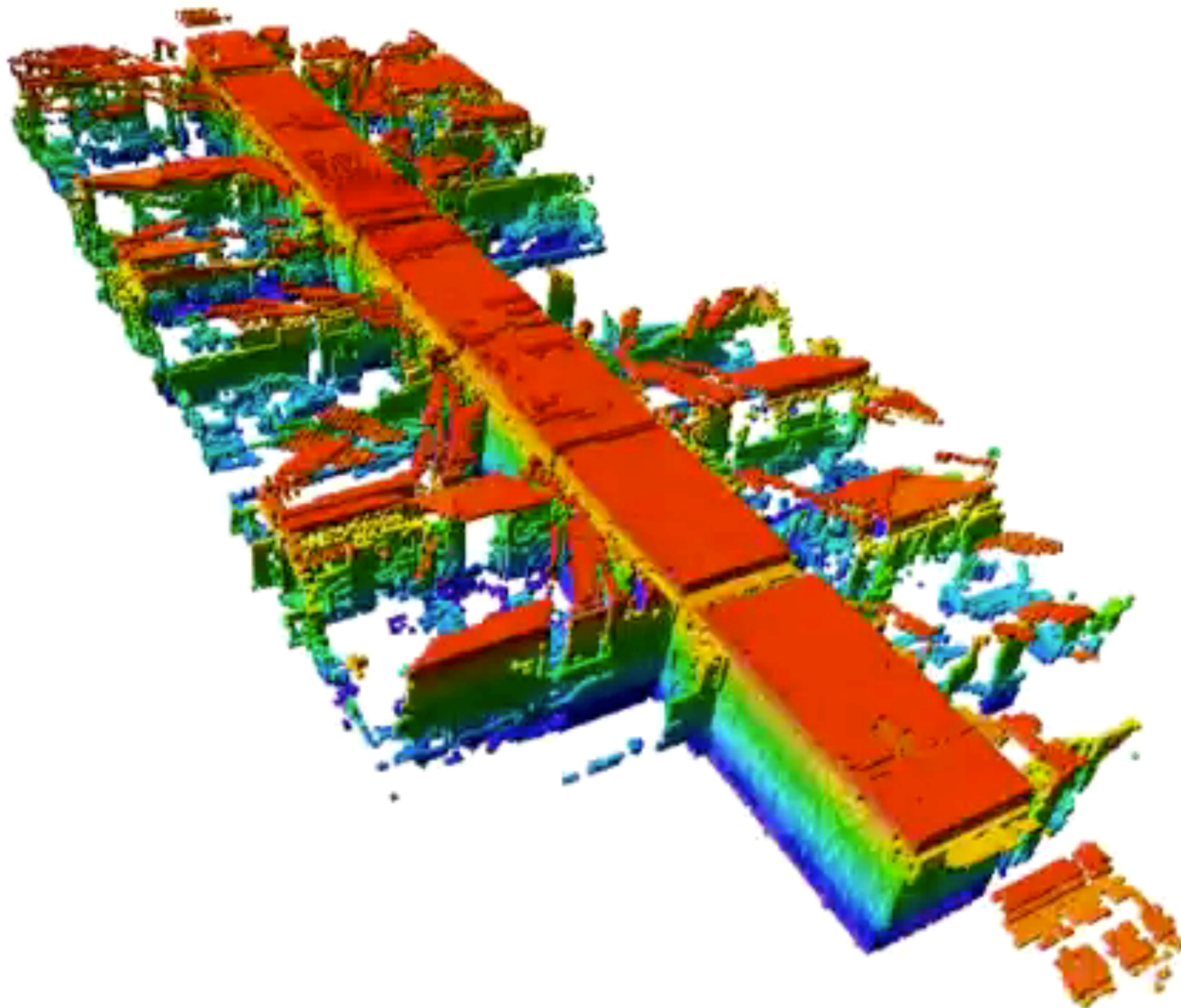


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Information	Same as occupancy grids Stores information at <b>multiple resolutions</b> . Useful for large scale exploration, multi-res planning.
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Update	
Memory	

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Memory	

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Query	<b>Little expensive</b> : $O(\log n)$ , where $n$ is the number of nodes in tree
Update	Similar to occupancy grids, <b>extra</b> $O(\log n)$ complexity
Memory	<b>Much smaller</b> than occupancy grids (proportional to amount of stuff in the world)

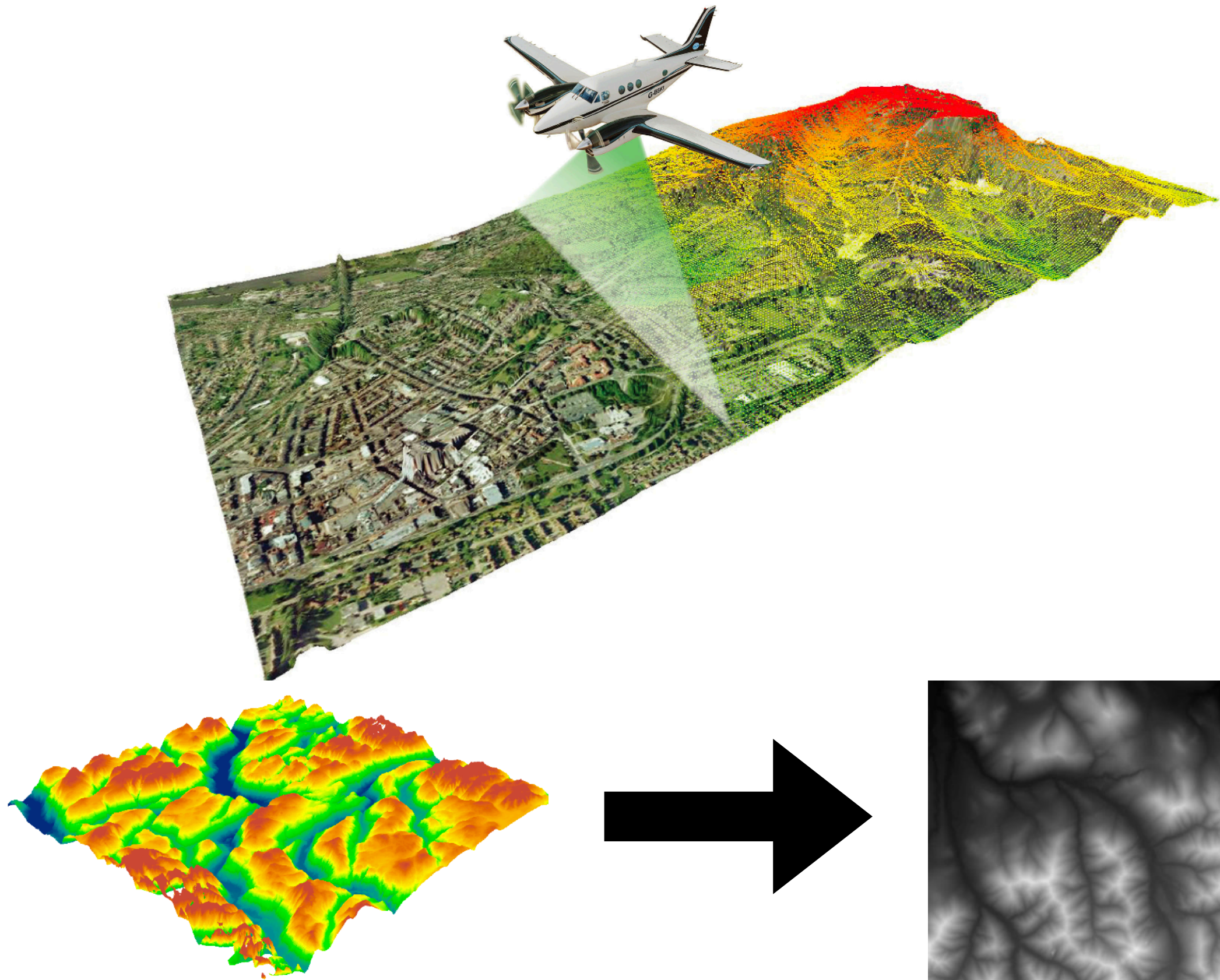
Is the world always 3D?

~~Is the world always 3D?~~

Do we care about 3D?



# Example 3: 2.5D height map



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---

Category	Details
Information	
Query	
Update	
Memory	

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Update	
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# Example 3: 2.5D height map

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Information	Image where each pixel denotes height. Useful for mapping terrain where for overhead flight. Don't use when flying underneath objects
Query	$O(1)$
Update	Can handle noisy measurements by defining a Bayes filter for height of each cell.
Memory	

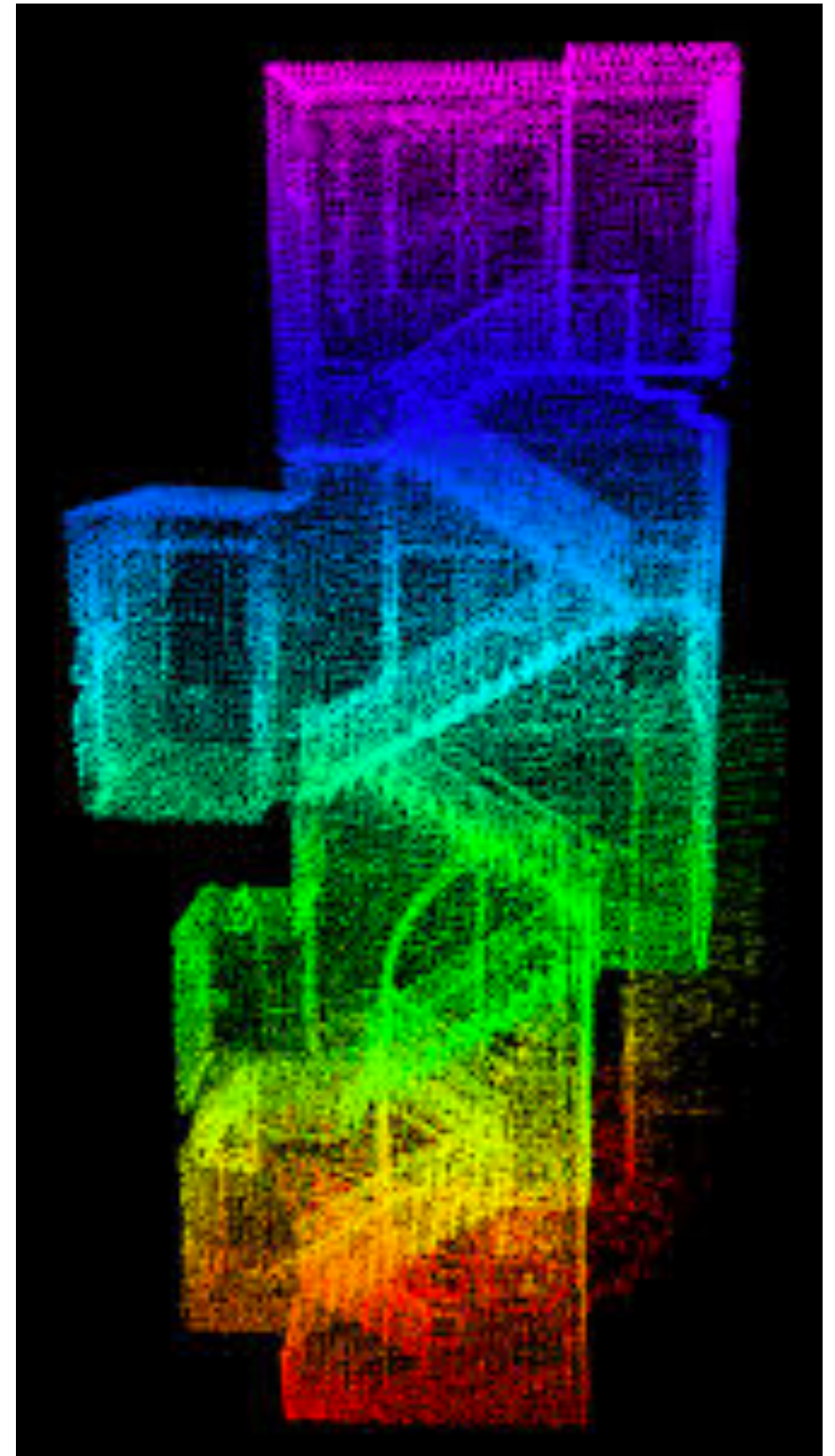
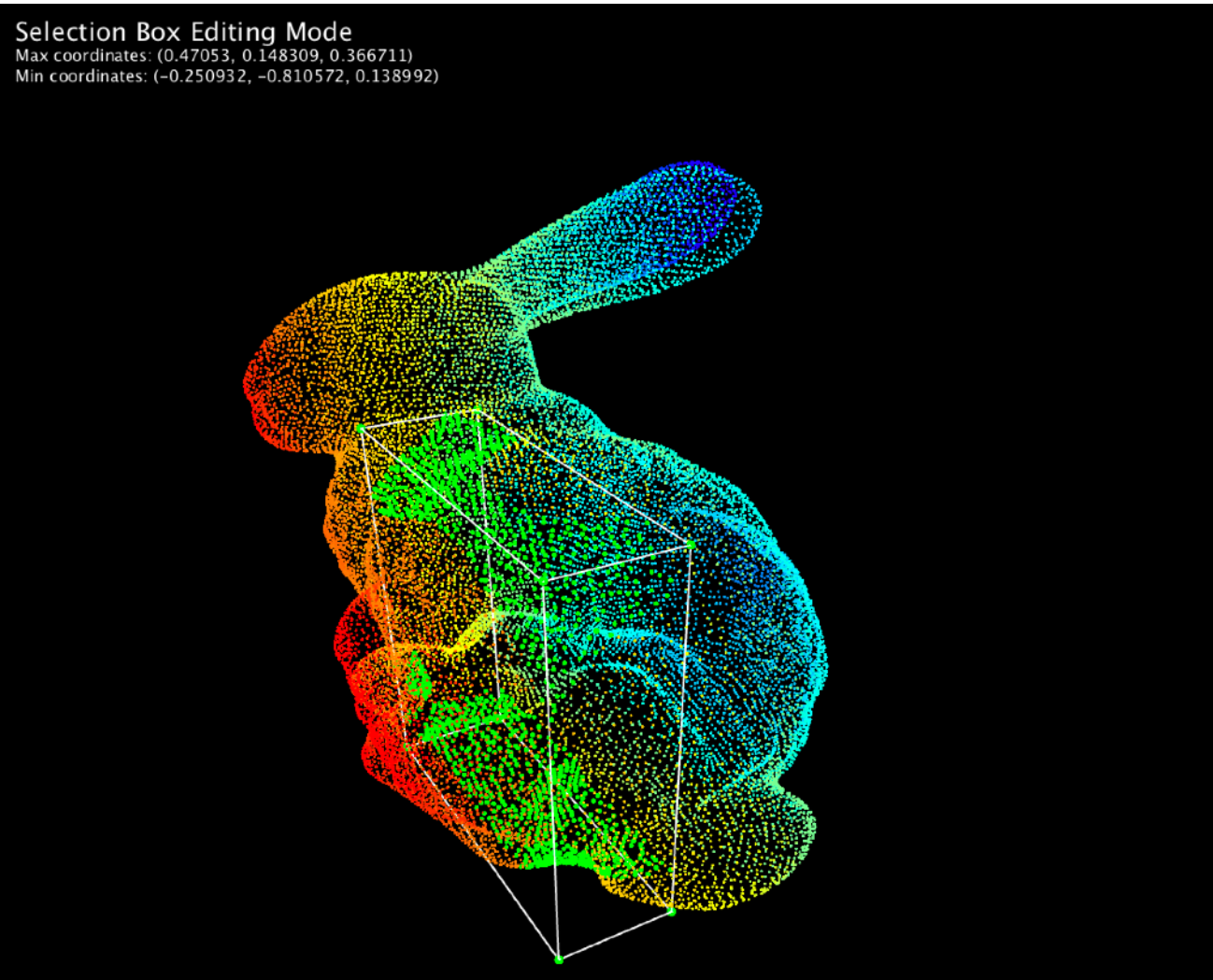
# Example 3: 2.5D height map

Category	Details
Information	Image where each pixel denotes height. Useful for mapping terrain where for overhead flight. Don't use when flying underneath objects
Query	$O(1)$
Update	Can handle noisy measurements by defining a Bayes filter for height of each cell.
Memory	Very cheap! (2D grid)

What are my options if I don't want  
to discretize?



# Example 4: Point cloud



courtesy Ji Zhang



# Example 4: Point cloud





# Example 4: Point cloud

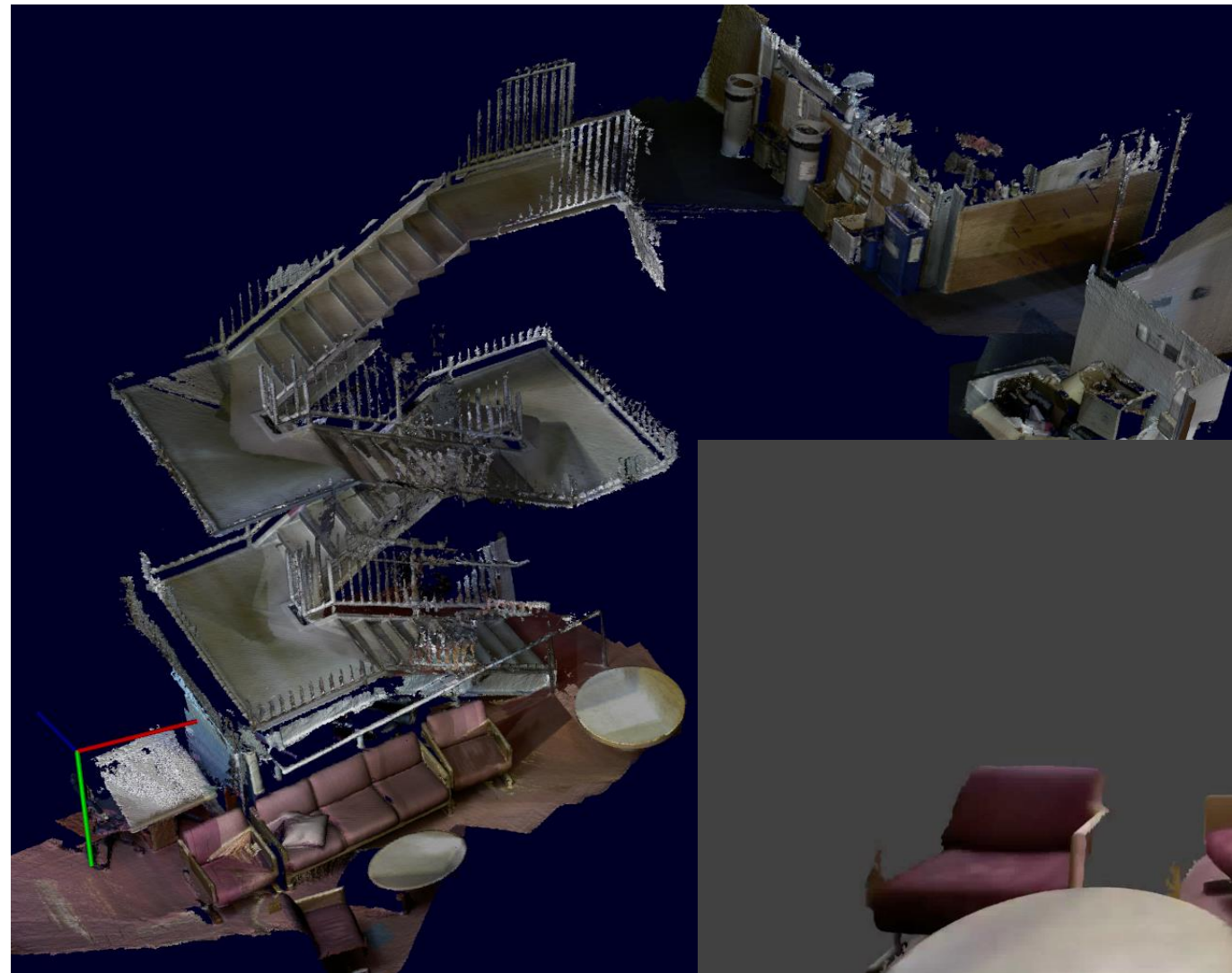
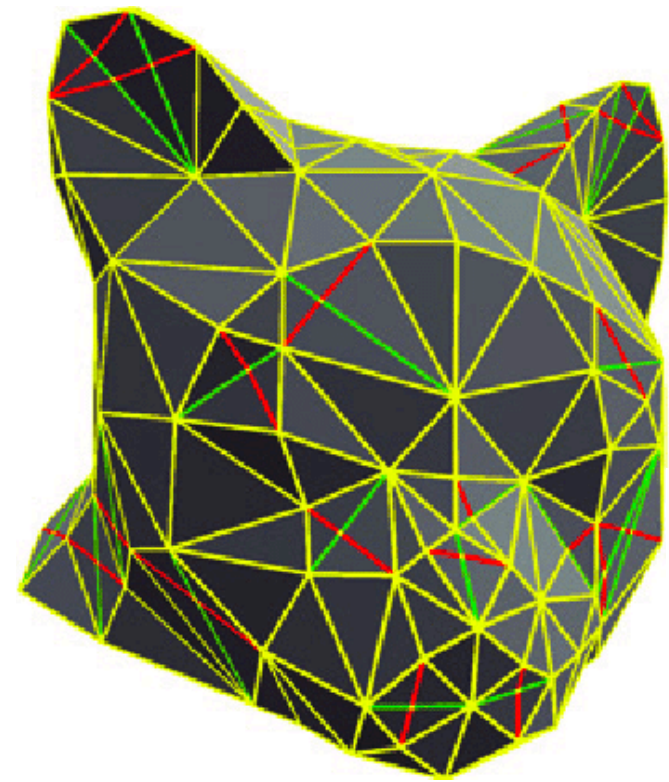




# Example 4: Point clouds

Category	Details
Information	Surface of obstacles (no discretization) Useful for 3D reconstruction Very accurate laser based odometry.
Query	Typical query - give me the closest point / set of points Naive query is $O(N)$ (remember $N$ is huge!!!)
Update	Easy to update (just dump points) Cannot deal with noisy measurements
Memory	Unbounded - can always keep adding points on top of each other indefinitely.

# Example 5: Surface representations



- Handheld RGB-D sensor (\$180)
- Real-time with GPU processing



courtesy M.Kaess

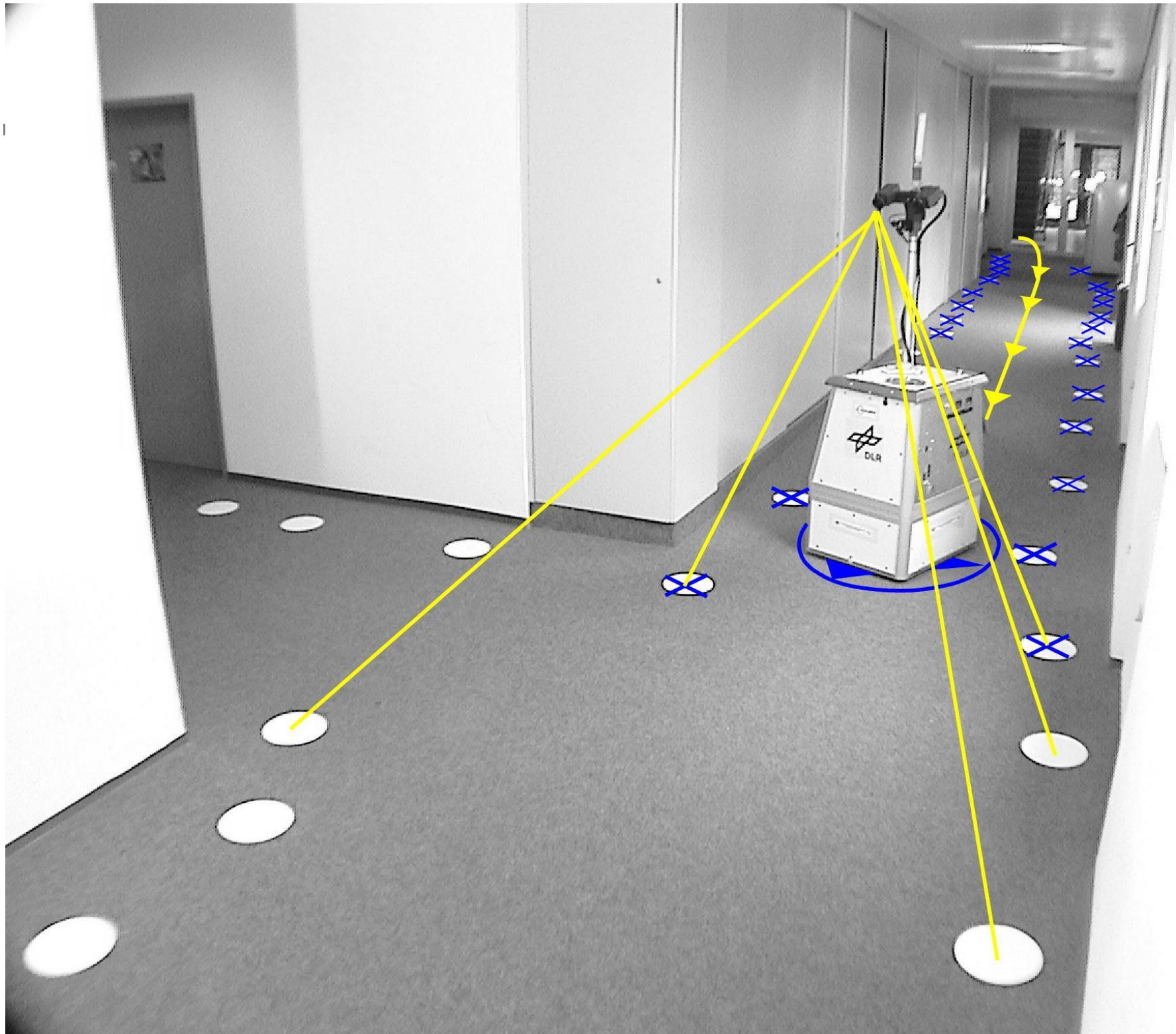
# Example 5: Surface representations

Category	Details
Information	List of triangles representing surface No discretization, arbitrary surfaces Used for computing object object interactions
Query	Find the closest surface. Very naively $O(N)$ but can get massive speedups
Update	Can be updated online (albeit non-trivial) Very susceptible to noisy sensors
Memory	Proportional to amount of surface

Maps that help robots localize



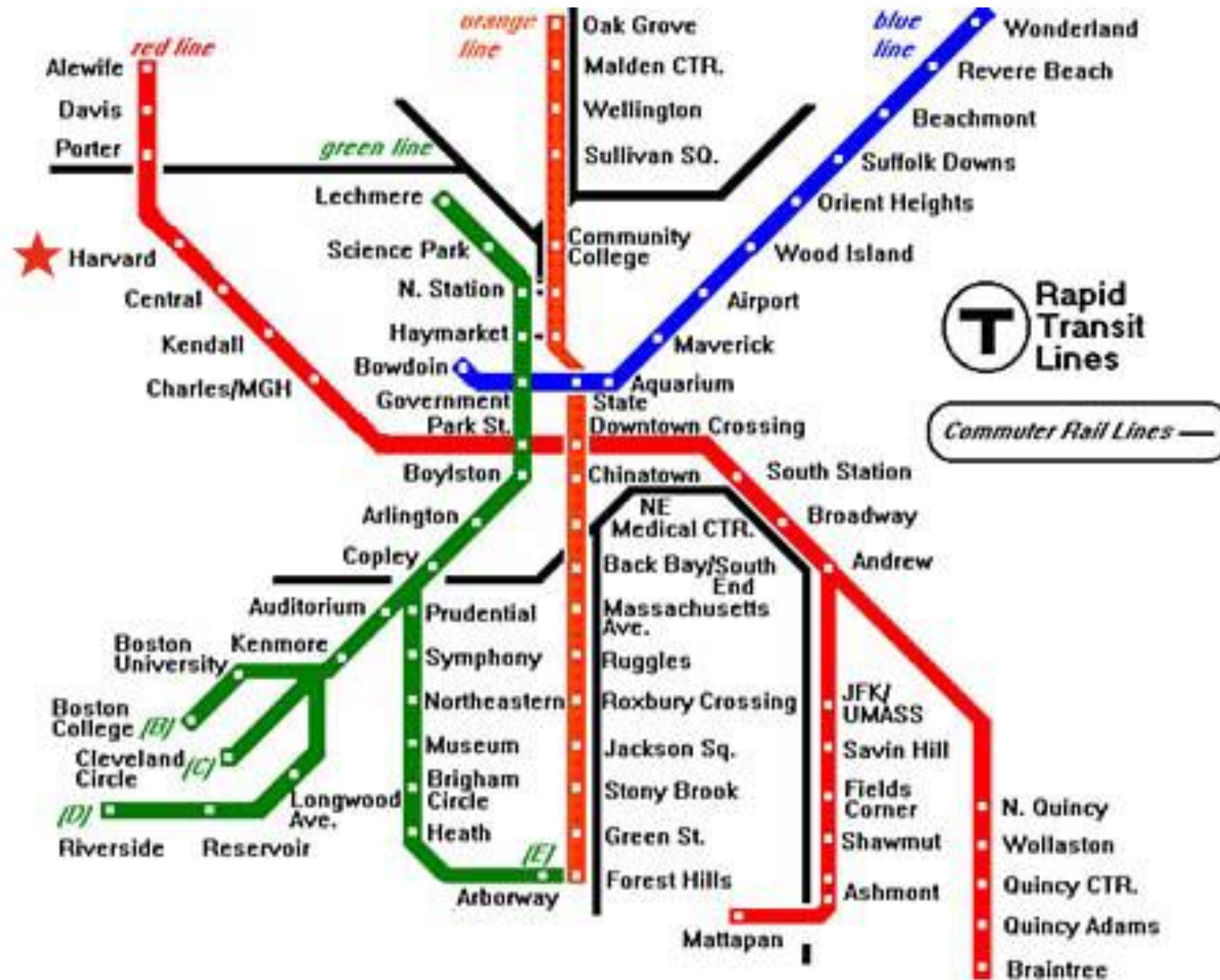
# Example 6: Landmark maps



# Example 6: Landmark maps

Category	Details
Information	Localization (correspondence between images at different timesteps)
Query	Typical query - give me the closest landmark Naive query is $O(N)$
Update	Easy to update (just dump landmarks) Need outlier rejection
Memory	Unbounded (but usually small as landmarks are sparse)

# Example 7: Topological representations

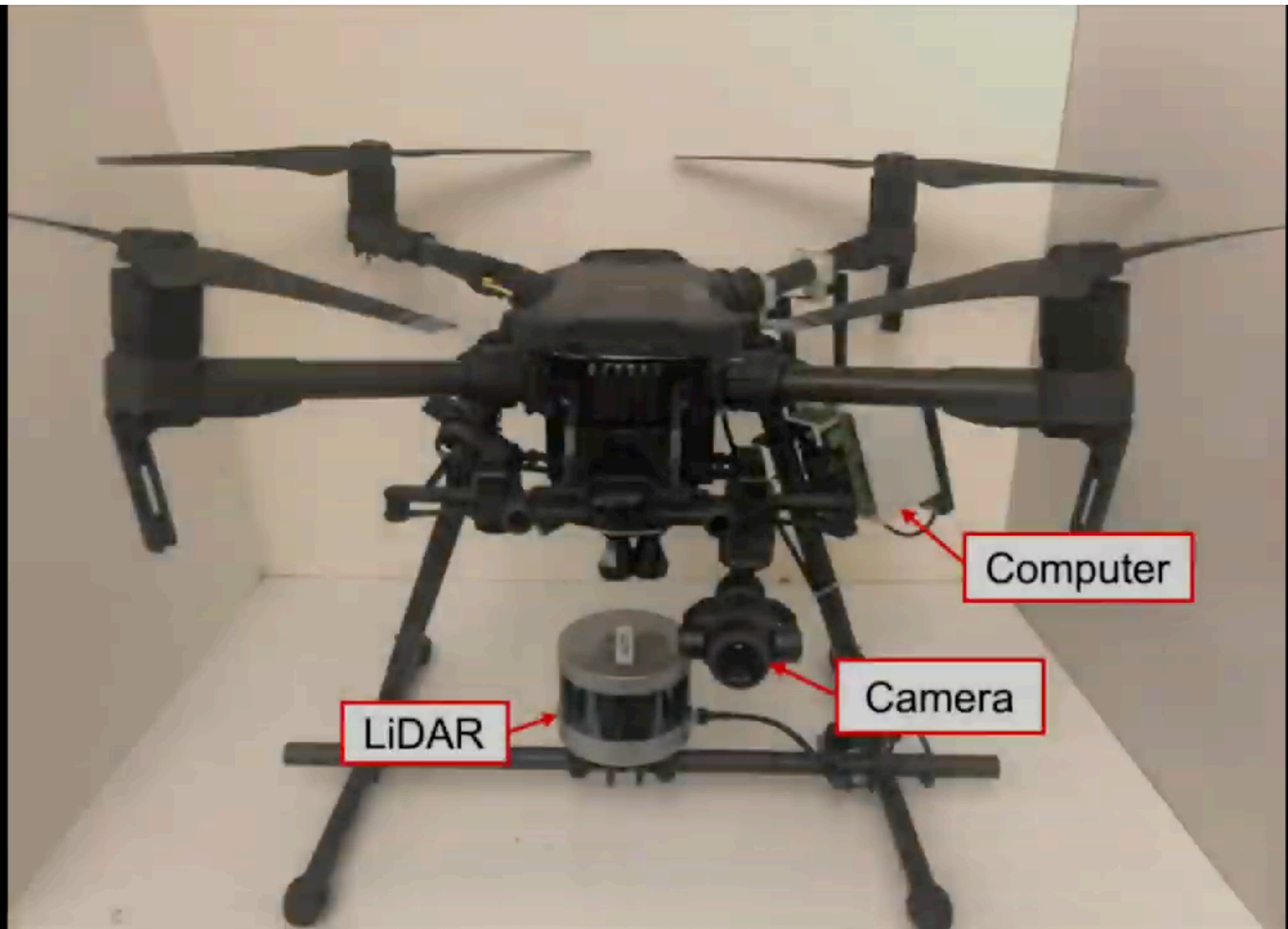




# Example 7: Topological representations

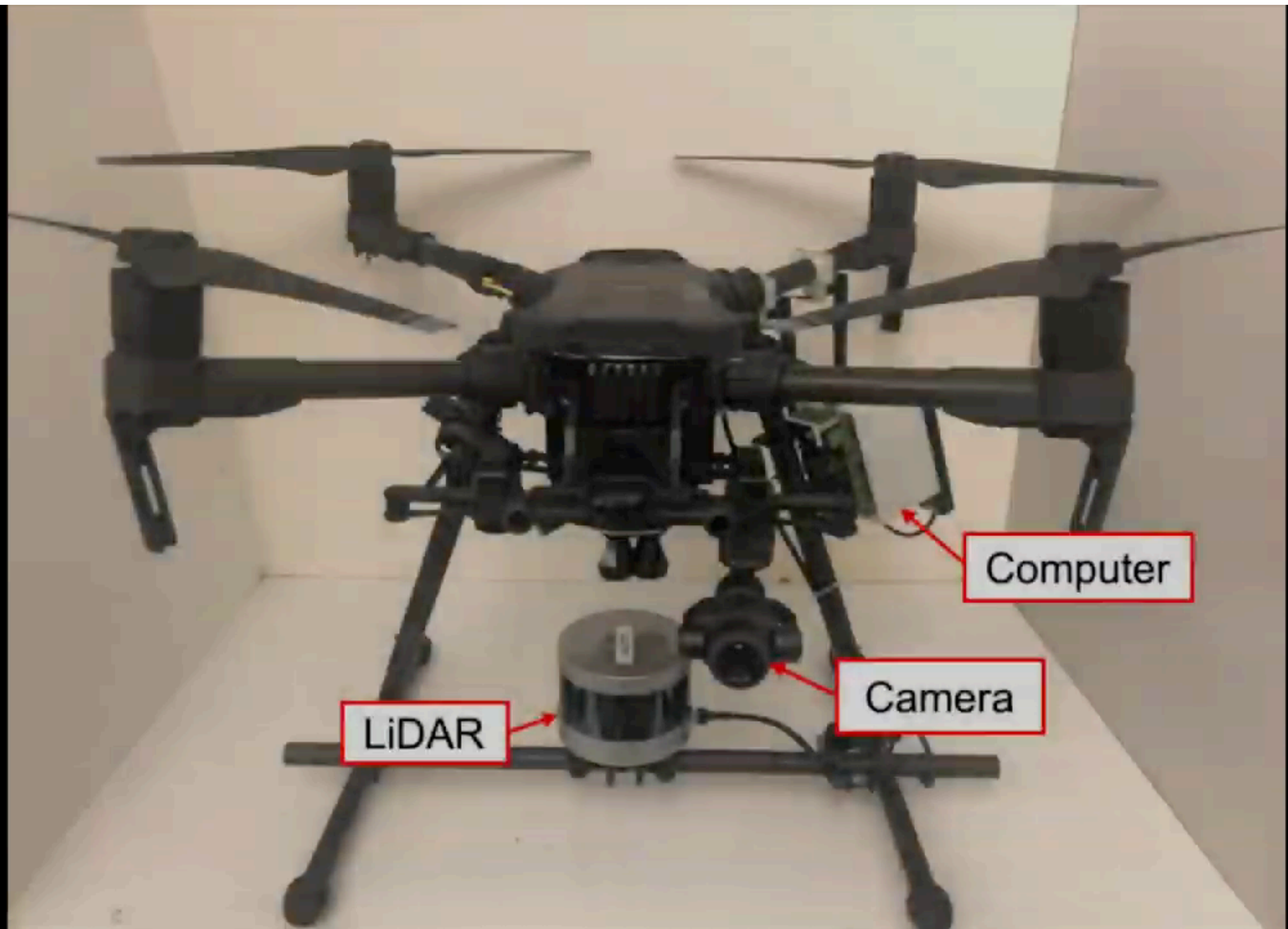
Category	Details
Information	Graph where vertices are landmarks (e.g. rooms in a building), and edges represent relationships (connections)
	High level navigation tasks which are specified on the topomap.
	Localize robot on the map by finding correspondence with vertices.
Query	Cheap graph query
Update	Non-trivial / mostly done offline
Memory	Low

# Applications with multiple map representations



Bonnatti et al. 2019

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Bonnatti et al. 2019

Maps are not just ways of storing  
sensor data

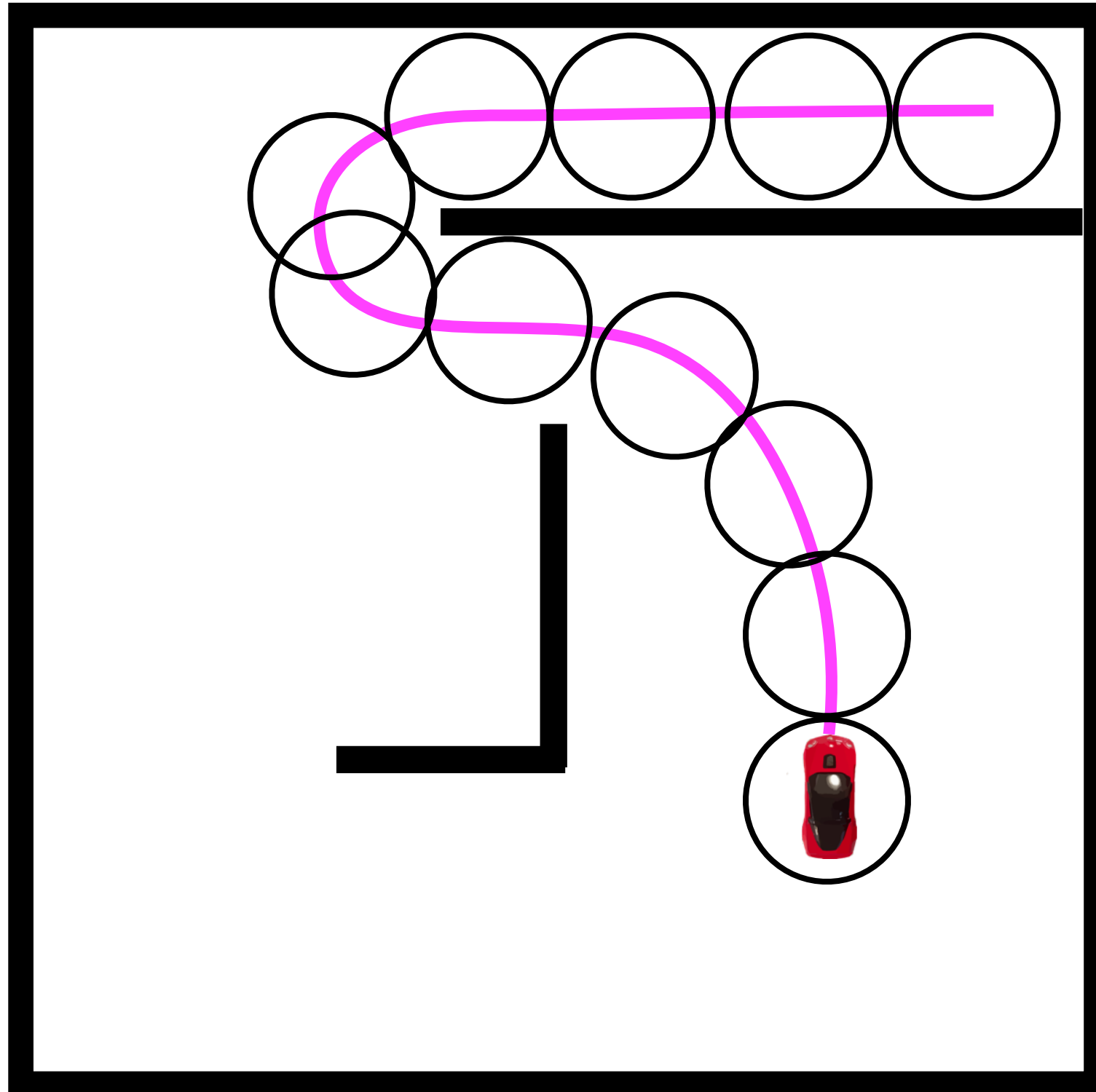
Some maps are computational  
operations on other maps



# Distance map

# Why do we need distance?

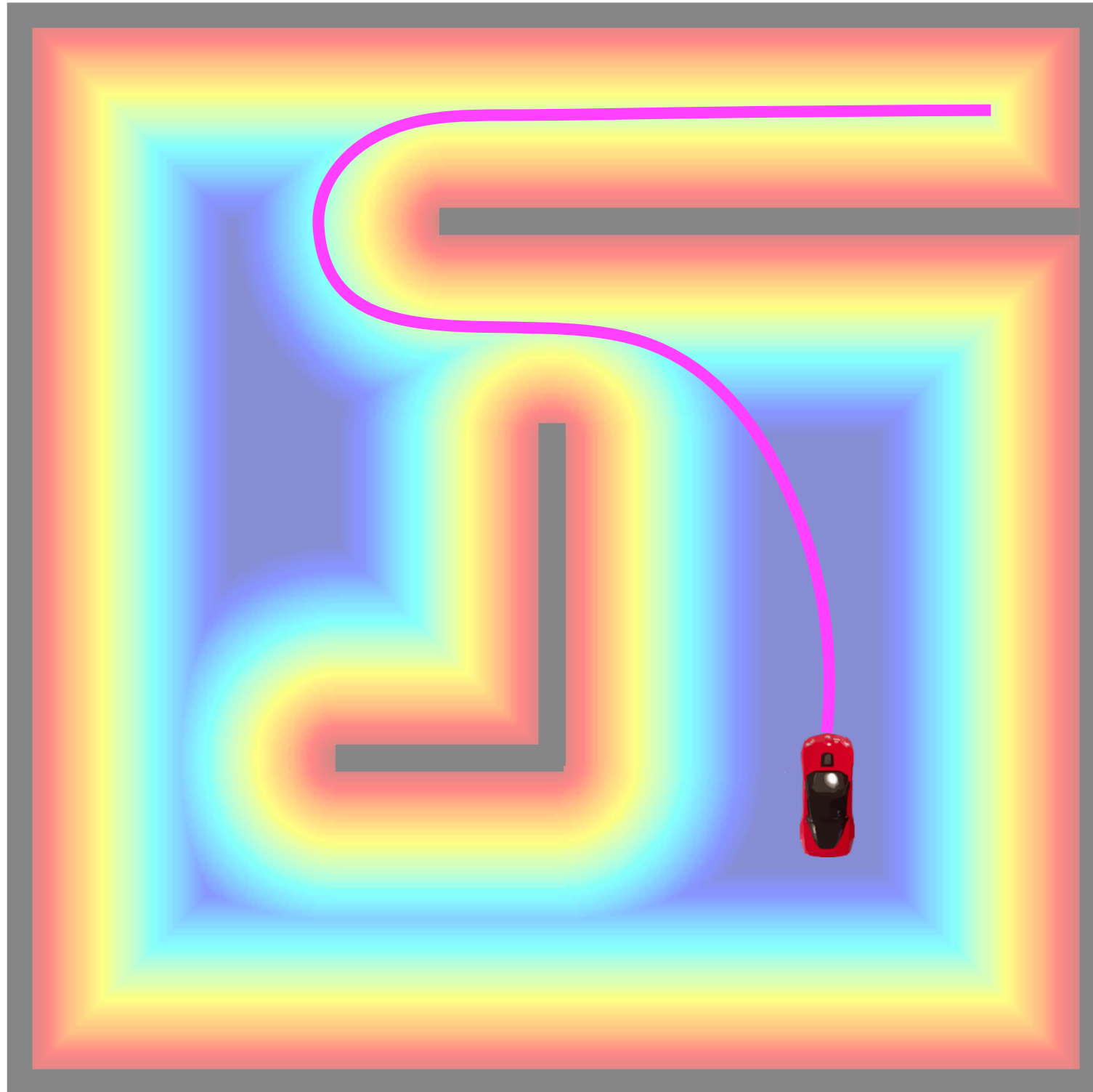
Plan a path  
that penalizes  
proximity to  
obstacles



Why do we  
need a map?

# Desiderata: Map storing (truncated) distance

Input:  
Binary map  
of the world



Output:  
Map of  
same size  
storing  
truncated  
distance

# Example 8: Distance map

Category	Details
Information	Truncated distance to obstacles
Query	$O(1)$
Update	We want to incrementally update this map Ideally $O(k)$ where $k$ is the number of cells which changed distance value
Memory	Same as the underlying occupancy grid

How do we efficiently calculate  
distance map?

# Dynamic programming to the rescue!

Initialize distance  $d(i)$  for free cells to Inf

Insert all boundary pixels to queue Q

**While** Q not empty

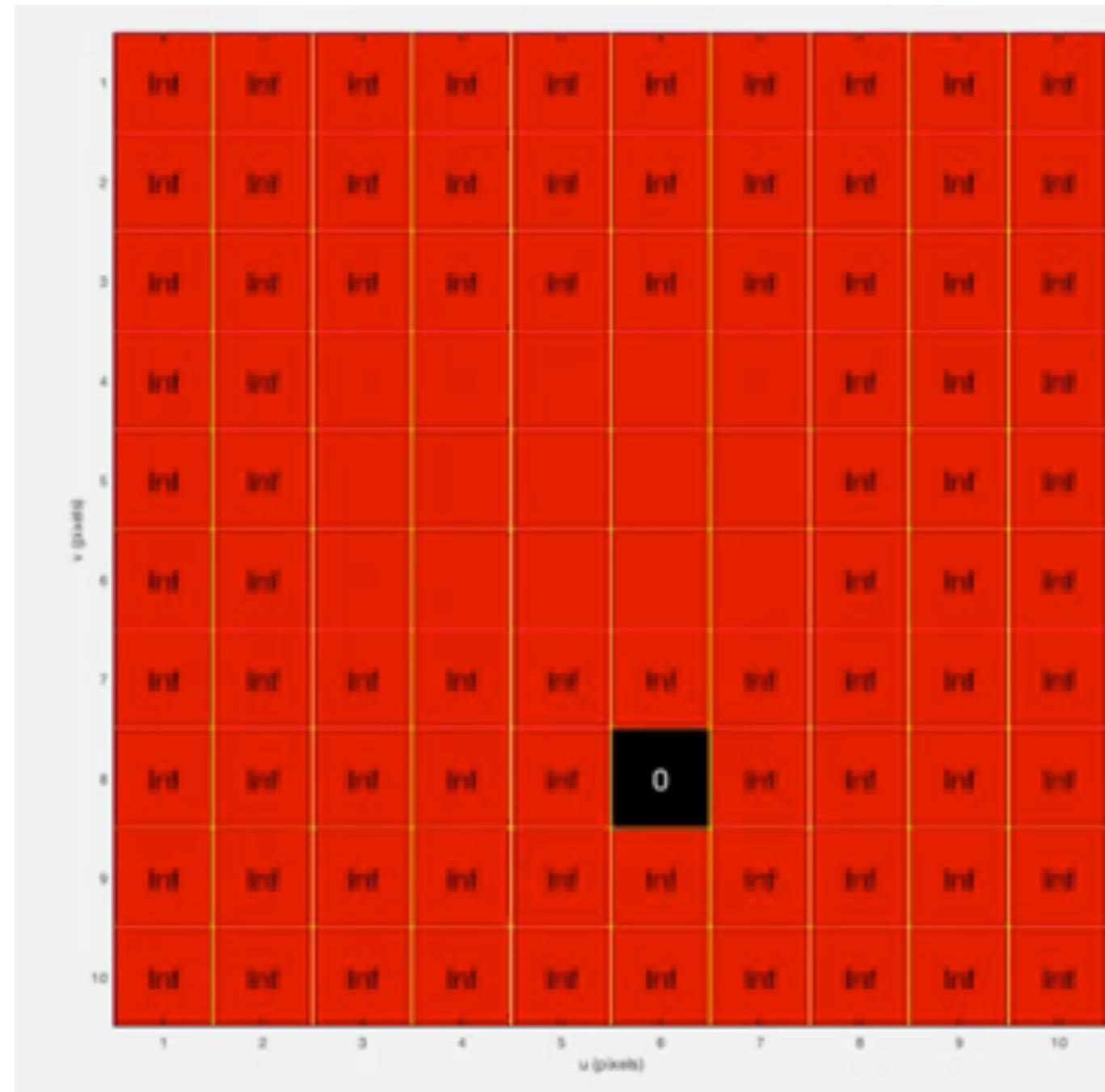
$x = Q.pop()$

**for each** n **in** Neighbour(x)

$d(n) = \min(d(n), v(x) + \text{dis}(x,n))$

**if**  $d(n) \leq d_{\max}$

        Q.insert(n)



# Dynamic programming to the rescue!

Initialize distance  $d(i)$  for free cells to Inf

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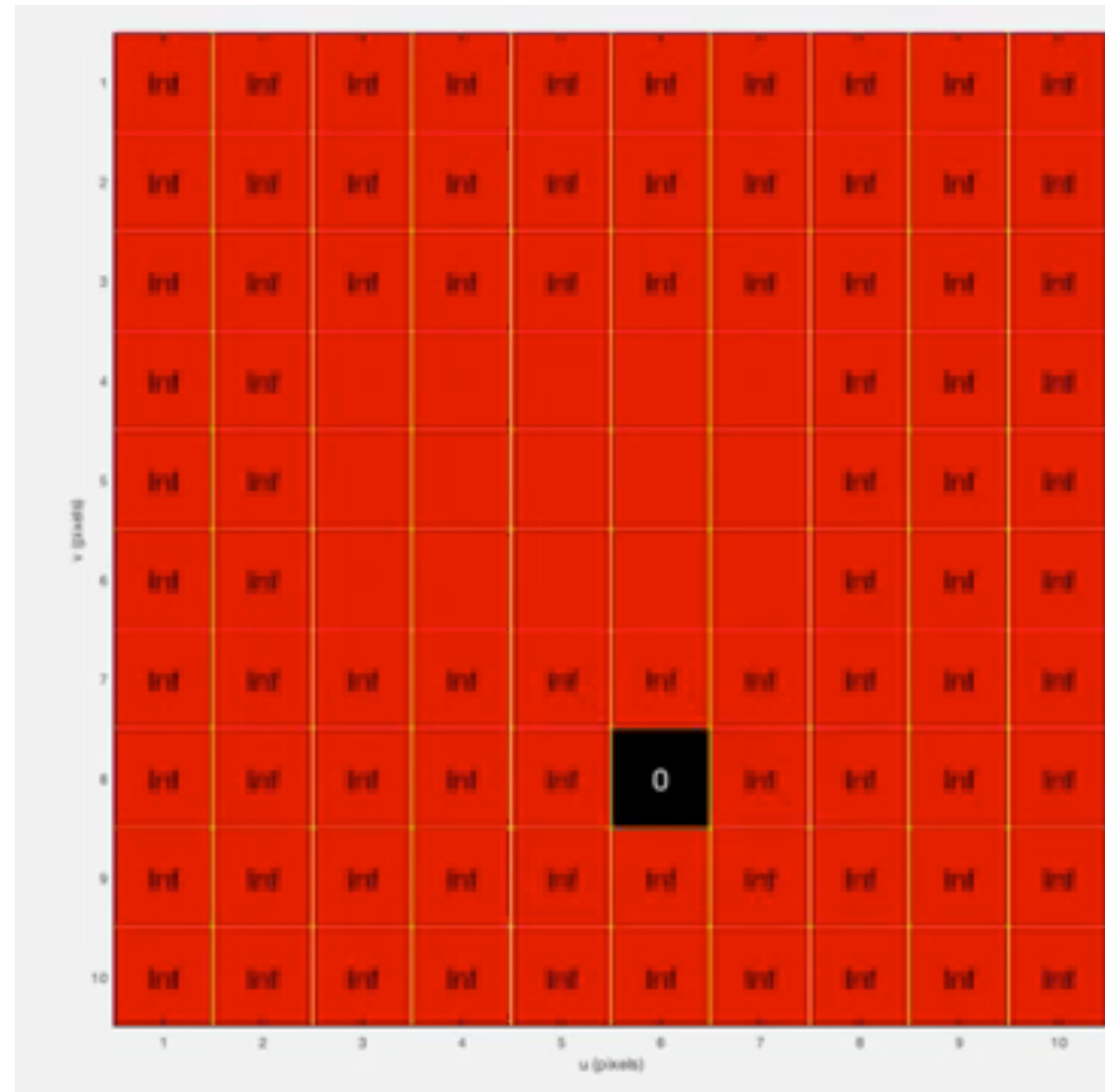
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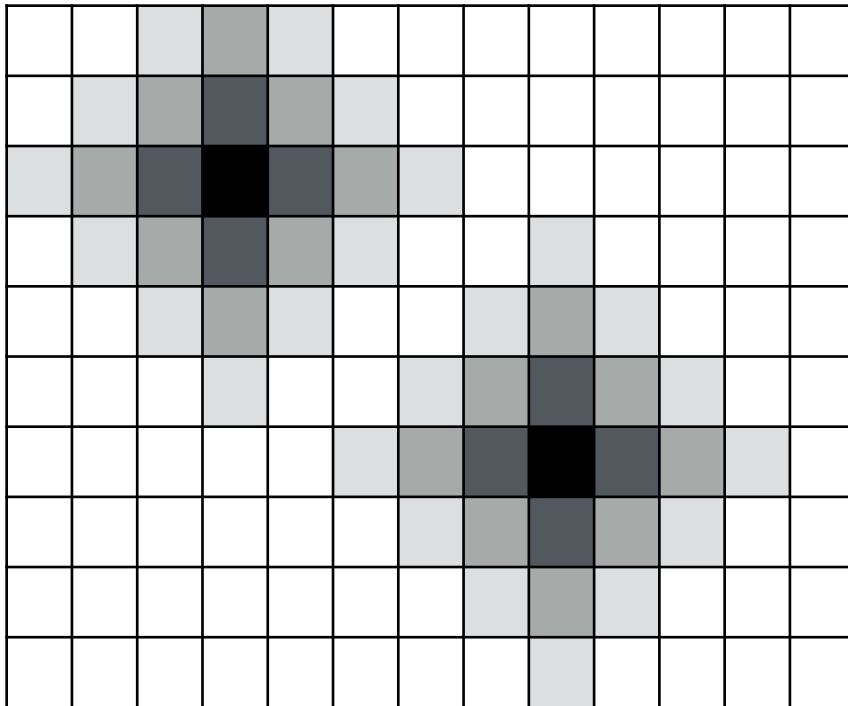
How can we incrementally update  
this map?

Tale of two wavefronts

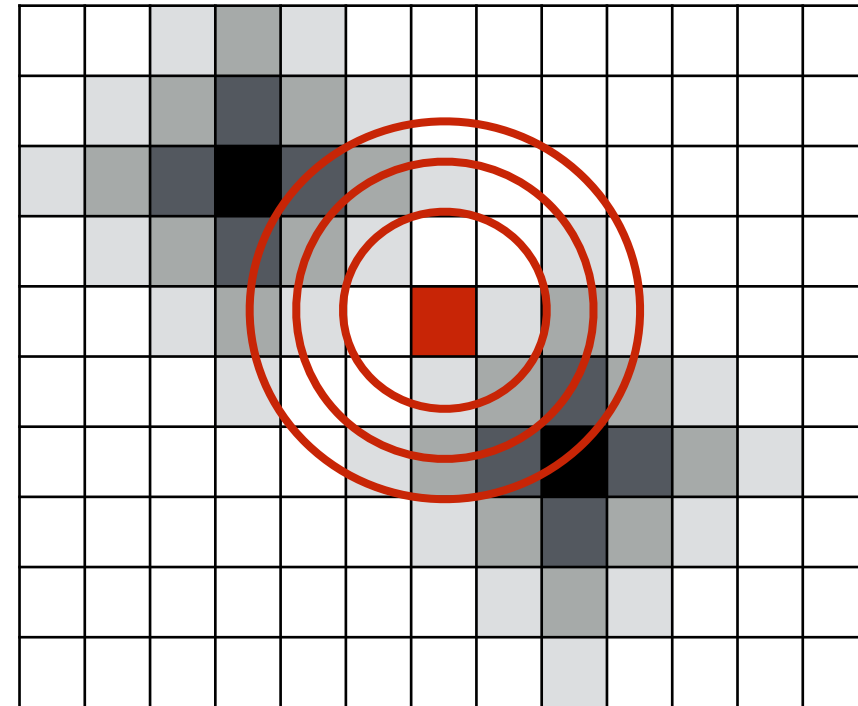
LOWER (when you add obstacle)

RAISE (when you delete obstacle)

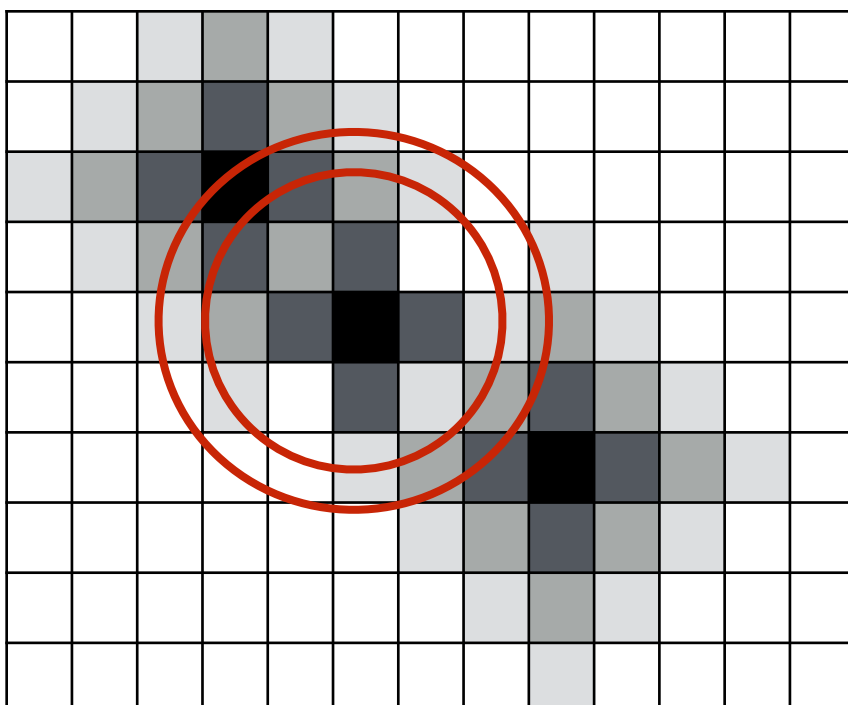
# When obstacle is added



Existing  
distance  
map

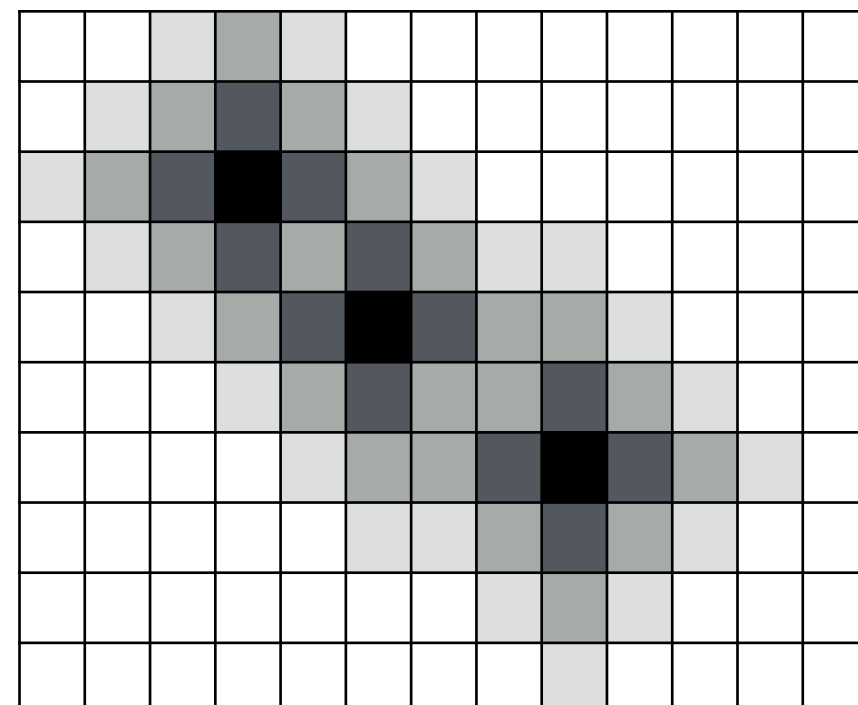


New obstacle  
added.  
LOWER  
wavefront  
started



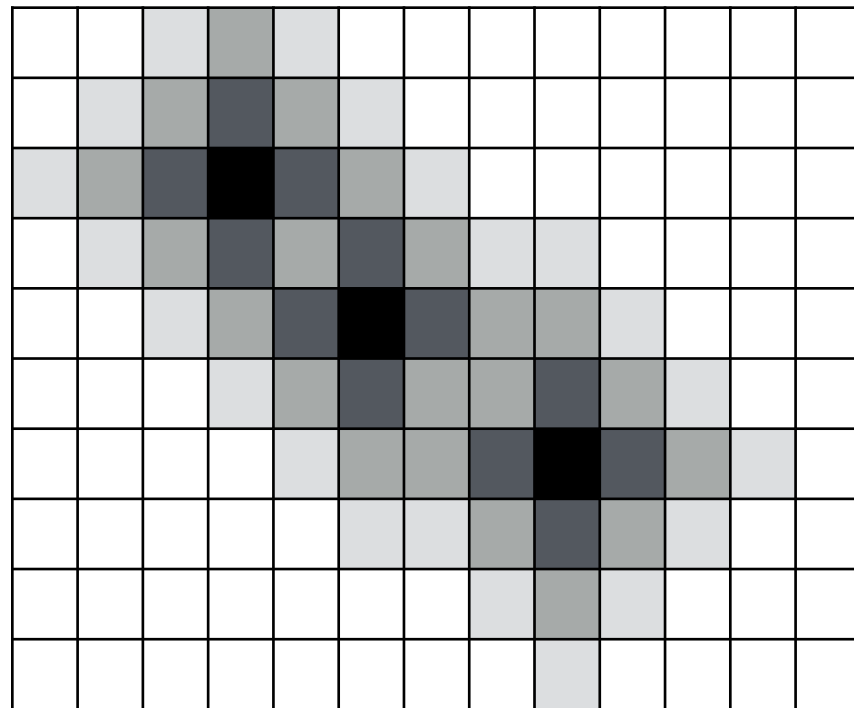
Overwrite  
distances  
if smaller  
value.

Remember  
closest  
obstacle

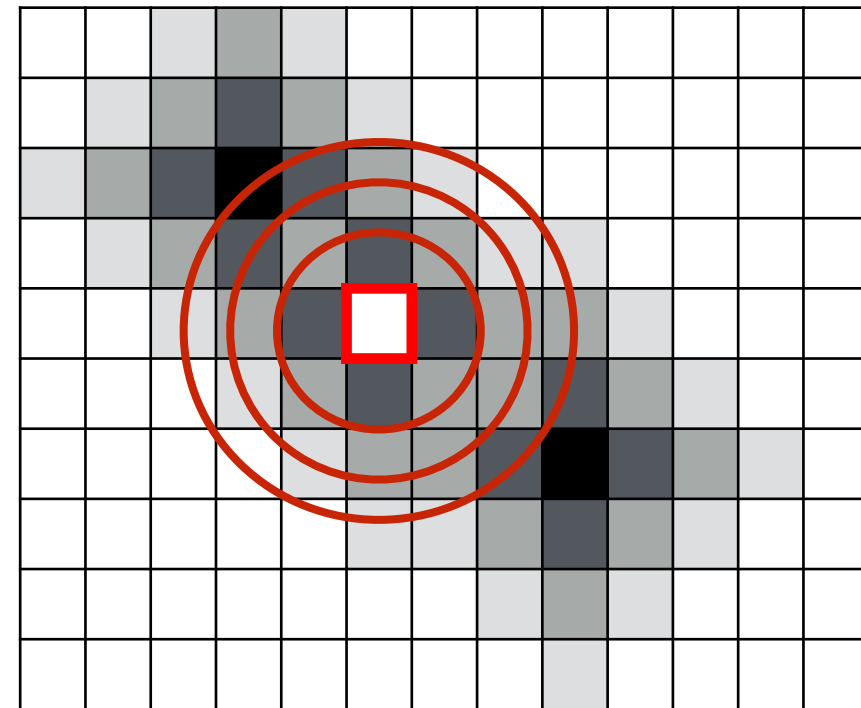


Stop  
wavefront  
whenever  
you meet  
higher  
distance

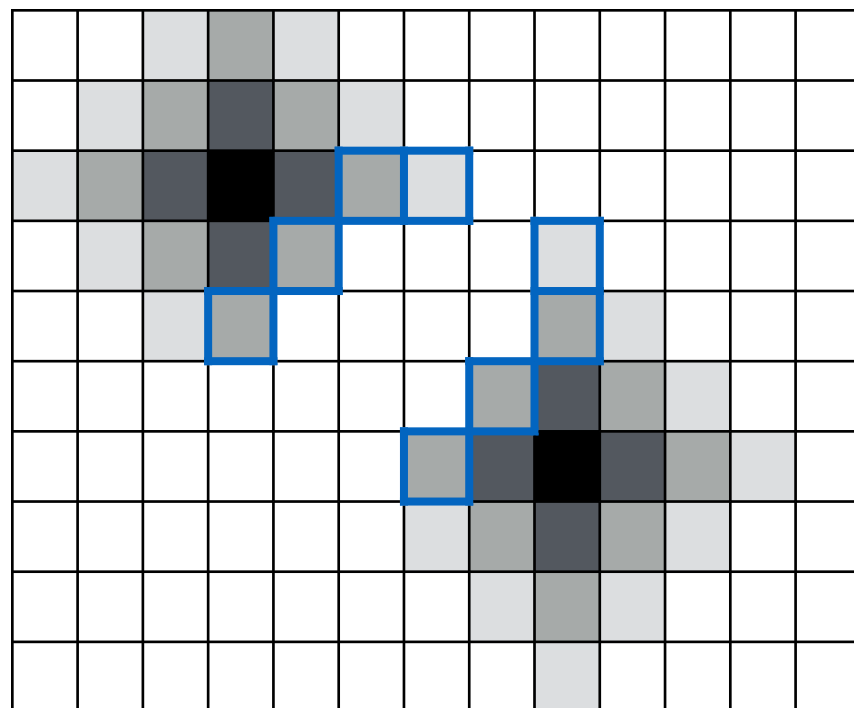
# When obstacle is deleted



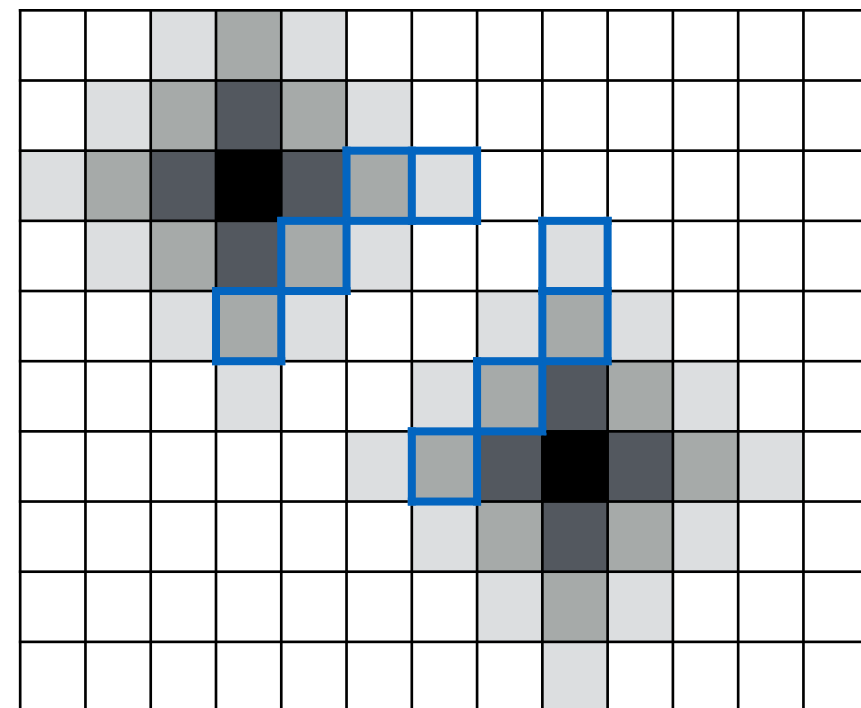
Existing  
distance  
map



Obstacle  
deleted  
RAISE  
wavefront  
started



Set  $d=d_{\max}$   
if closest  
obstacle was  
deleted.  
  
Stop  
wavefront  
otherwise.



Boundary  
cells  
trigger  
LOWER  
wavefront

# Template for incremental dynamic programming

Input: Cells which changed status (obstacles added / removed)

Insert all changed cells into a queue  $Q$

While  $Q$  not empty

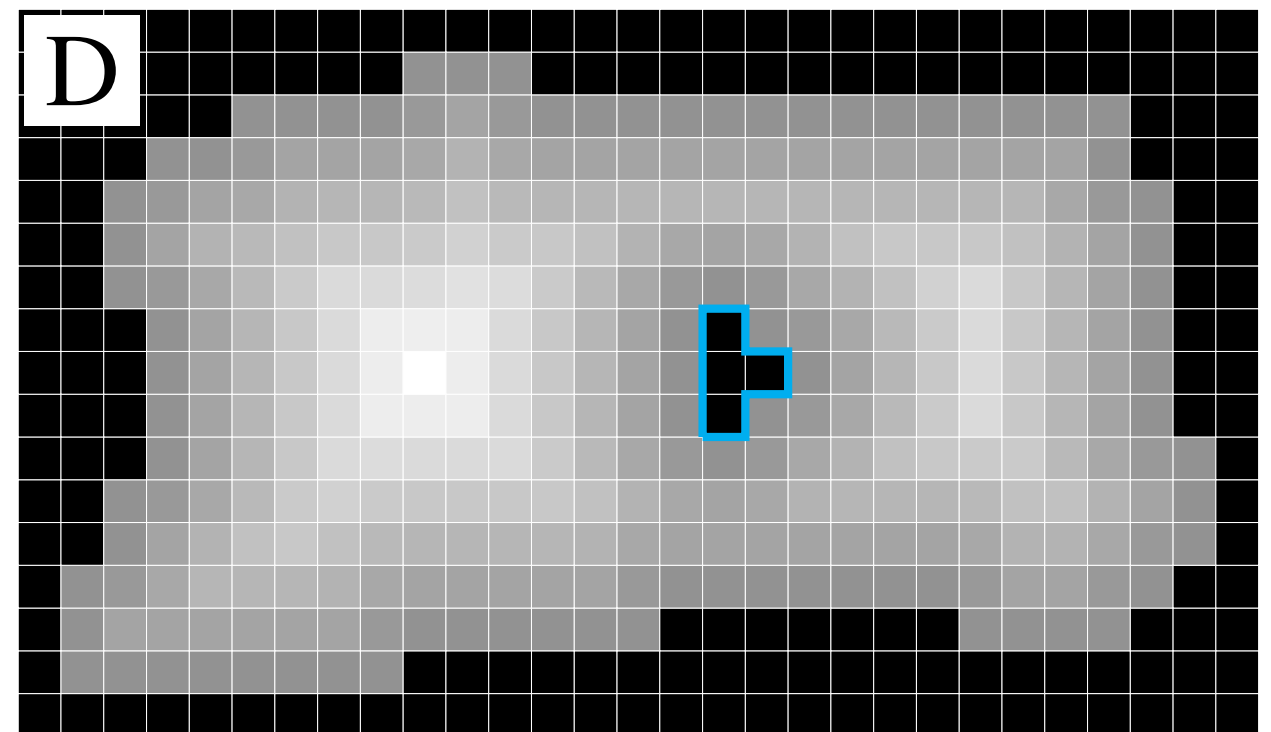
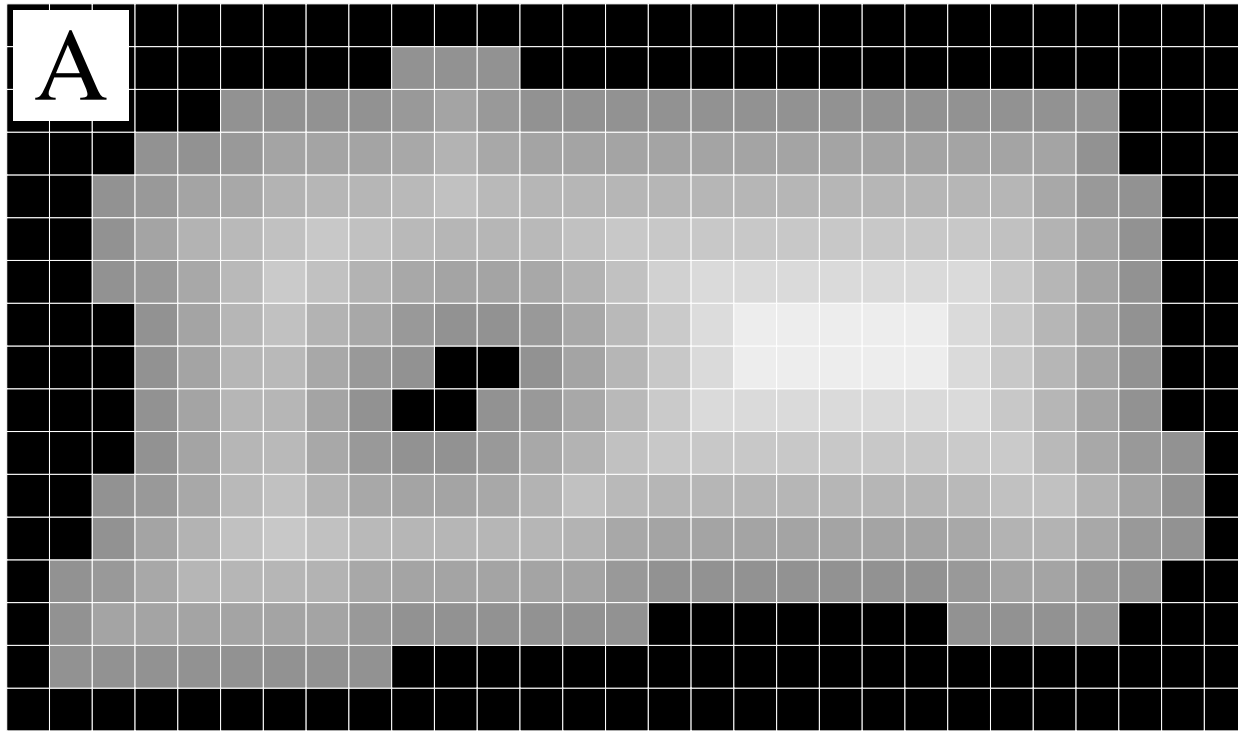
Node  $n = Q.pop()$

If  $n$  is over consistent ( $d\_old > d\_new$ ), lower value

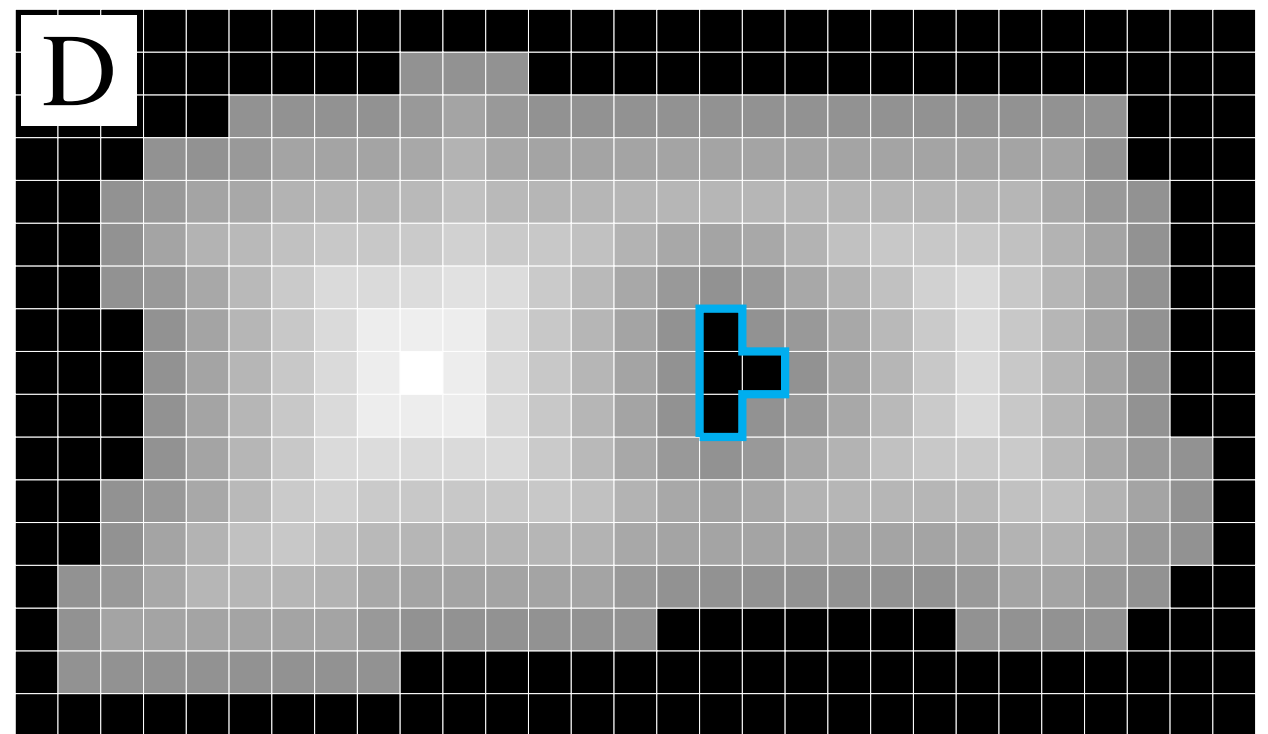
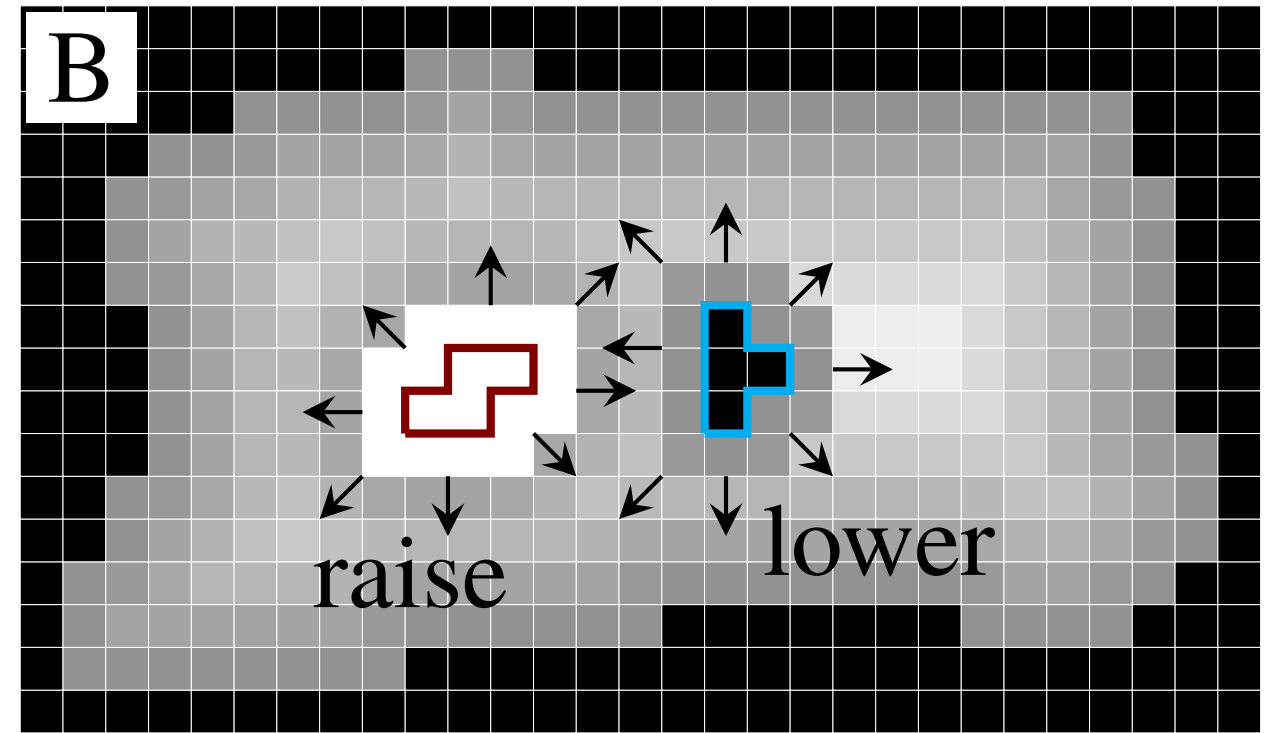
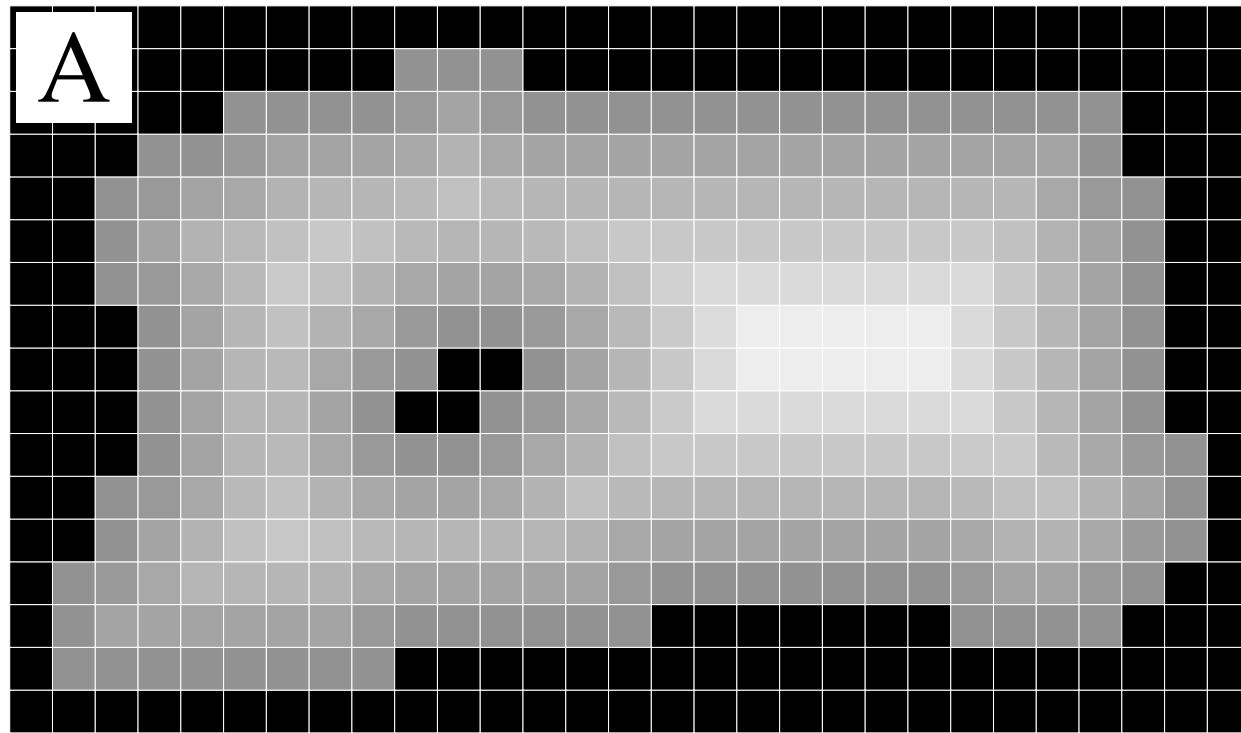
If  $n$  is under consistent ( $d\_old < d\_new$ ), raise value

Add neighbors whose values need to be changed.

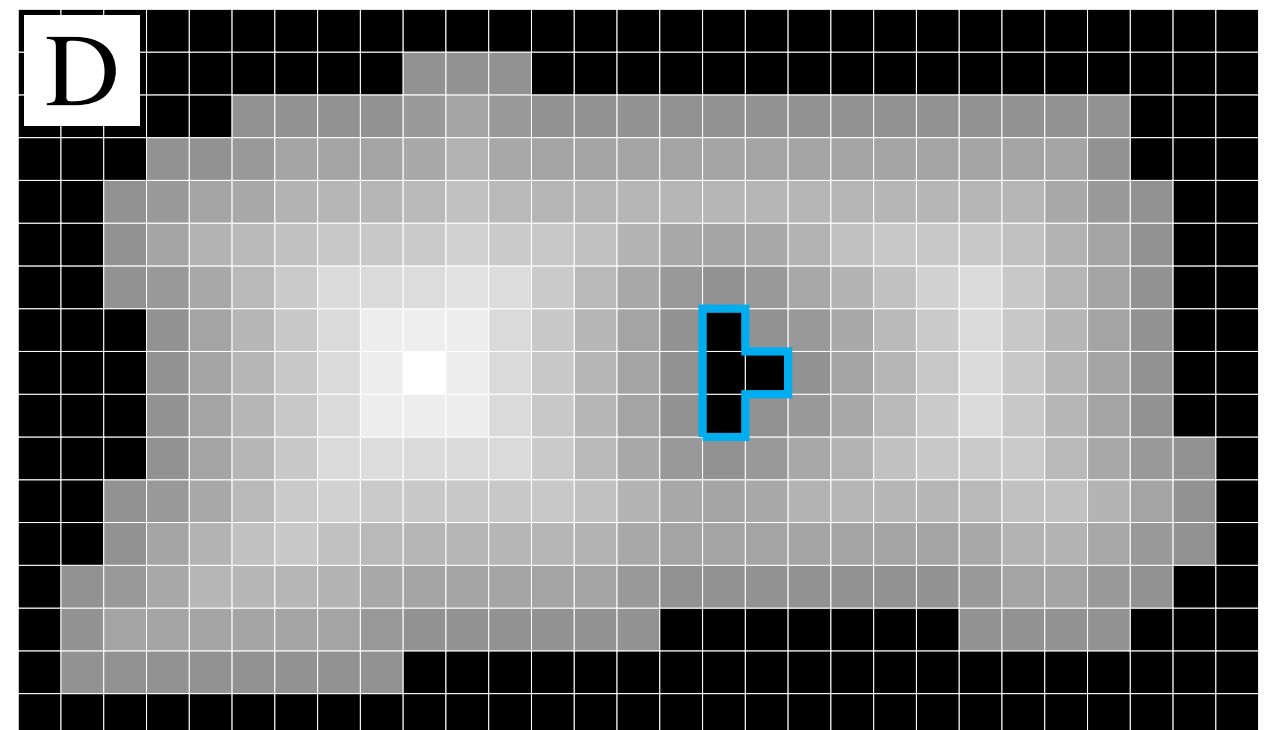
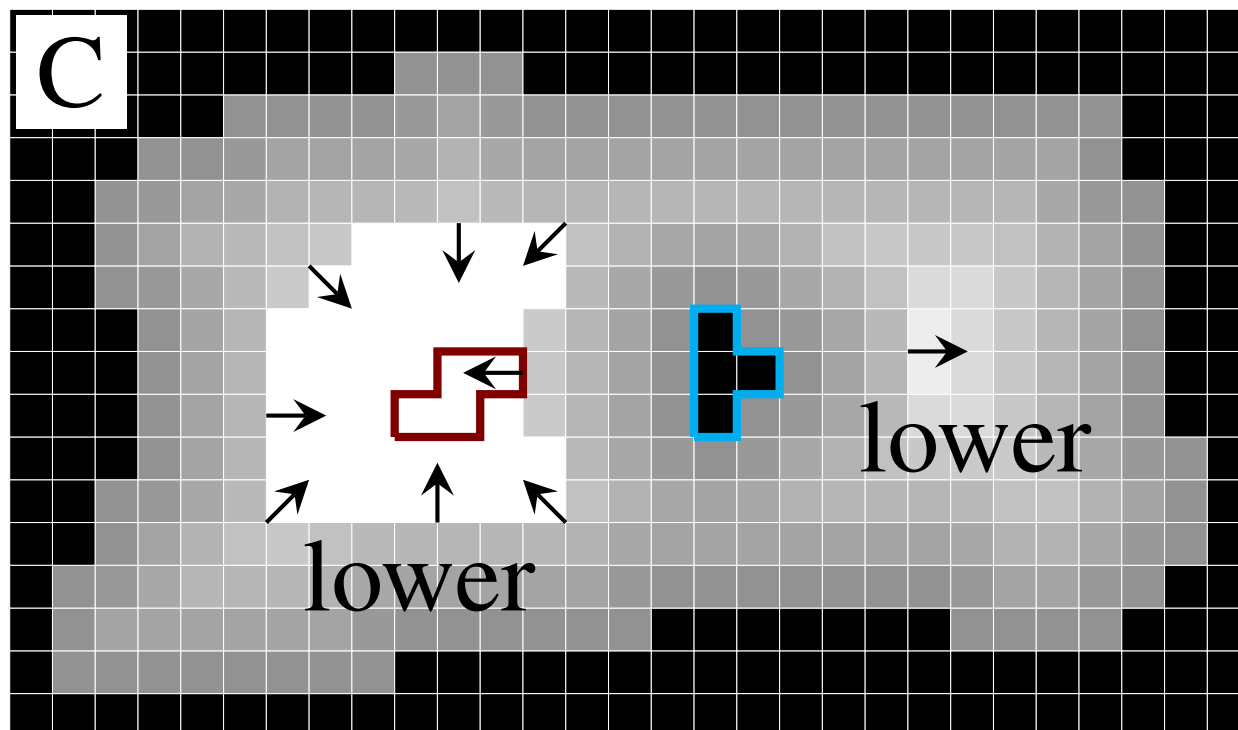
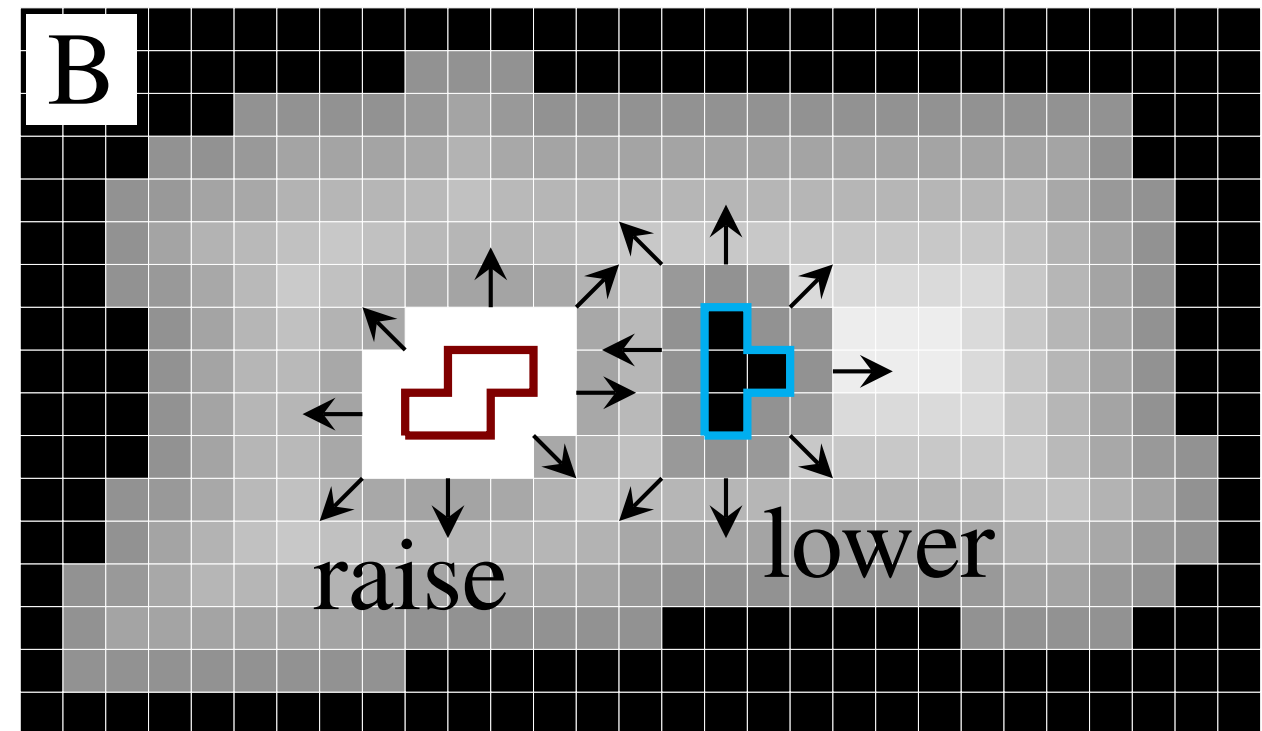
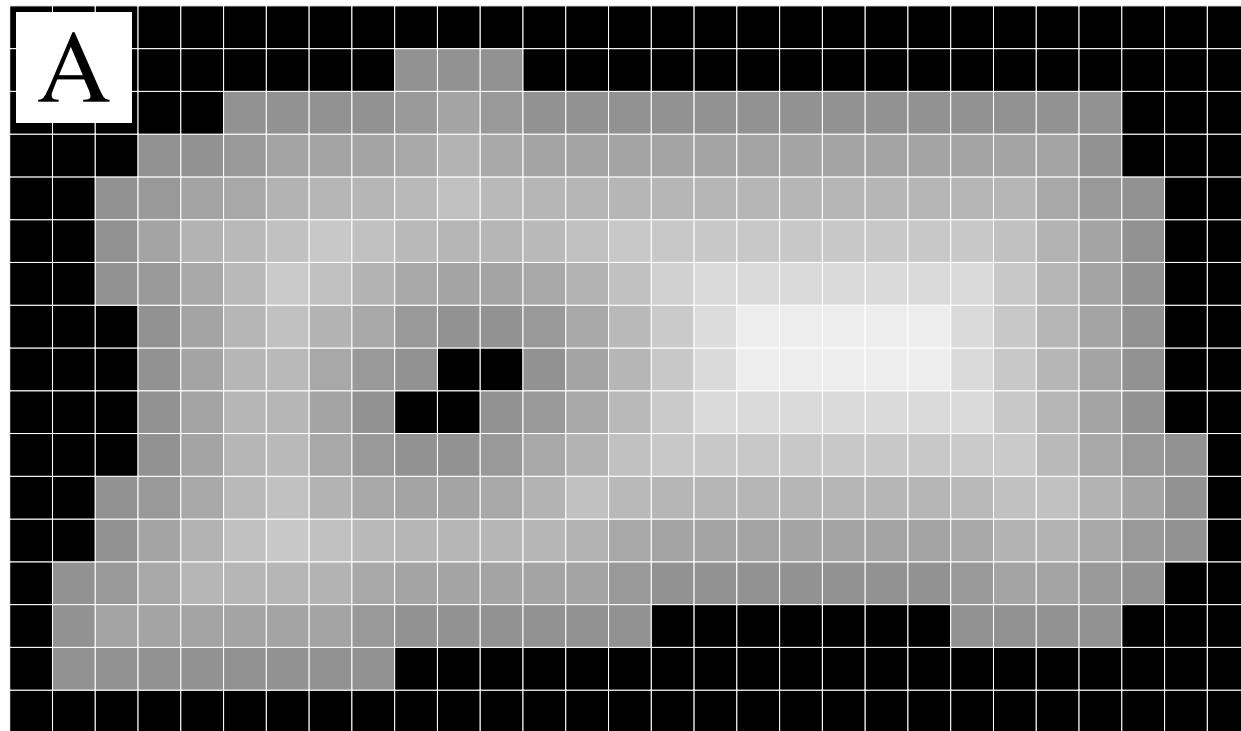
# Incremental Euclidean Distance Mapping



# Incremental Euclidean Distance Mapping



# Incremental Euclidean Distance Mapping





# Truncated signed distance map

We can easily modify this algorithm to tell us distance  
inside an object.

Signed distance - negative inside object, positive outside

Signed distance important in motion planning!