Signed Distance Function Representation, Tracking, and Mapping

Tanner Schmidt

Overview

- Explicit and implicit surface representations
- SDF fusion
- SDF tracking
- SDF limitations
- Related research
  - KinectFusion
  - KinTinuous
  - BundleFusion
  - DART
  - DynamicFusion

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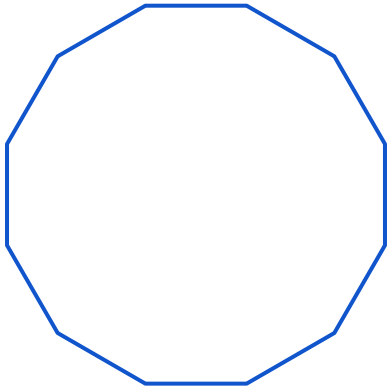
Explicit Surface Representations

- Geometry is stored **explicitly** as a list of points, triangles, or other geometric fragments
  - e.g. meshes, point clouds

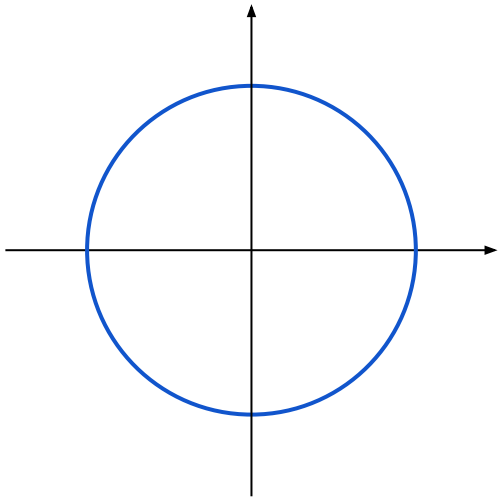


Vertices: [(x0, y0, z0), (x1, y1, z1), …, (xn, yn, zn)]

Indices: [(i0, i1), (i2, i3), …, (in-1, in)]

Implicit Surface Representation

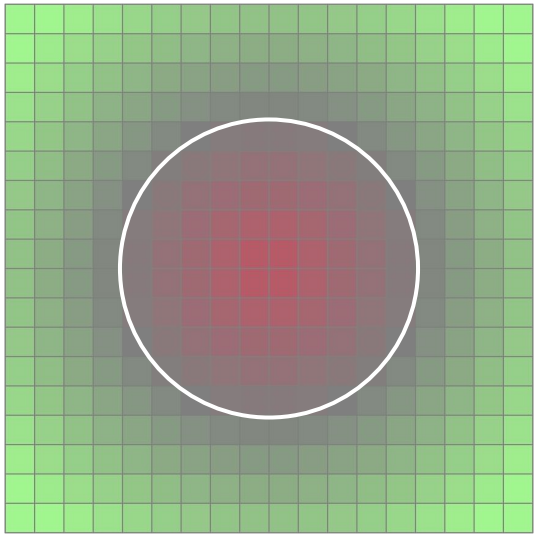
- Geometry is not stored explicitly but rather defined as a level set of a function defined over the space in which the geometry is embedded
  - There are **parametric** representations:



$$f(x, y) = x^2 + y^2 - r^2$$

Implicit Surface Representation

- Geometry is not stored explicitly but rather defined as a level set of a function defined over the space in which the geometry is embedded
  - And there are **nonparametric** representations:



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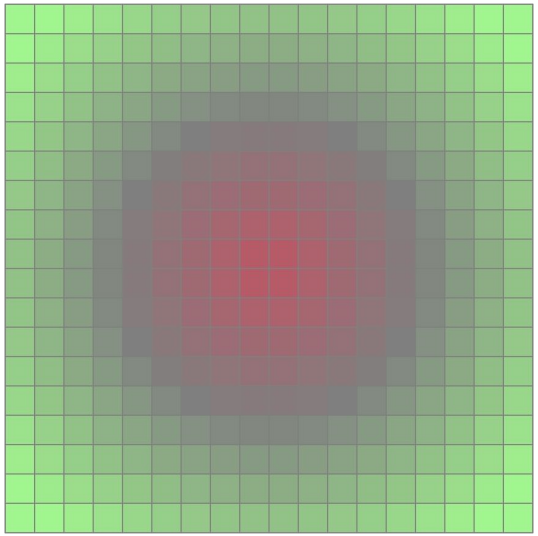
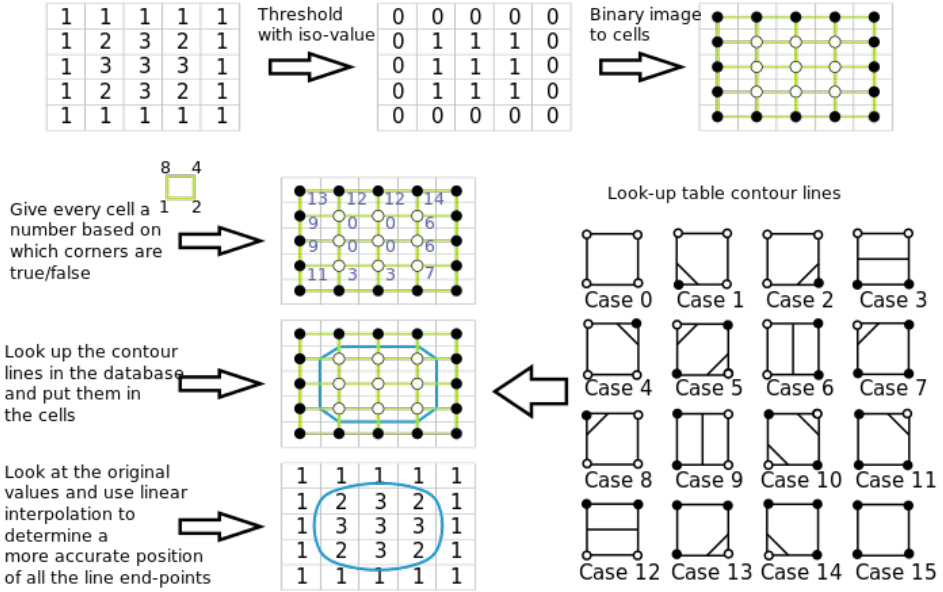


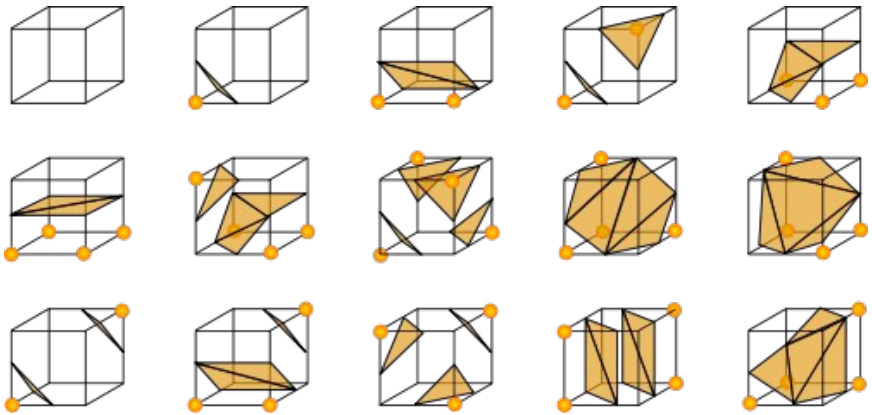
Implicit to Explicit Conversion

- In two dimensions, we can use an algorithm called **marching squares**



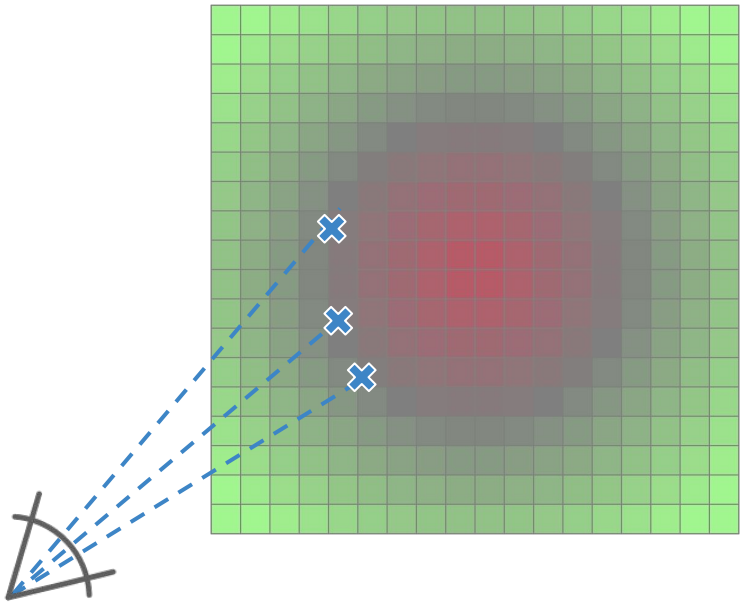
Implicit to Explicit Conversion in 3D

- Typically done using **marching cubes**, a 3D analogue to marching squares



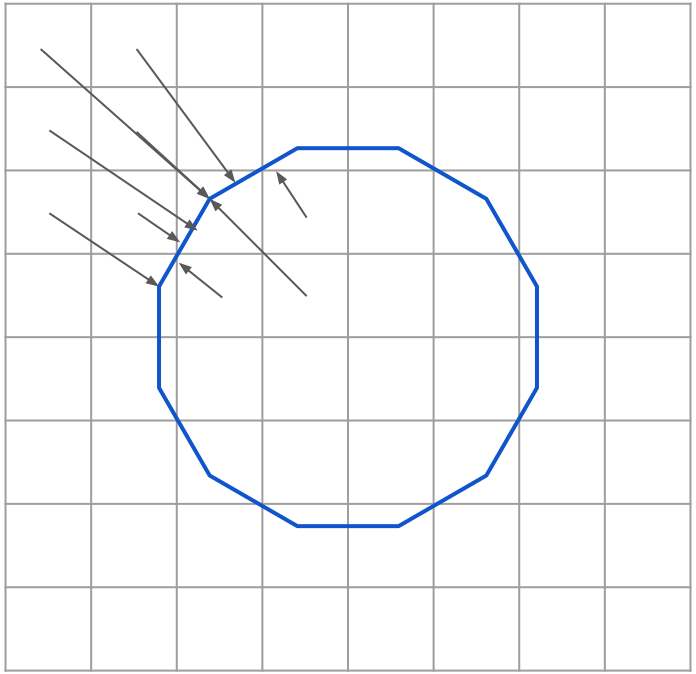
Implicit to Explicit Conversion in 3D

- Can also be done by **raycasting** for a view-dependent partial surface



Explicit to Implicit Conversion

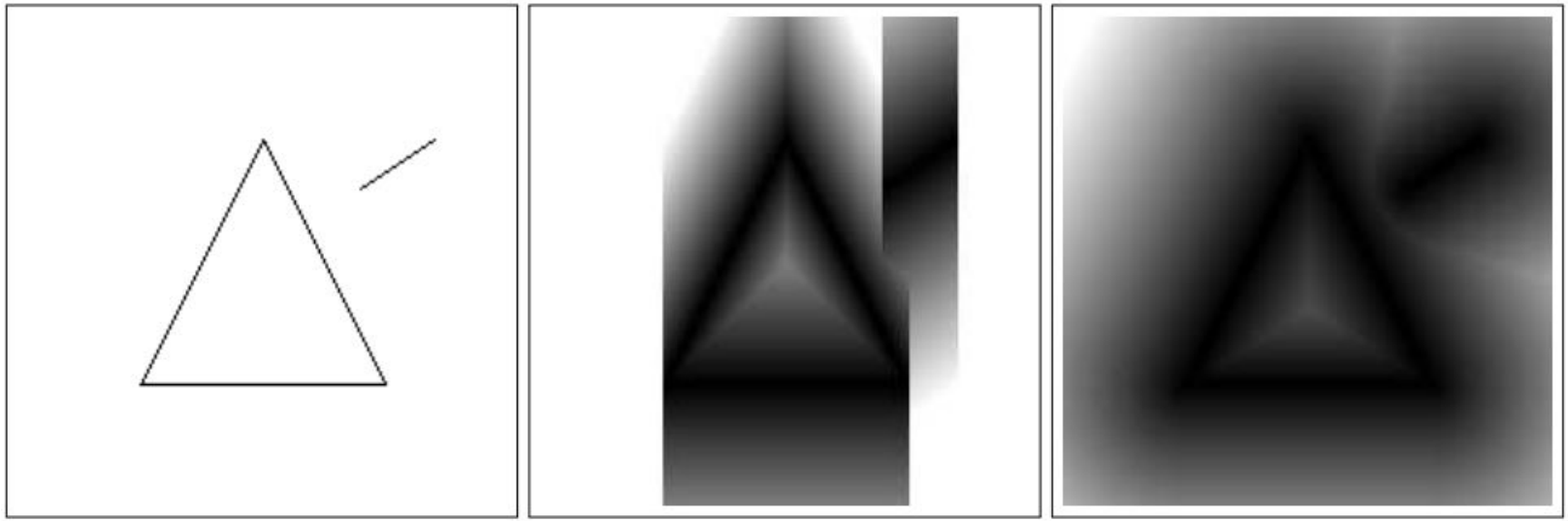
- Can be done by finding the closest point between each discrete location and any part of the geometry



Explicit to Implicit Conversion

- Can also be done with a **distance transform**

What if we want to build surface representations from raw (noisy) observations?



$f(x, y)$

$$\mathcal{D}(f(x,y)) = \min_{x',y'} f(x',y') + (x-x')^2 + (y-y')^2$$

$$\mathcal{D}(f(x,y)) = \min_{x'}(x-x') + \min_{y'} f(x',y') + (y-y')^2$$

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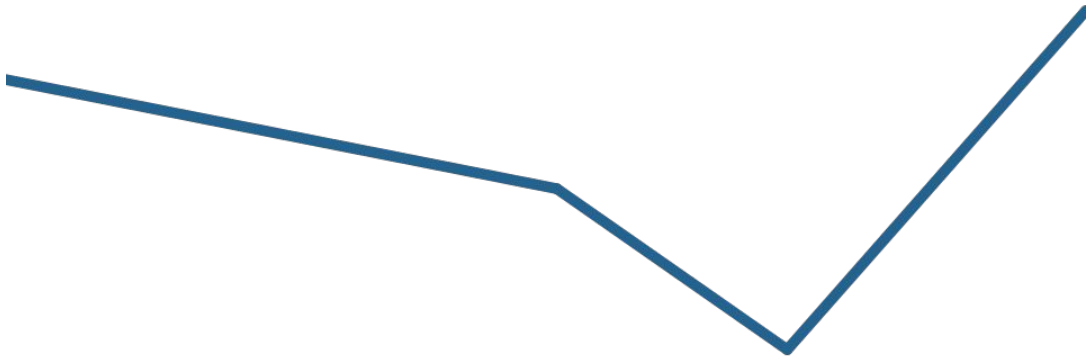
Signed Distance Function Fusion

$$
\begin{aligned}
\mathrm{F}_{\mathrm{R}_k}(\mathbf{p}) &= \Psi\left(\lambda^{-1}\|(\mathbf{t}_{g,k} - \mathbf{p}\|_2 - \mathrm{R}_k(\mathbf{x})\right) , \\
\lambda &= \|\mathrm{K}^{-1}\dot{\mathbf{x}}\|_2 , \\
\mathbf{x} &= \left\lfloor \pi\left(\mathrm{K}\mathrm{T}_{g,k}^{-1}\mathbf{p}\right)\right\rfloor , \\
\Psi(\eta) &= \begin{cases} \min\left(1, \frac{\eta}{\mu}\right)\mathrm{sgn}(\eta) & \text{iff } \eta \geq -\mu \\ \mathit{null} & \mathit{otherwise} \end{cases}
\end{aligned}
$$

Signed Distance Function Fusion



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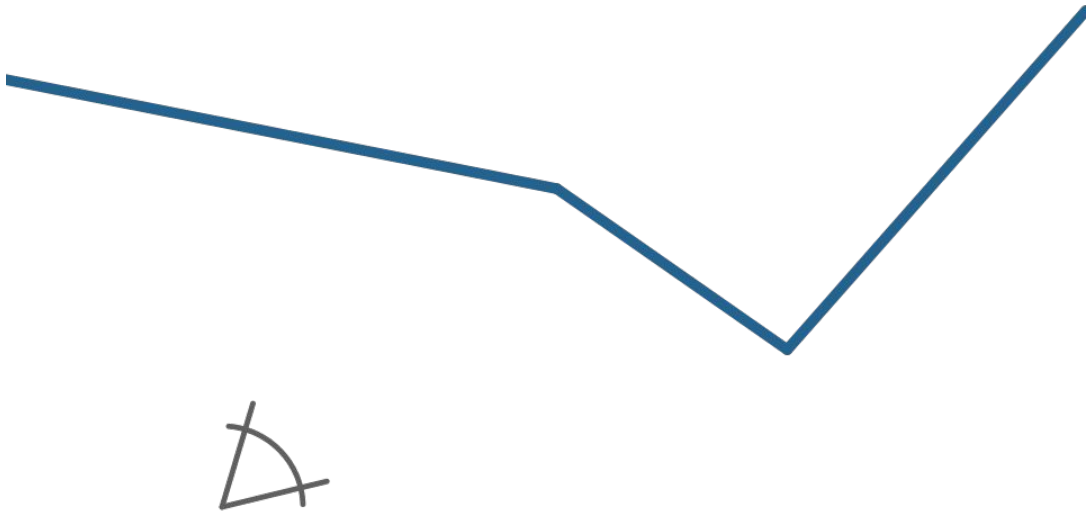
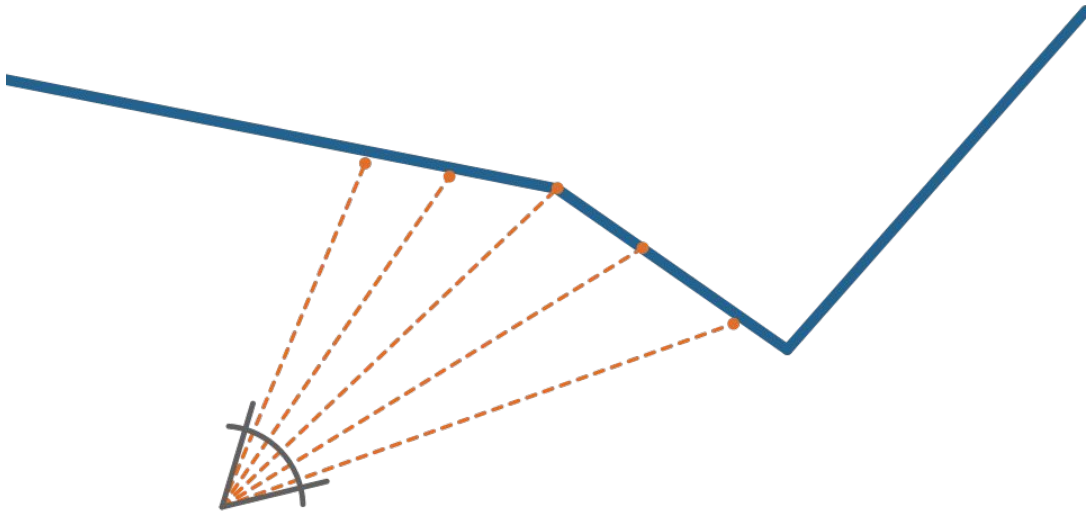


1.356	1.384	1.422	1.511	1.607

Signed Distance Function Fusion



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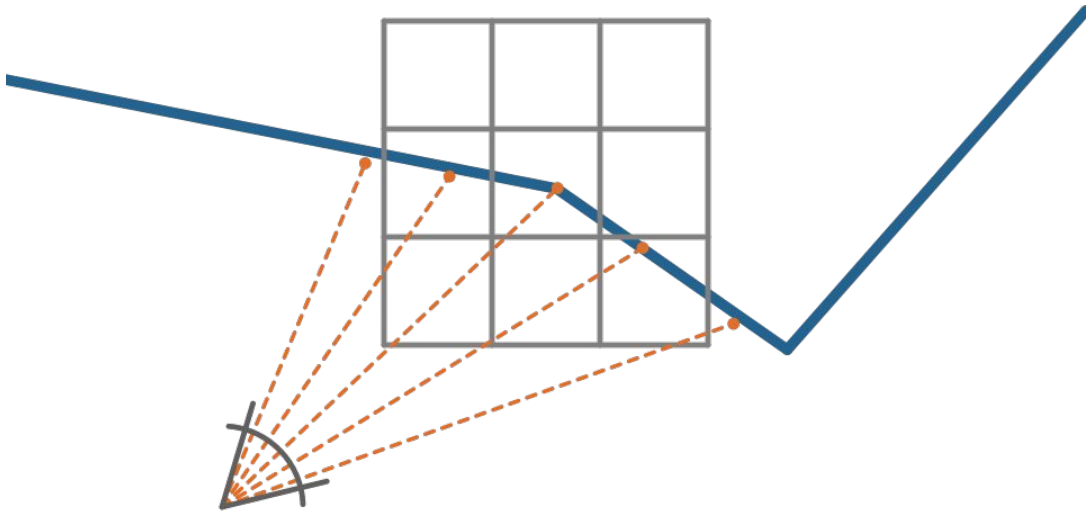

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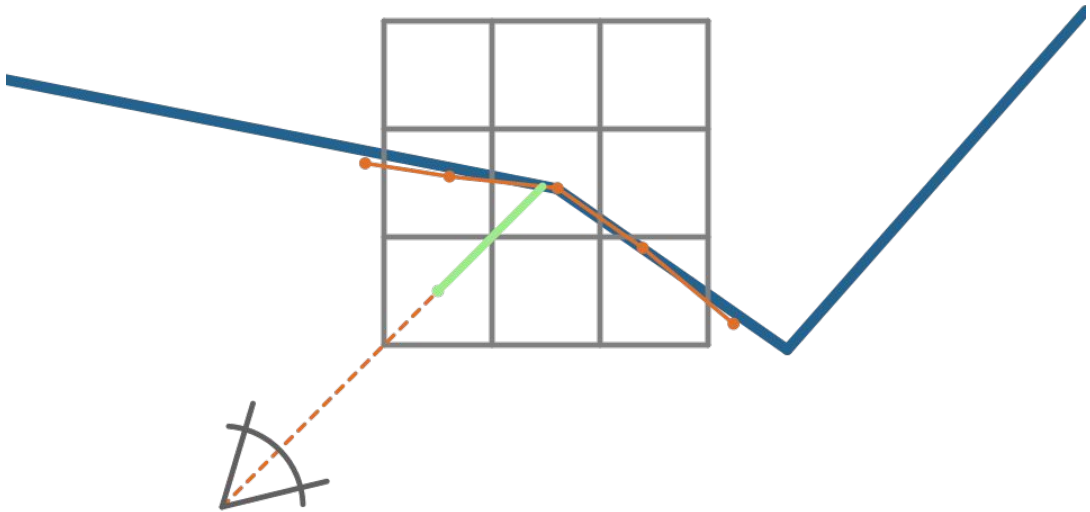

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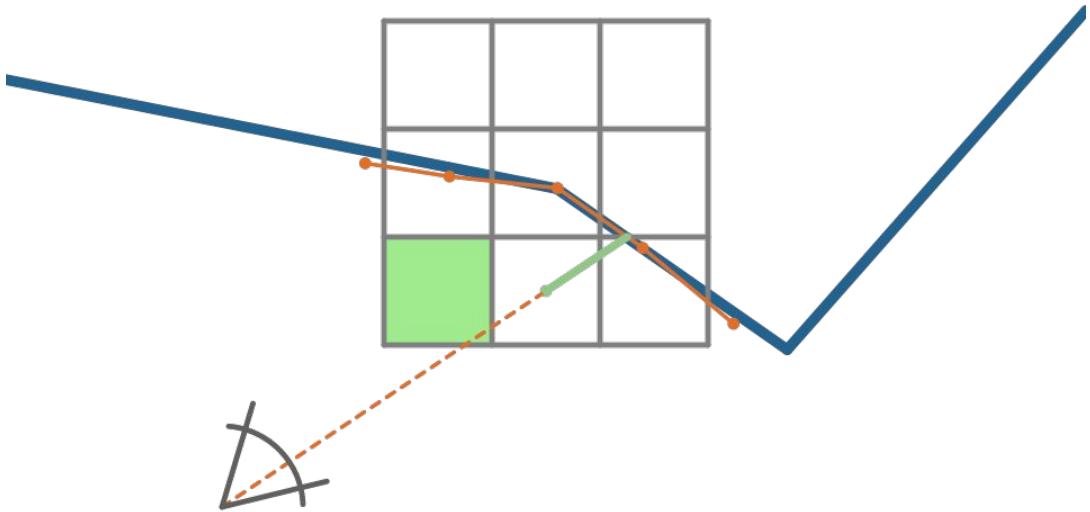

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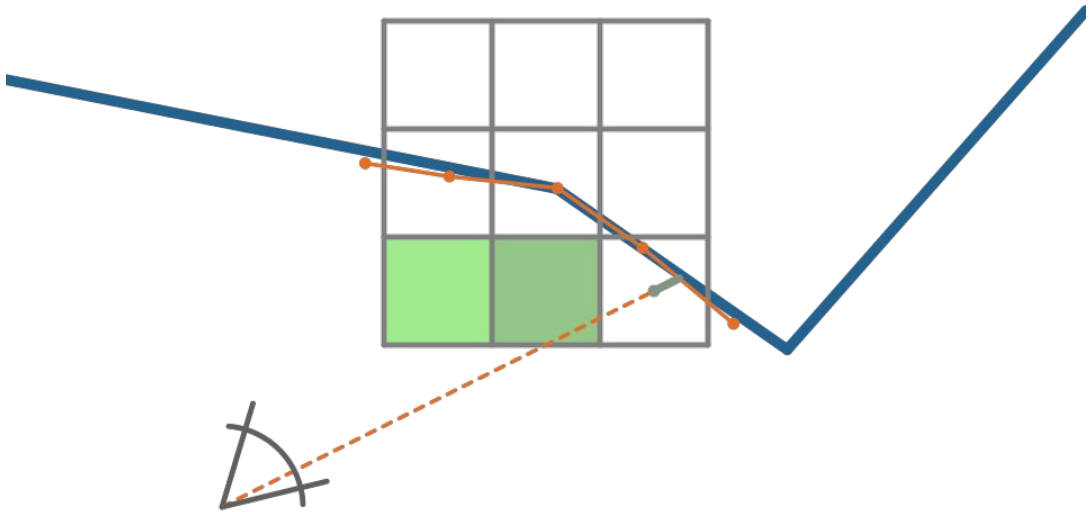

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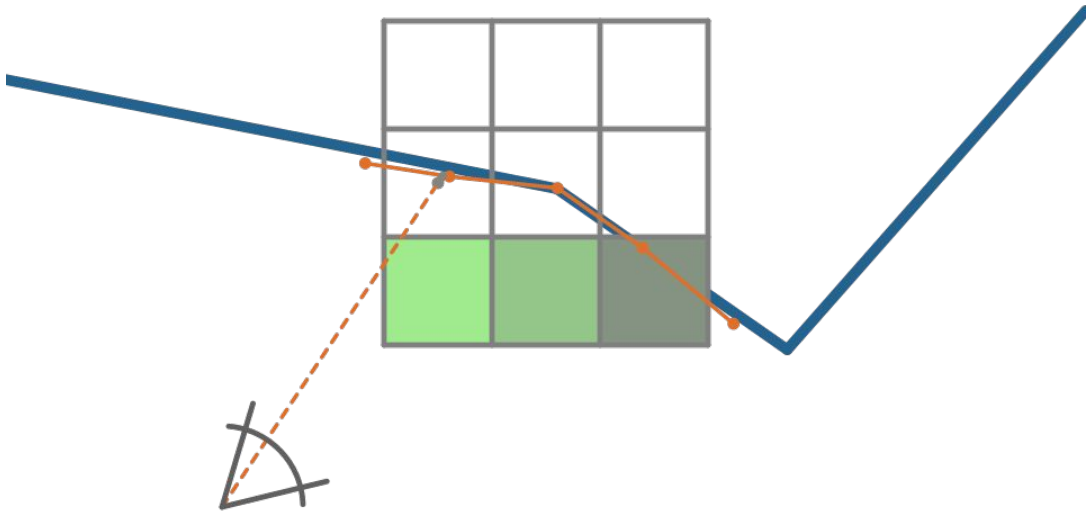

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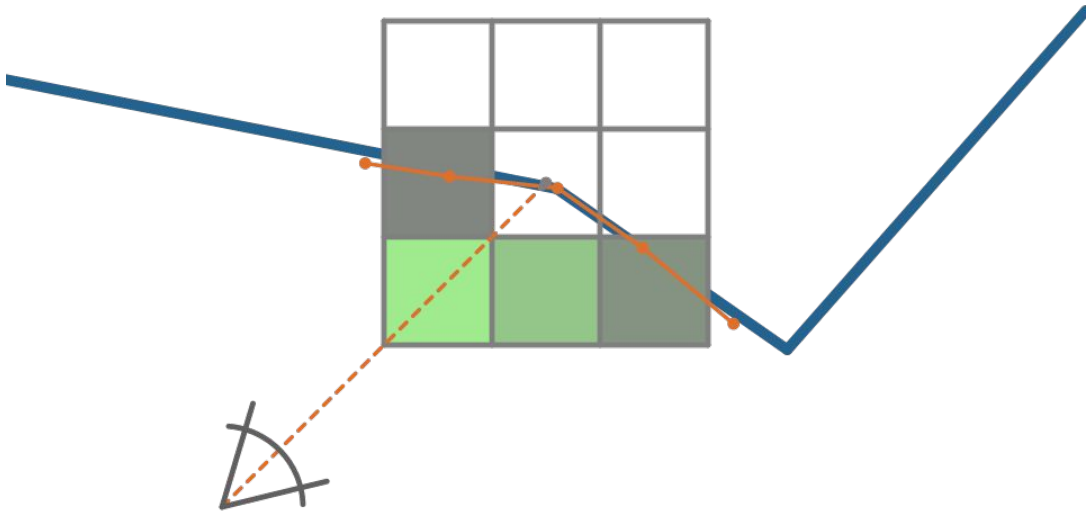

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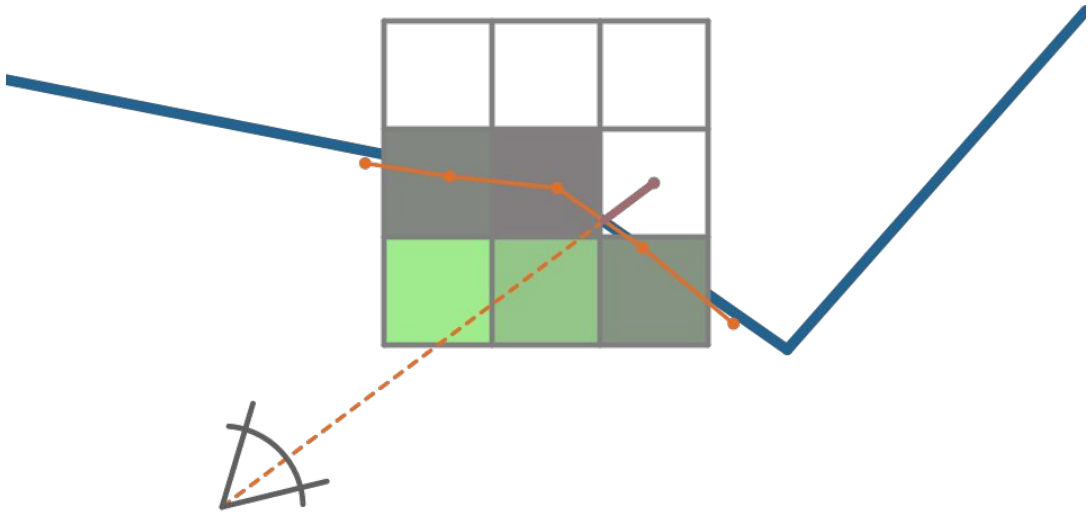

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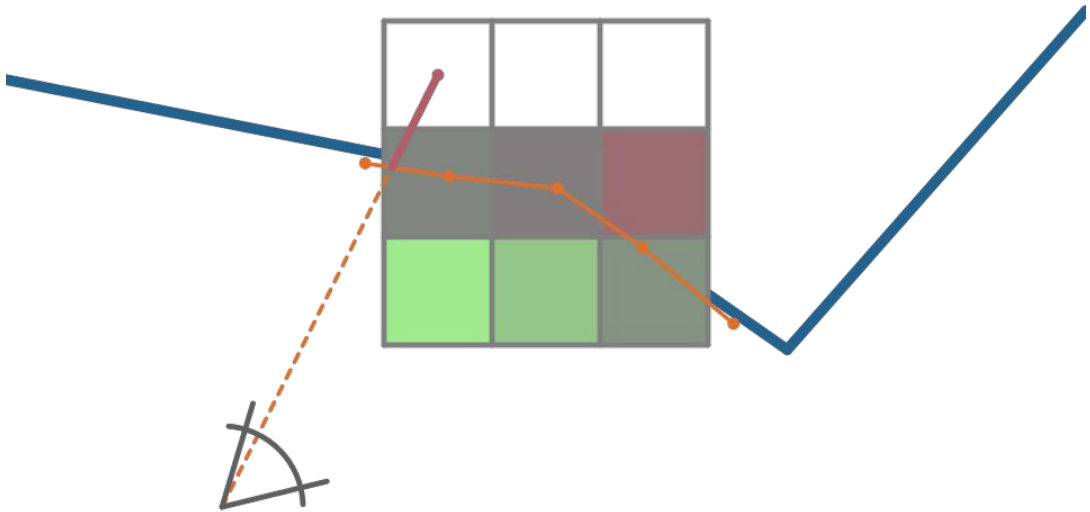

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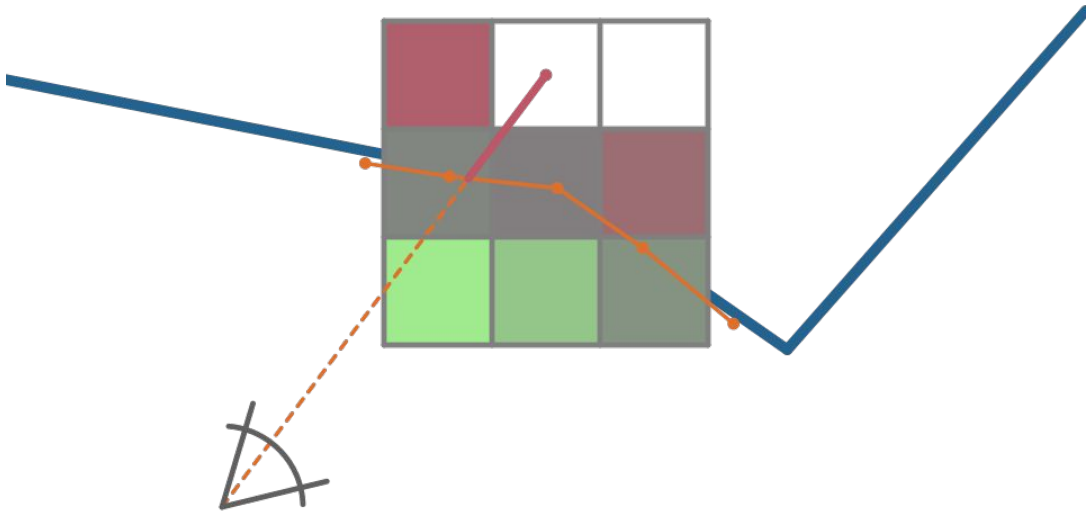

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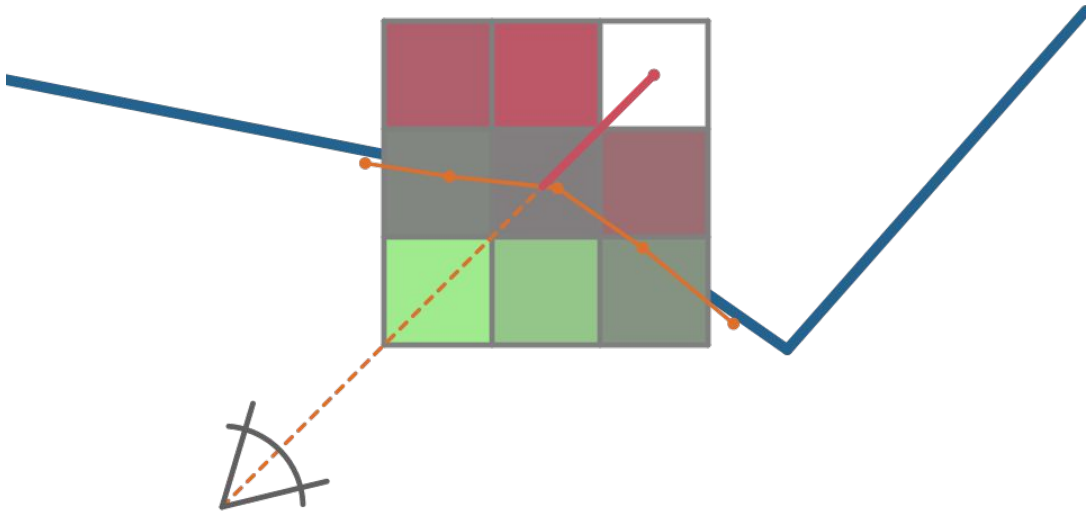

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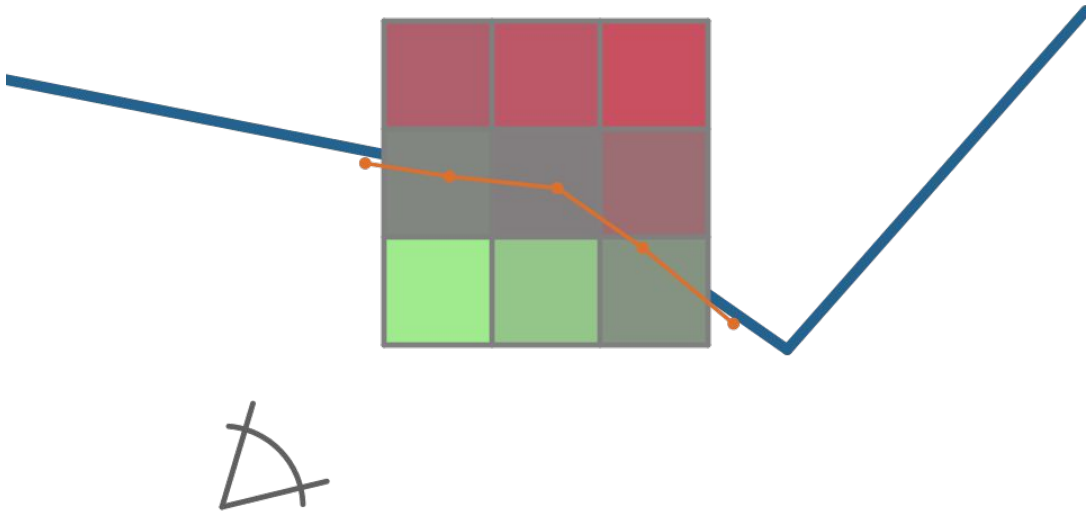

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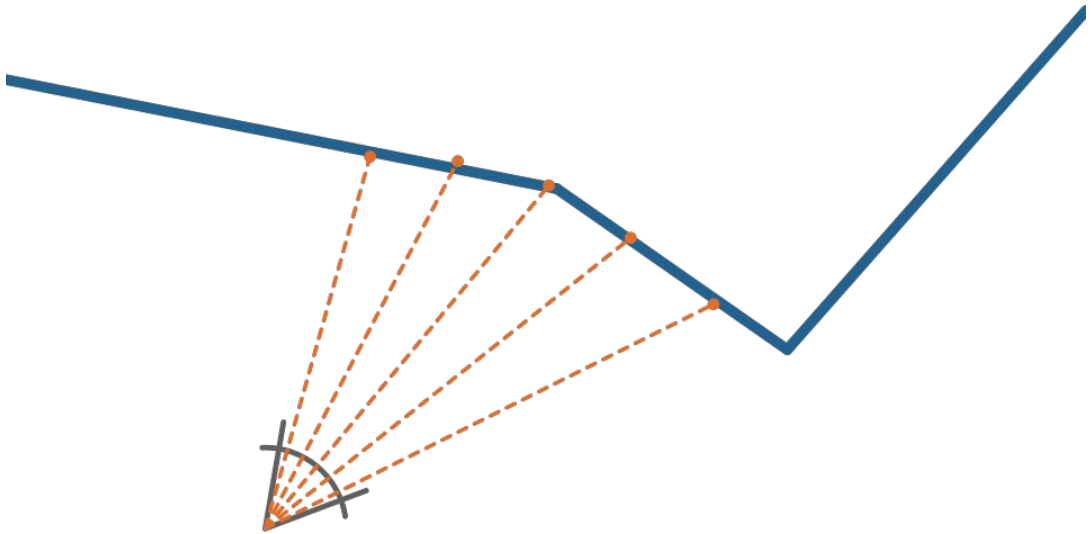

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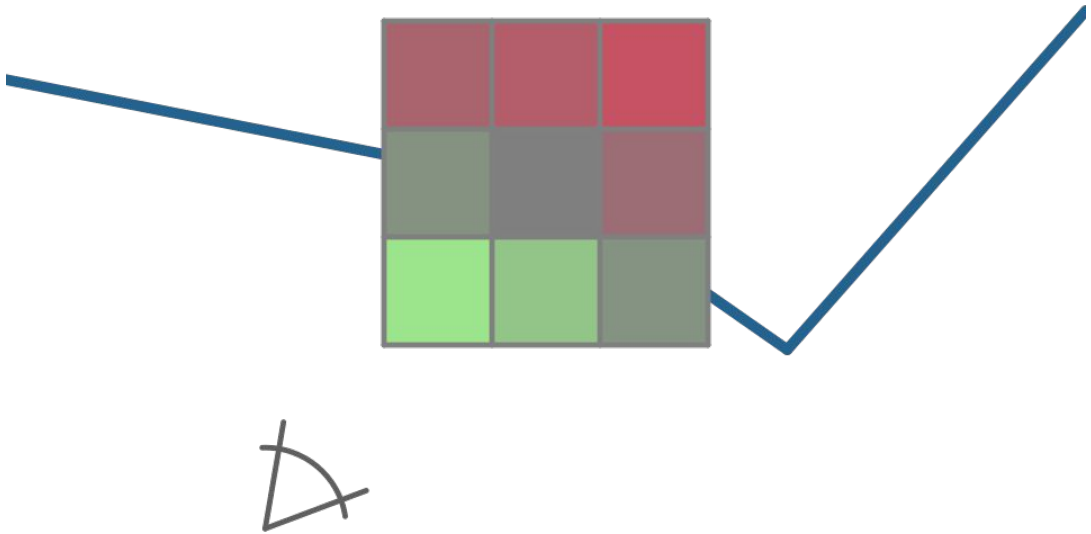

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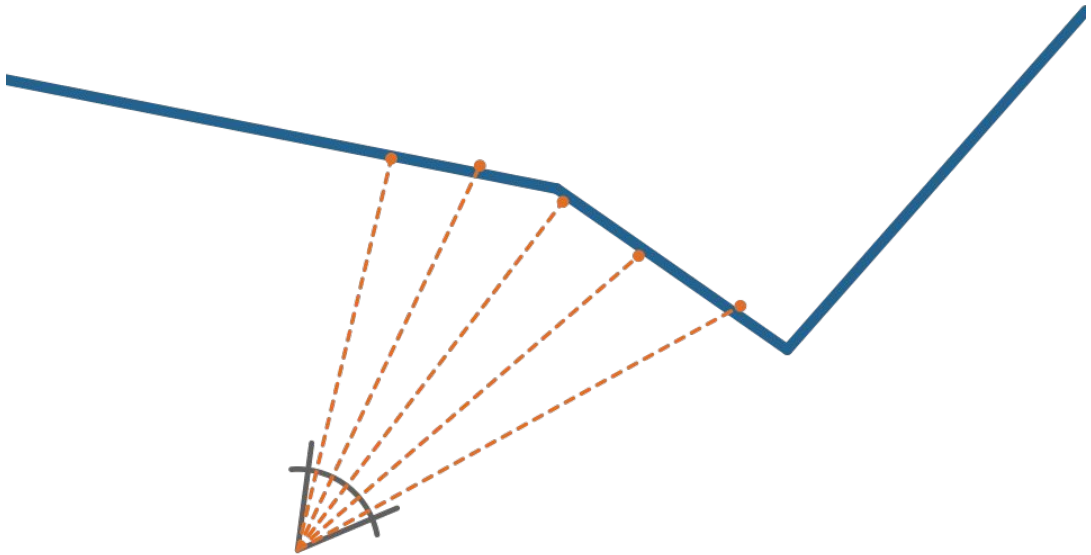

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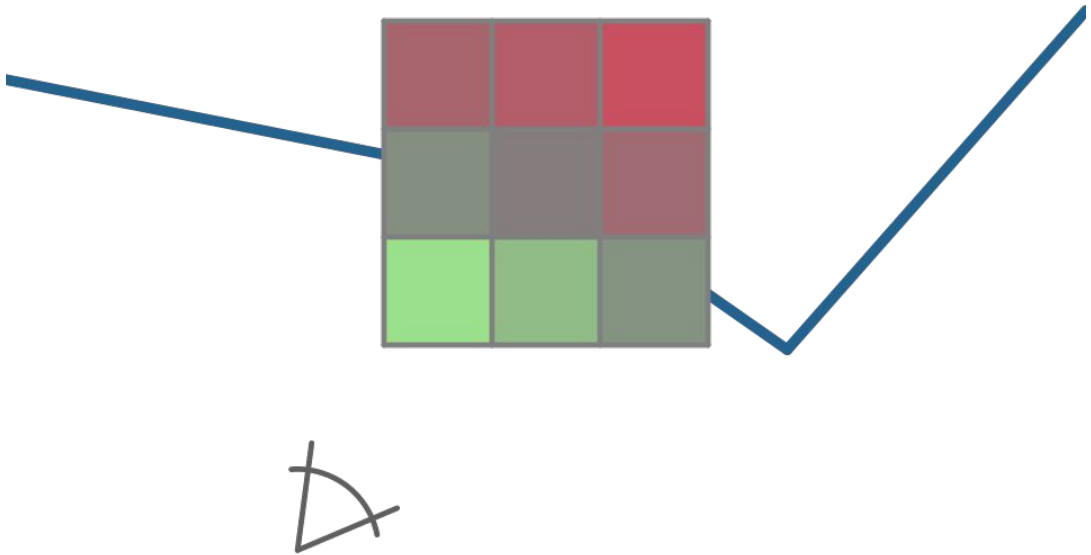

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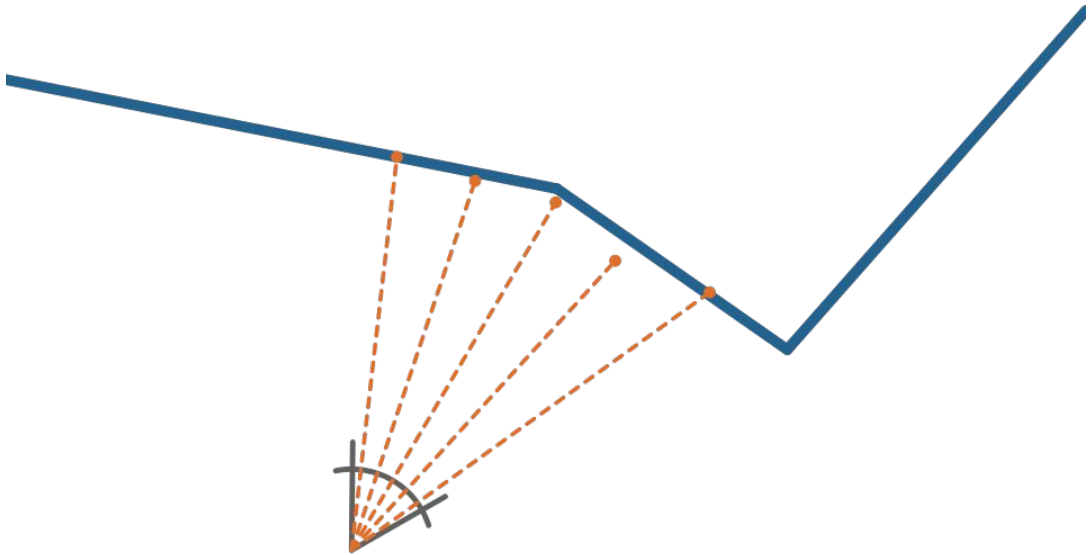

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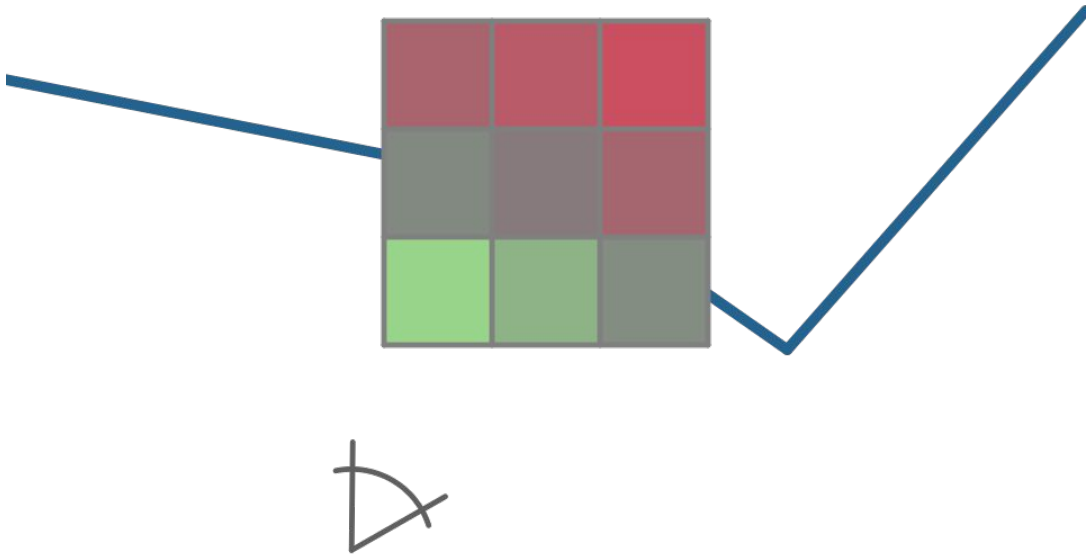

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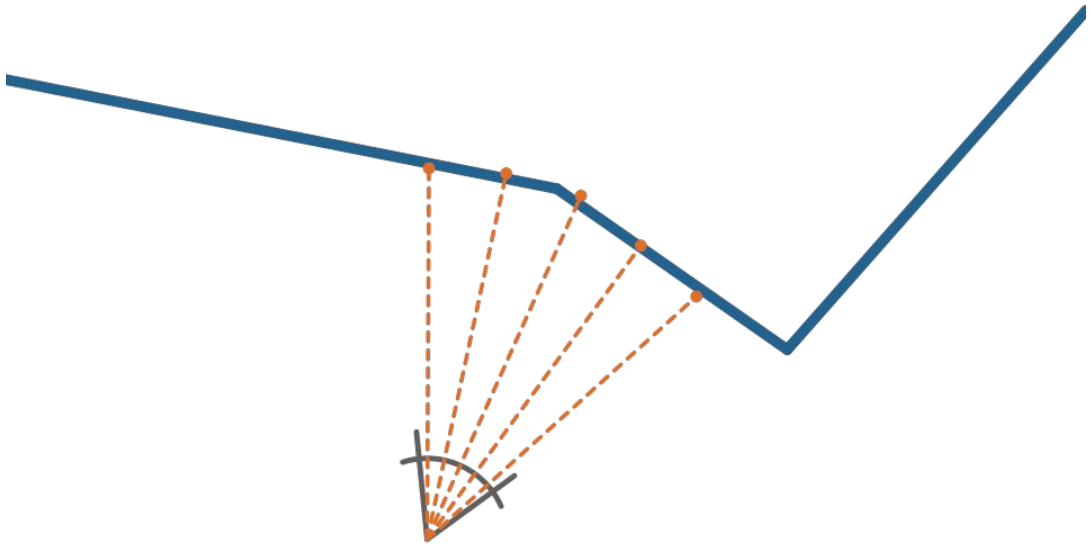

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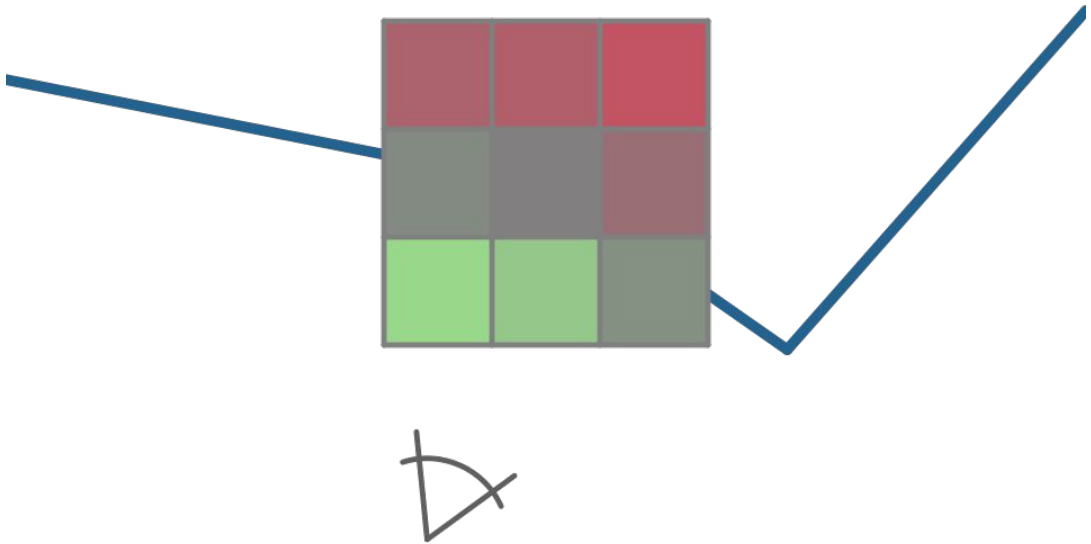

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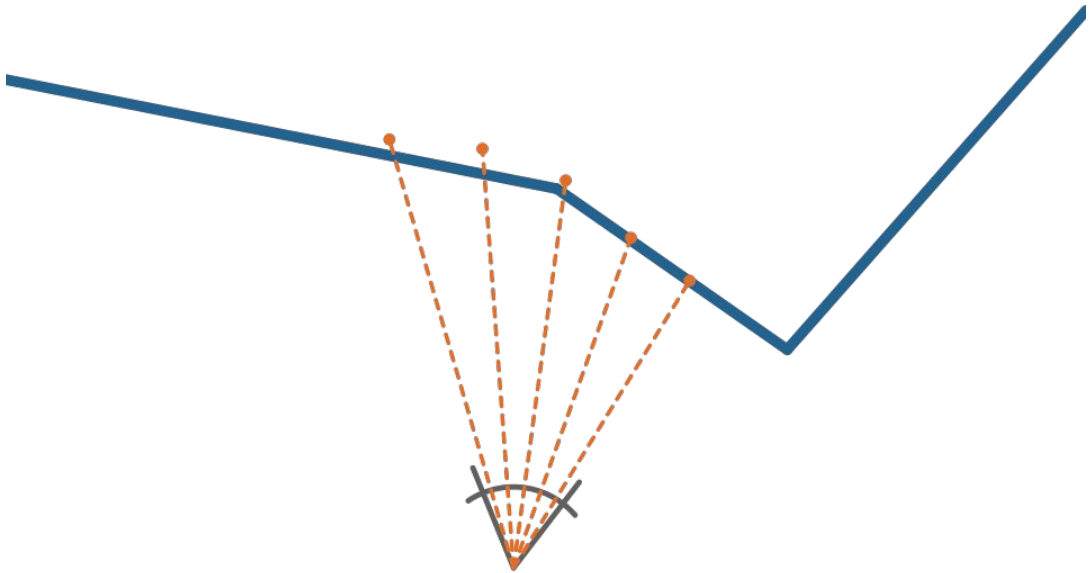

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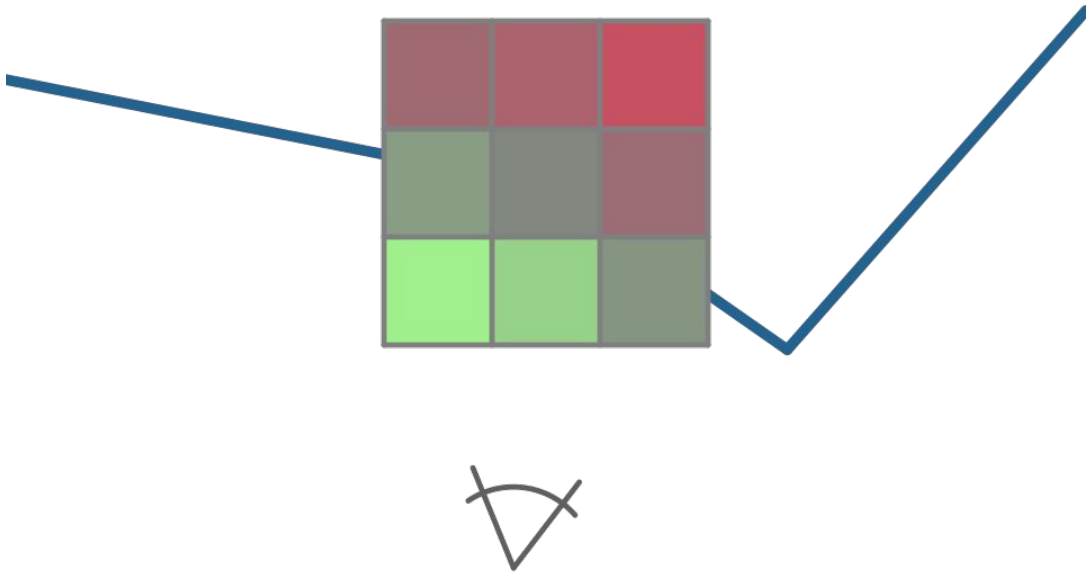

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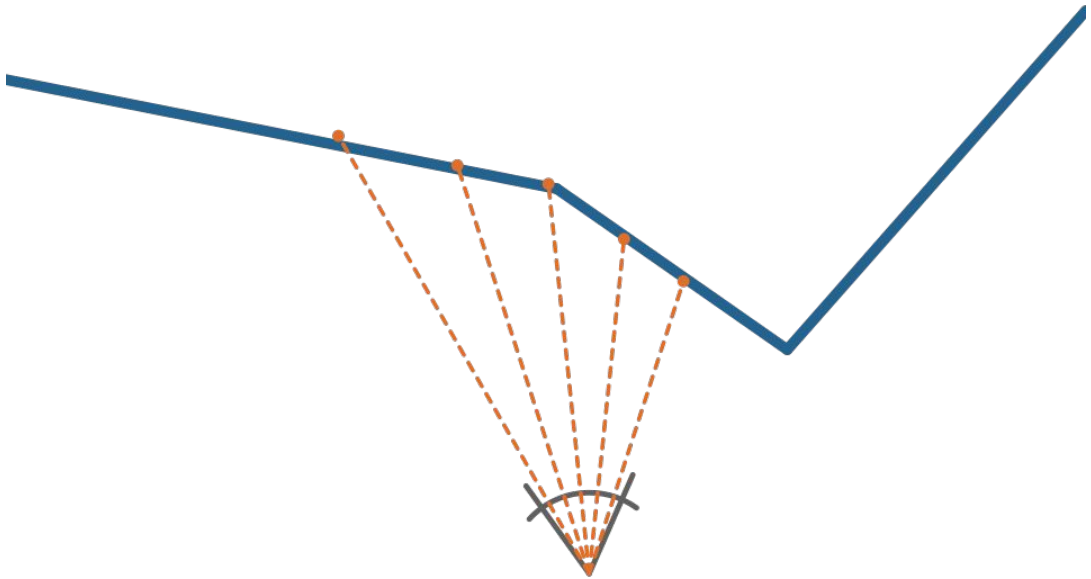

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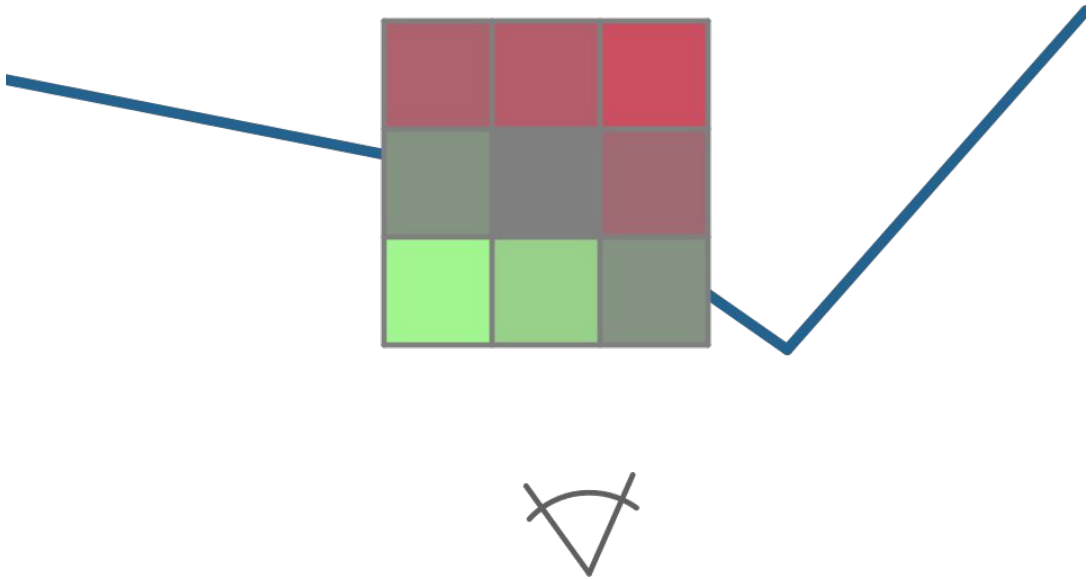

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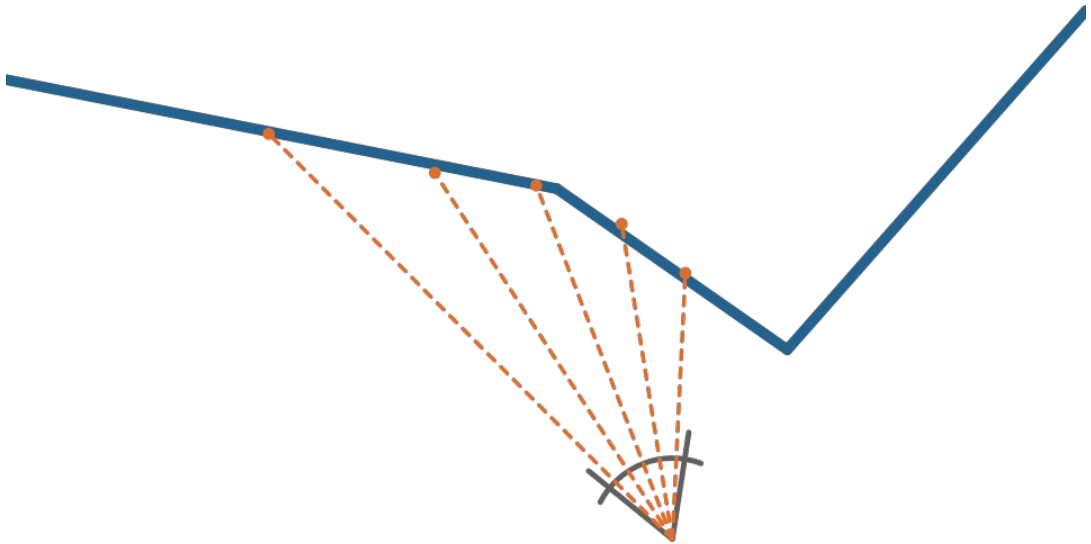

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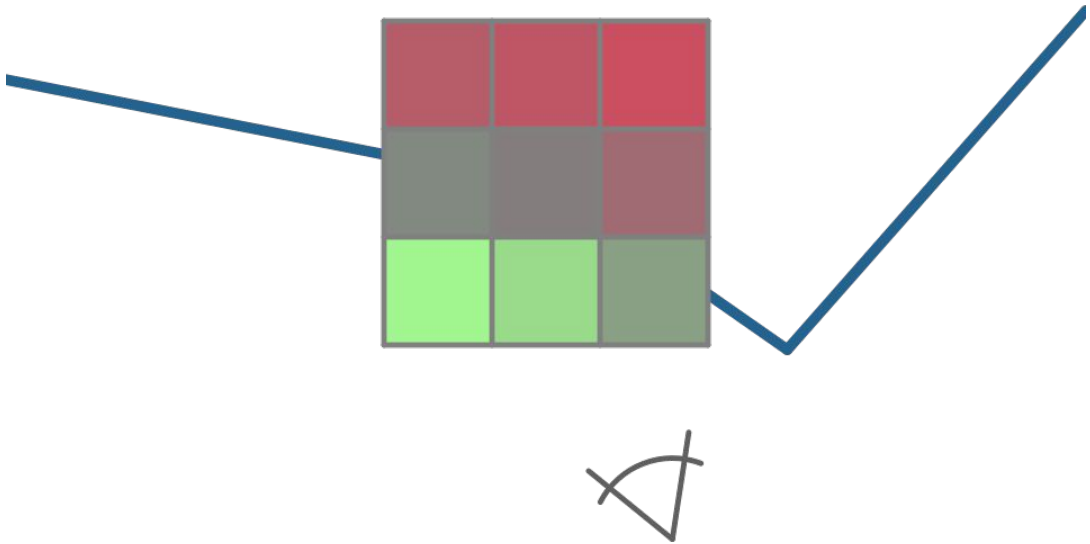

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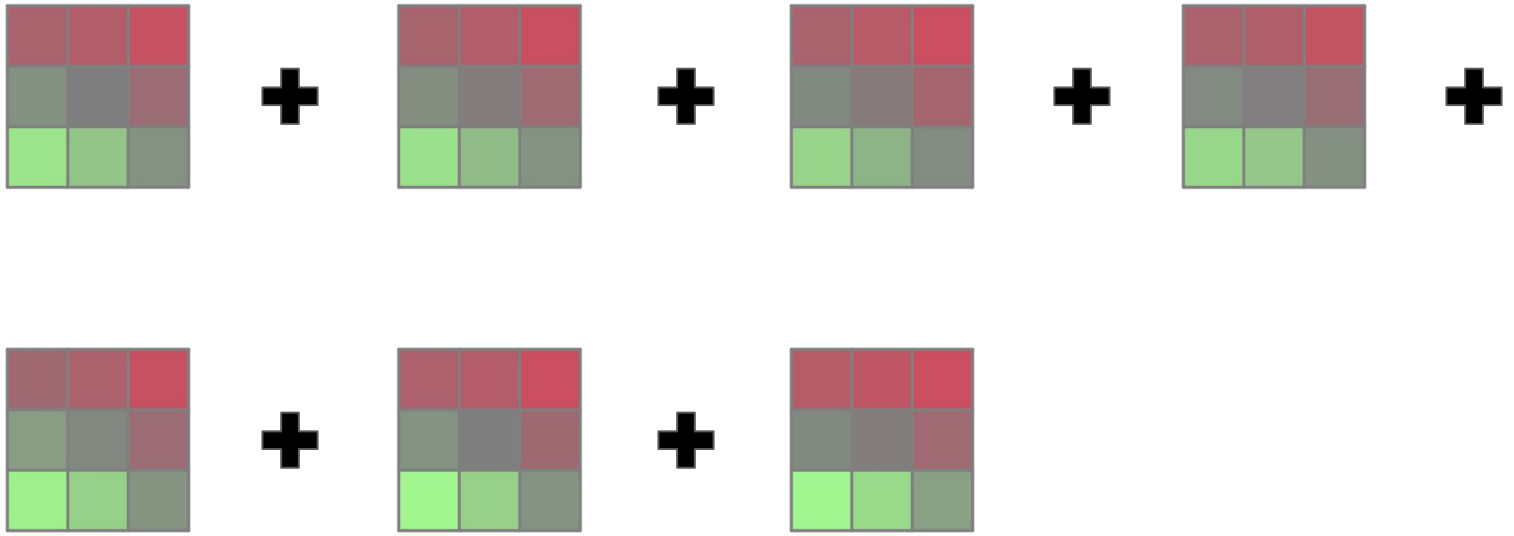

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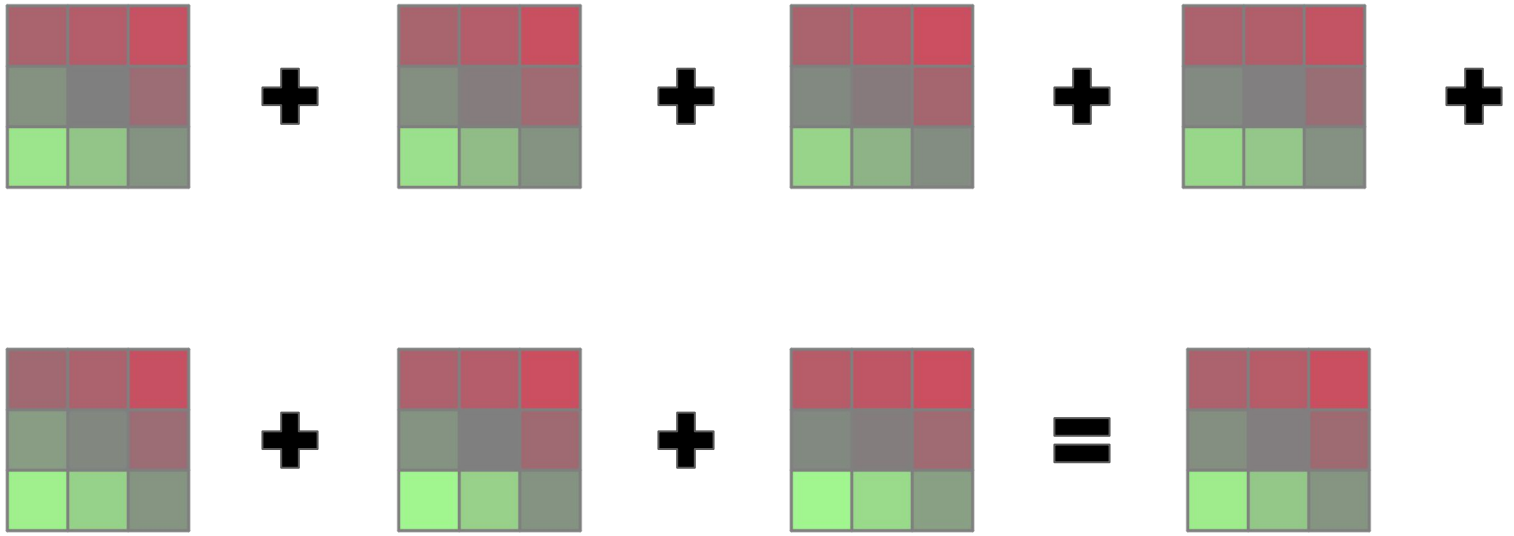

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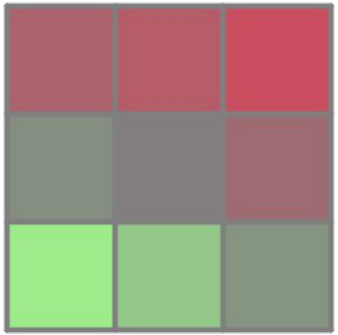

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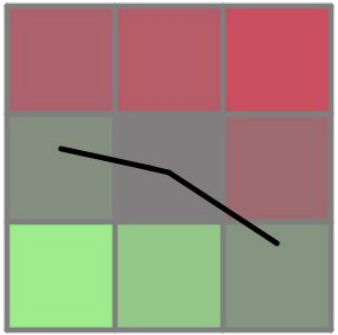

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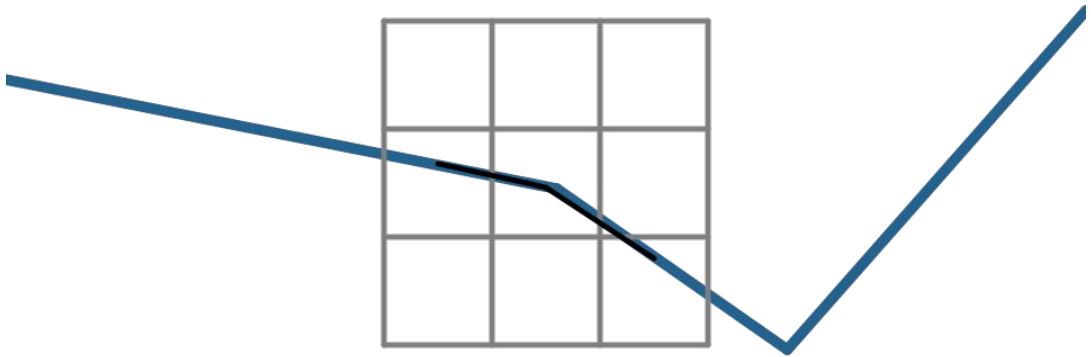
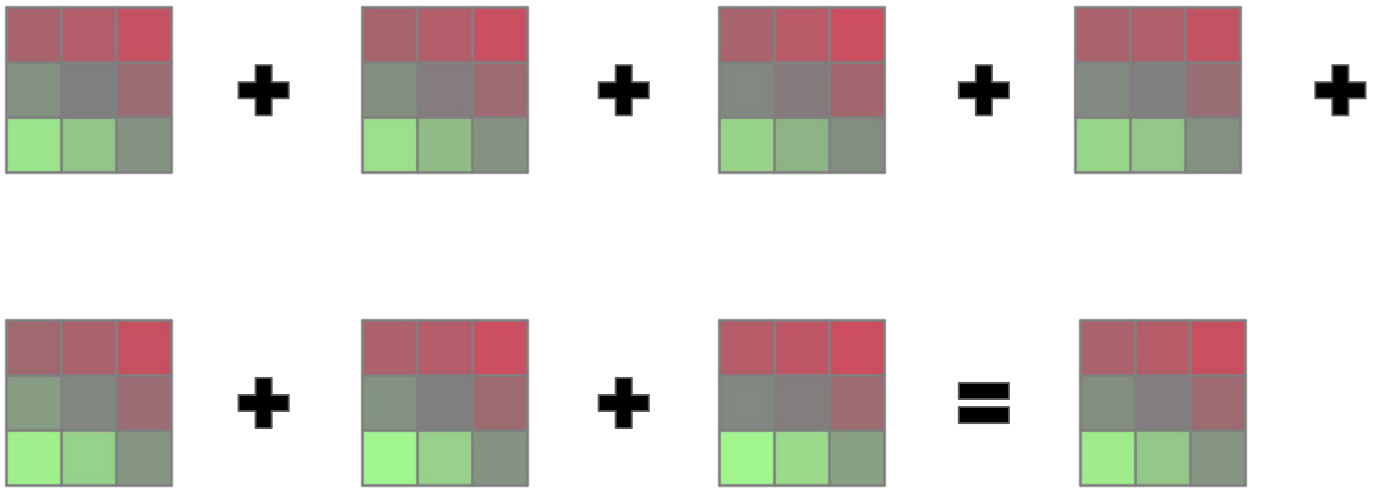


Signed Distance Function Fusion

- This addition requires the per-frame projected truncated signed distance volumes to be globally registered



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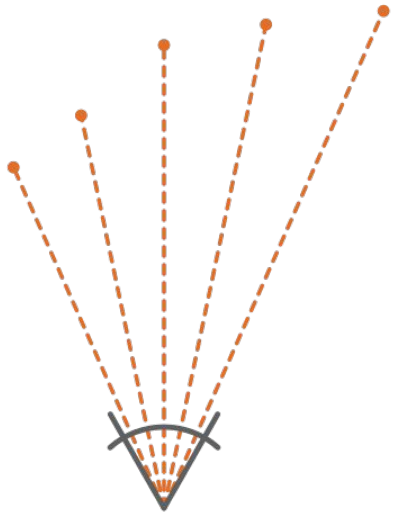
Signed Distance Function Tracking



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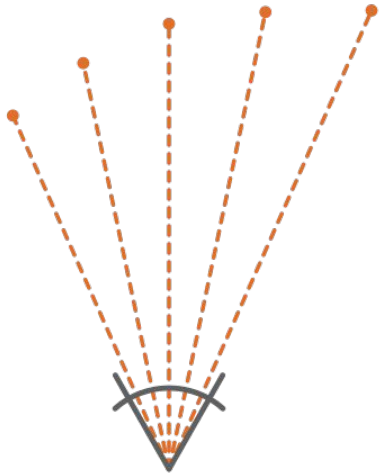

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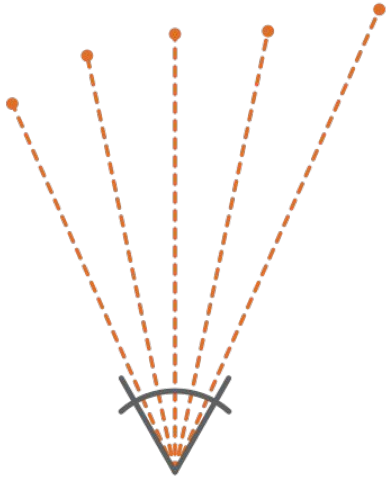

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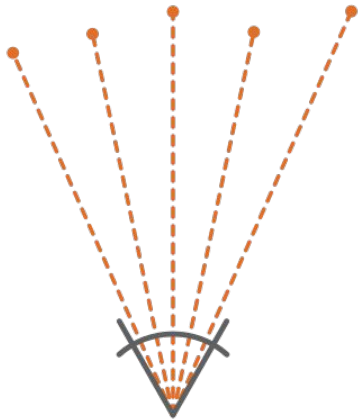

Signed Distance Function Tracking

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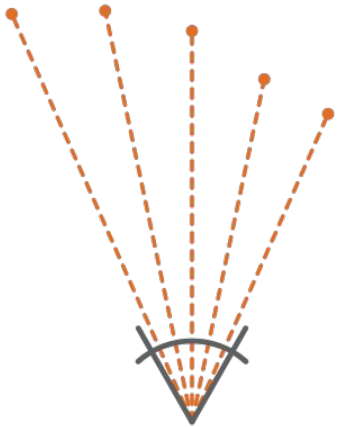

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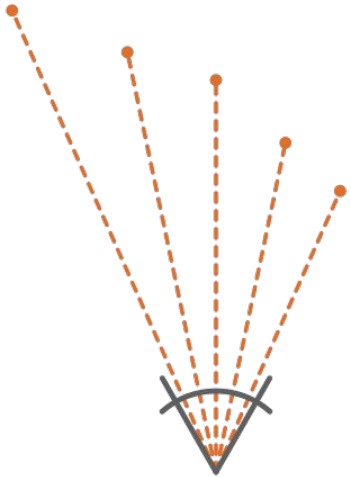

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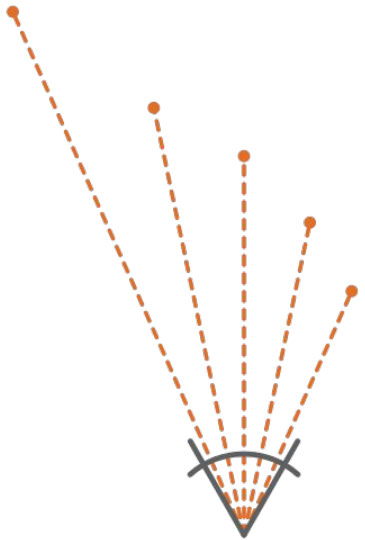
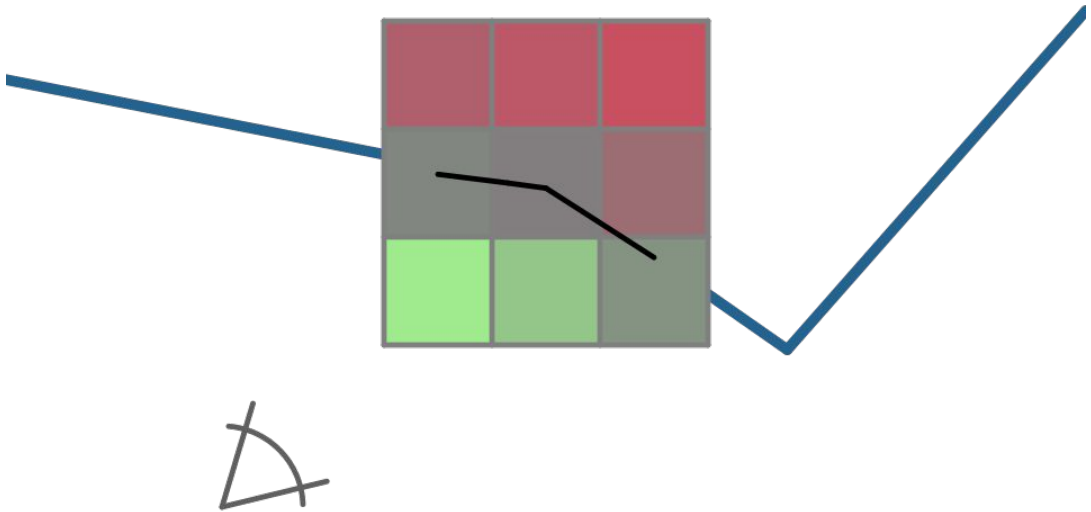


Point-plane Iterative Closest Point (ICP)



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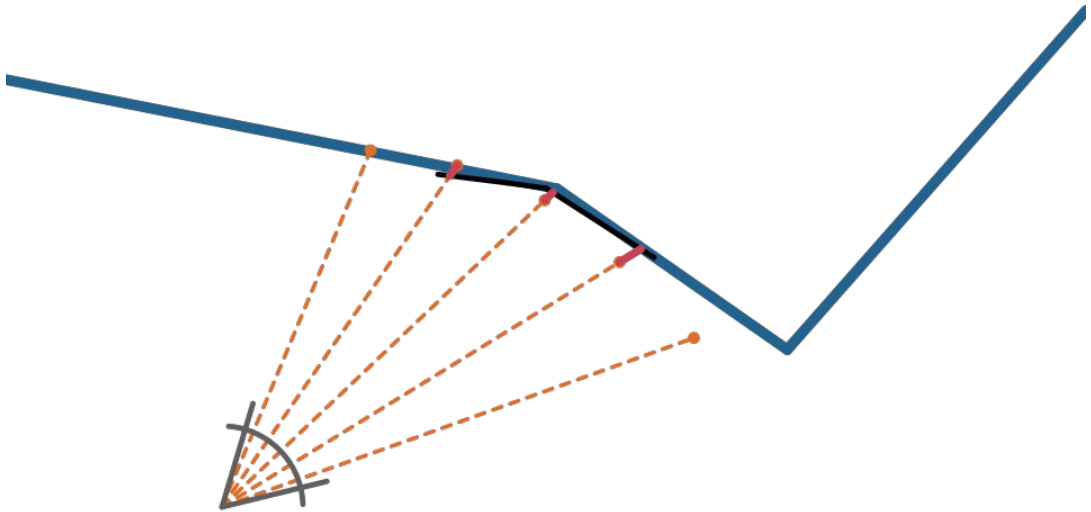
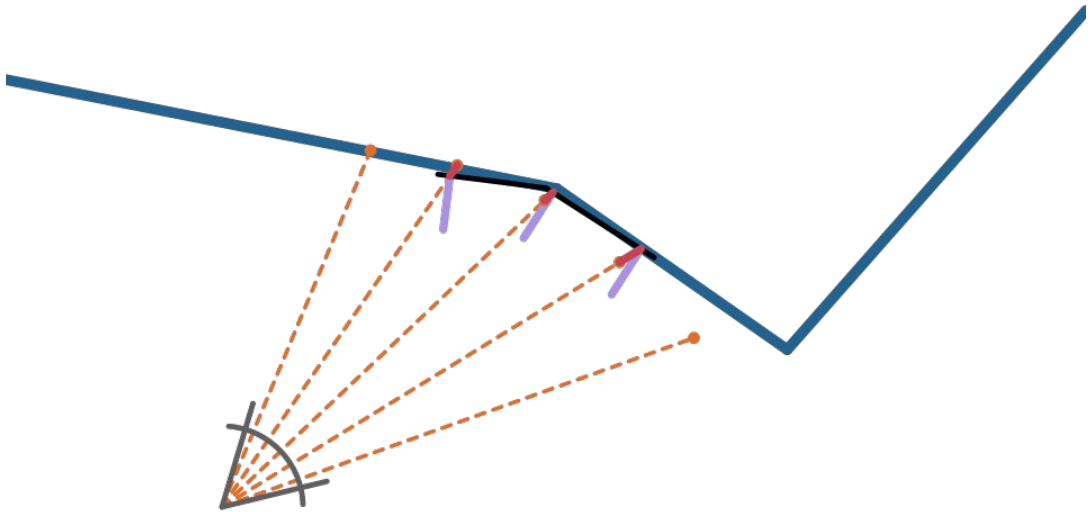


Point-plane Iterative Closest Point (ICP)



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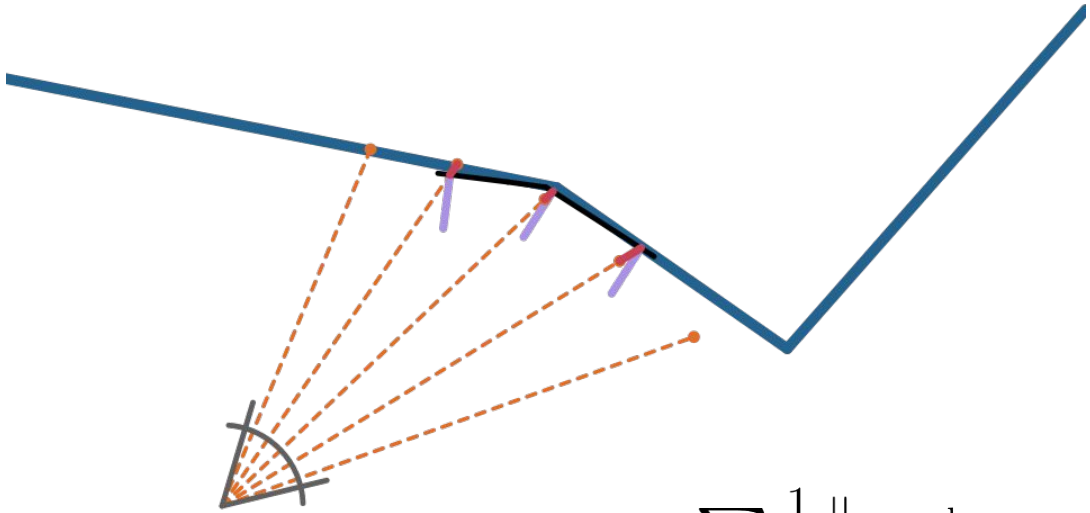
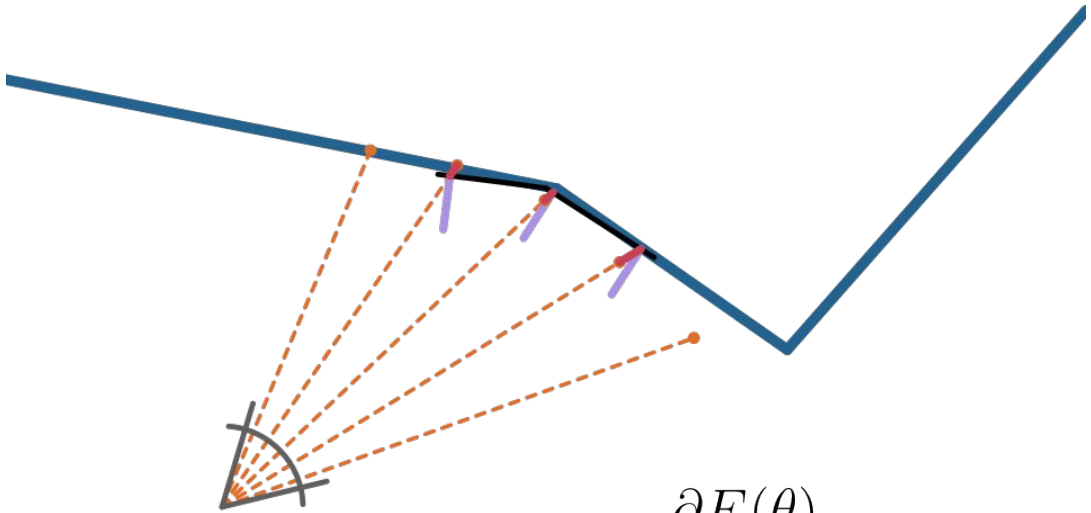


$$E(\theta) = \sum_i \frac{1}{2} \left\| n_i^{\text{pred}} \cdot (T_{gl}(\theta) x_i^{\text{obs}} - x_i^{\text{pred}}) \right\|_2^2$$

Point-plane Iterative Closest Point (ICP)



$$\frac{\partial E(\theta)}{\partial \theta} = \sum_i n_i^{\text{pred}} \cdot \frac{\partial T_{gl}(\theta) x_i^{\text{obs}}}{\partial \theta}$$

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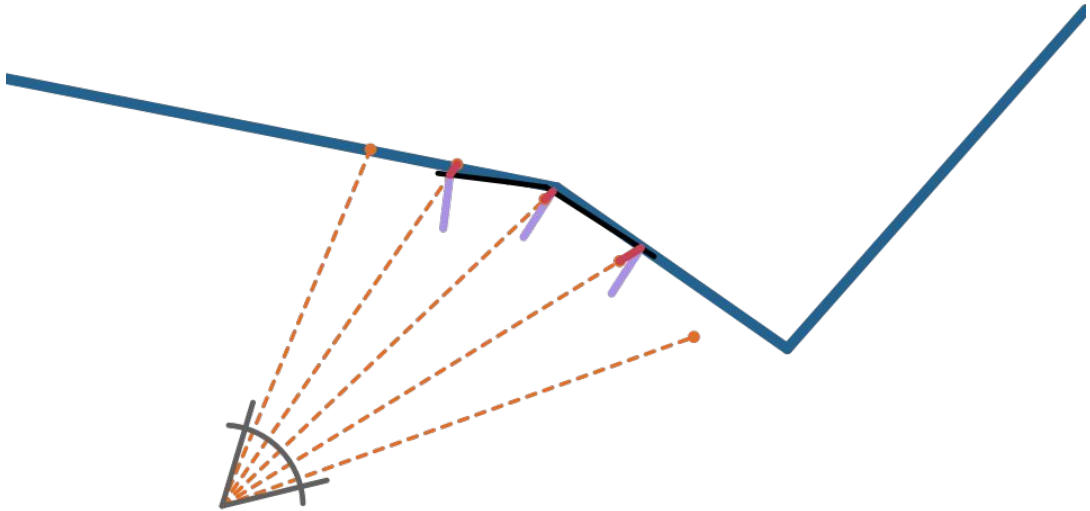


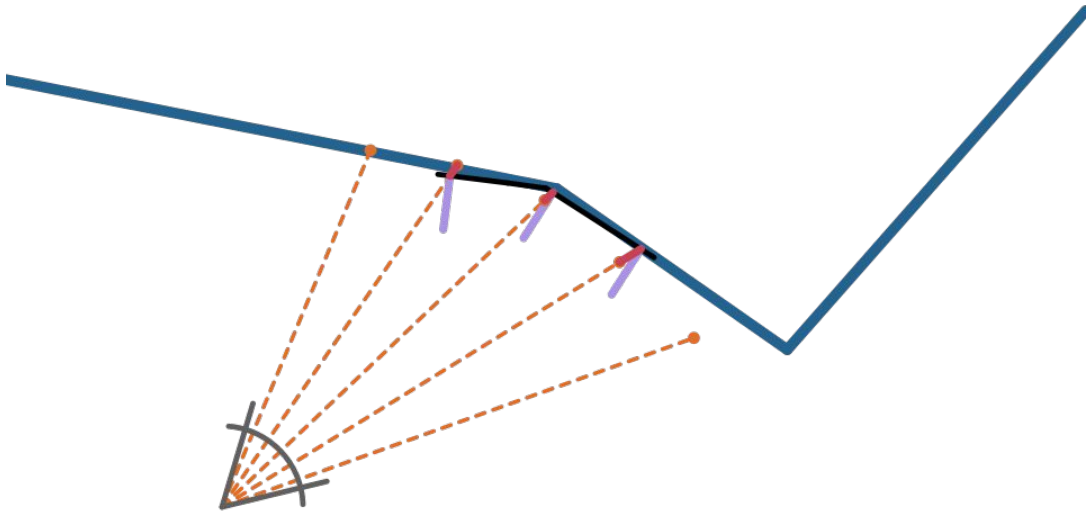
$$\frac{\partial E(\theta \oplus \Delta\theta)}{\partial \Delta\theta} = \sum_i n_i^{\text{pred}} \cdot \frac{\partial}{\partial \Delta\theta} T_{gl}(\theta \oplus \Delta\theta) x_i^{\text{obs}}$$

Point-plane Iterative Closest Point (ICP)

$$\frac{\partial}{\partial \Delta\theta} T(\theta \oplus \Delta\theta) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & z & -y \\ 0 & 1 & 0 & -z & 0 & x \\ 0 & 0 & 1 & y & -x & 0 \end{bmatrix}$$

$$T(\theta \oplus \Delta\theta) = e^{\Delta\theta} T(\theta)$$

Point-plane Iterative Closest Point (ICP)



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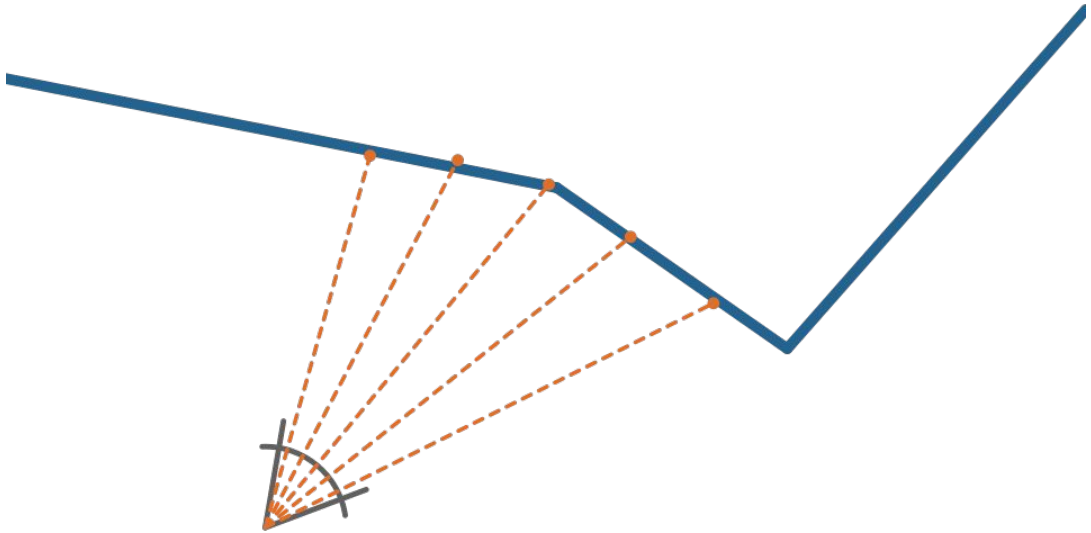
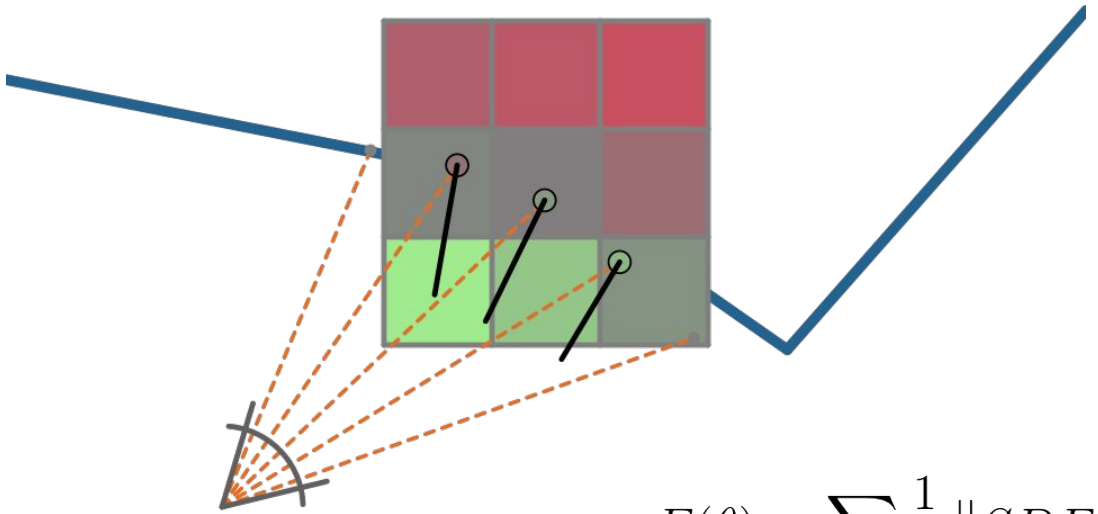


Direct Signed Distance Function Tracking



$$E(\theta) = \sum_i \frac{1}{2} \left\| SDF(T_{gl}(\theta) x_i^{\text{obs}}) \right\|_2^2$$

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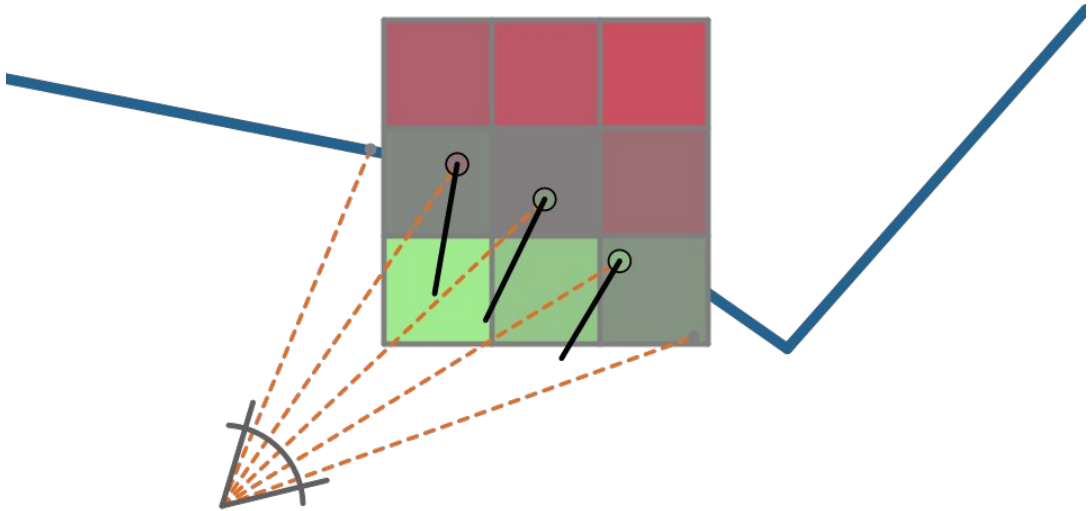
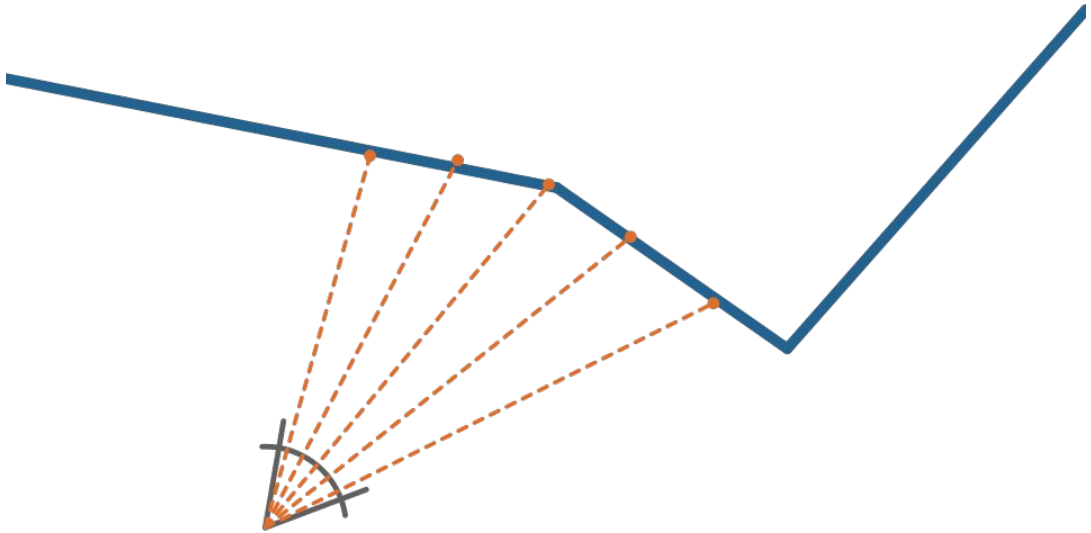


$$\frac{\partial E(\theta \oplus \Delta\theta)}{\partial \Delta\theta} = \sum_i \nabla SDF(T_{gl^{t-1}}(\theta) x_i^{\text{obs}}) R_{gl^{t-1}} \frac{\partial T_{l^{t-1}l^t}(\Delta\theta)}{\partial \Delta\theta}$$

Direct Signed Distance Function Tracking



Online fusion

- Tracking requires the fused SDF volume for all frames up to the current frame

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Online fusion

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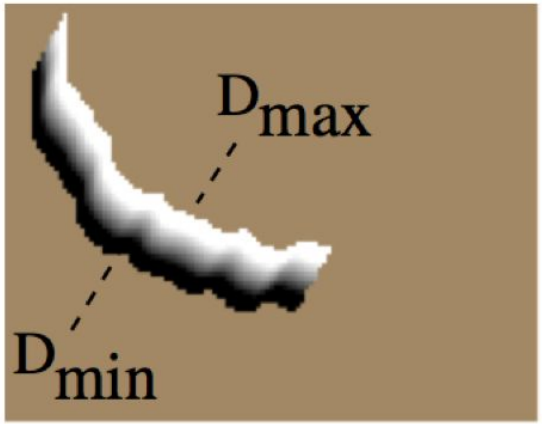
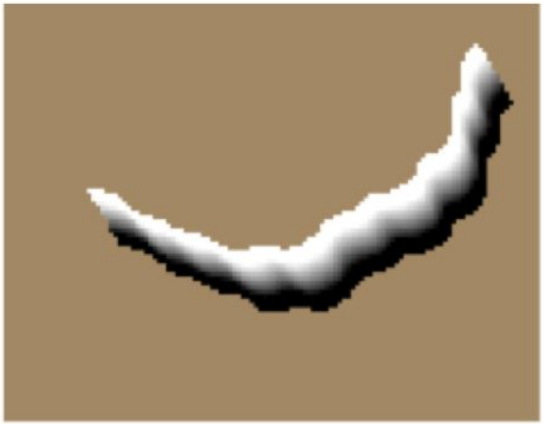
- We must maintain a **running average** SDF value at each cell
- Each cell stores both an SDF value and a weight

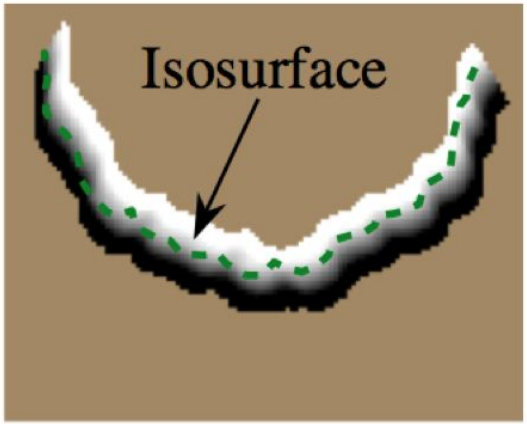
$$SDF^{0:t} \leftarrow \frac{SDF^{0:t-1} w^{0:t-1} + SDF^{t} w^{t}}{w^{0:t-1} + w^{t}}$$

$$w^{0:t} \leftarrow w^{0:t-1} + w^{t}$$

Truncated Signed Distance Function







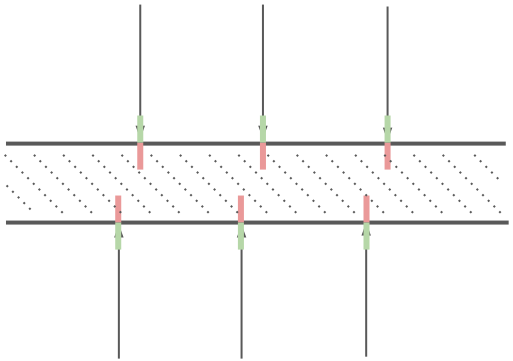
$$\Psi(\eta) = \begin{cases} \min\left(1, \frac{\eta}{\mu}\right) \operatorname{sgn}(\eta) & \text{iff } \eta \geq -\mu \\ null & otherwise \end{cases}$$

Overview

- Explicit and implicit surface representations
- SDF fusion
- SDF tracking
- **SDF limitations**
- Related research
  - KinectFusion
  - KinTinuous
  - BundleFusion
  - DART
  - DynamicFusion

SDF Limitations

- **Size:**
  - In general, scales linearly with the volume of the computed distance field, and exponentially in the resolution
- **Thin surface representation:**
  - If the resolution is insufficient, observations from opposing sides of thin surfaces can “cancel out”



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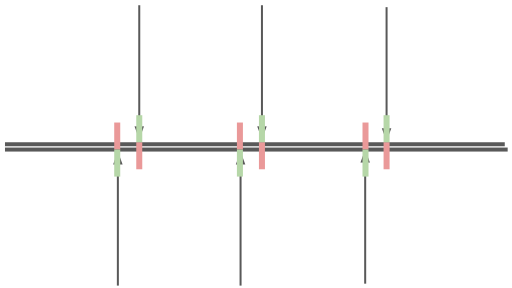

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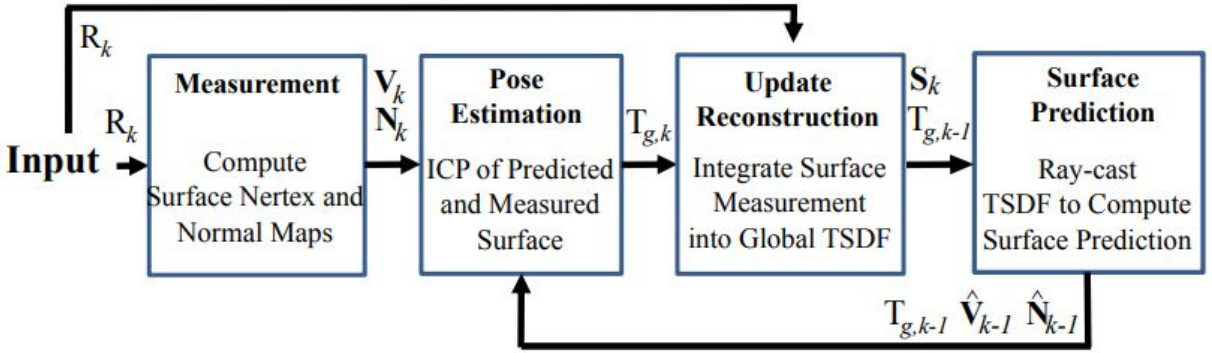
KinectFusion [Newcombe et al.]




Kinect RGB
Raw Normal Map
Composited Kinect RGB
3D Reconstruction +
GPU physics simulation

Overview

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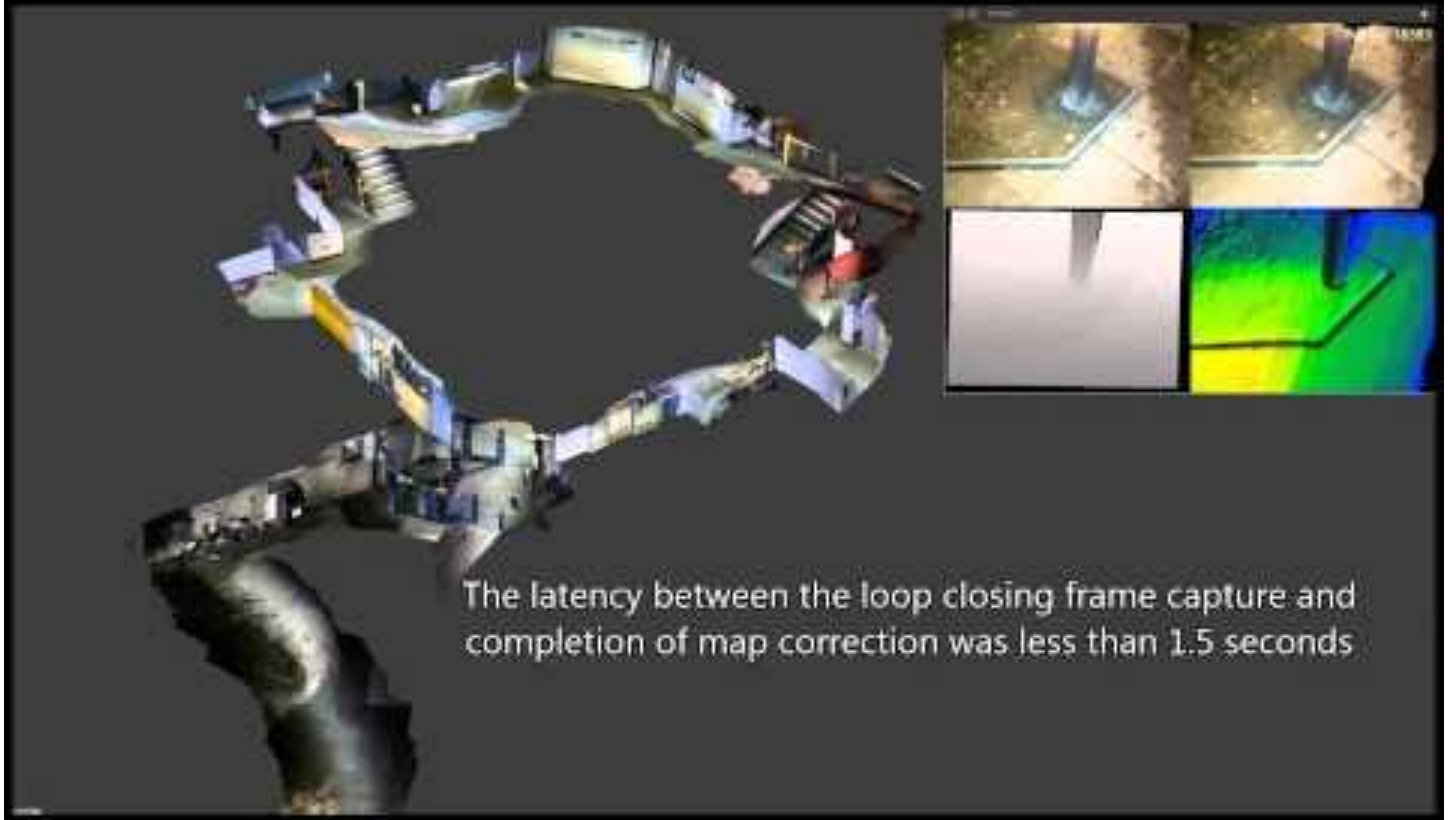
The latency between the loop closing frame capture and
completion of map correction was less than 1.5 seconds

Overview

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Resulting Scan

Overview

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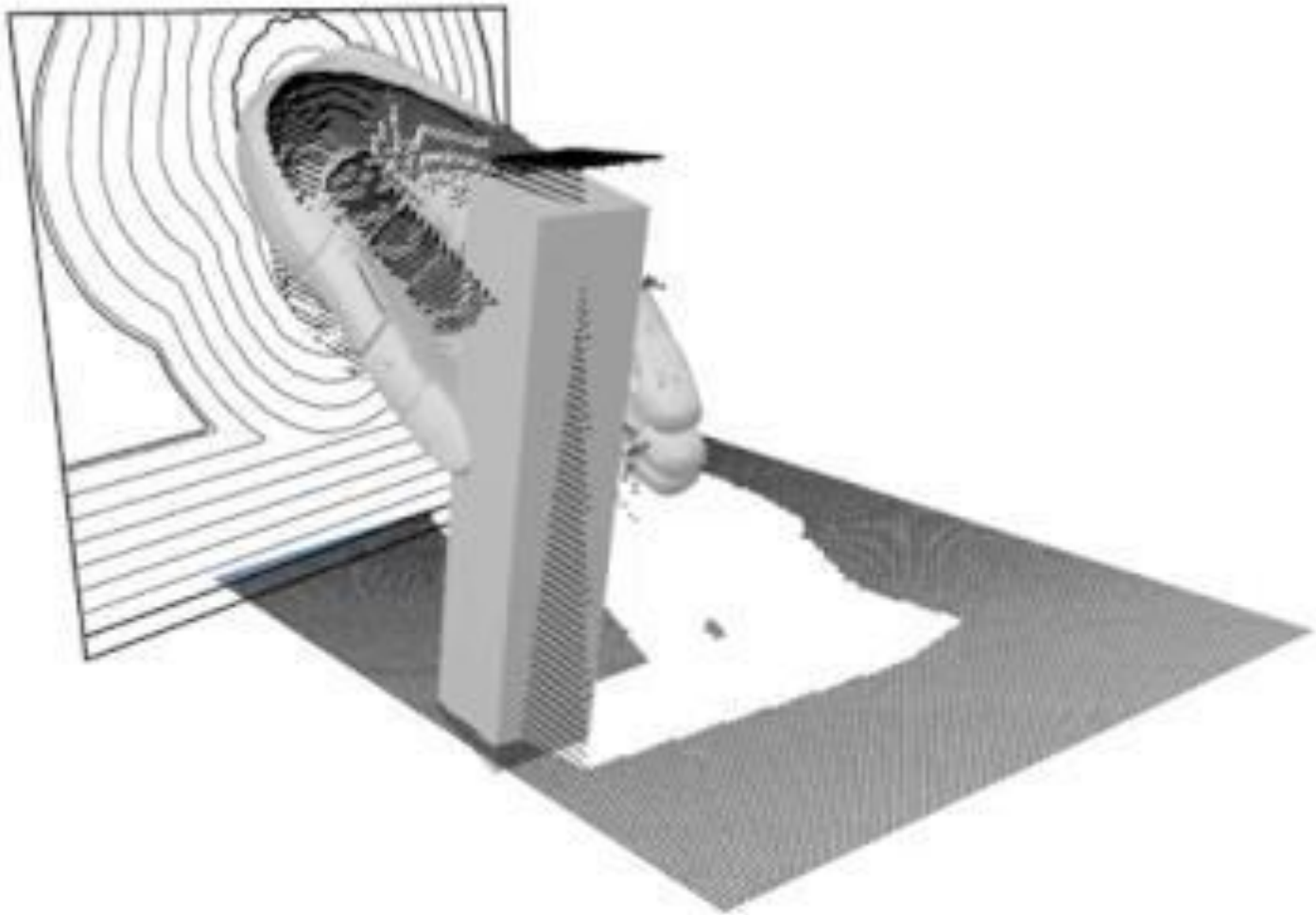
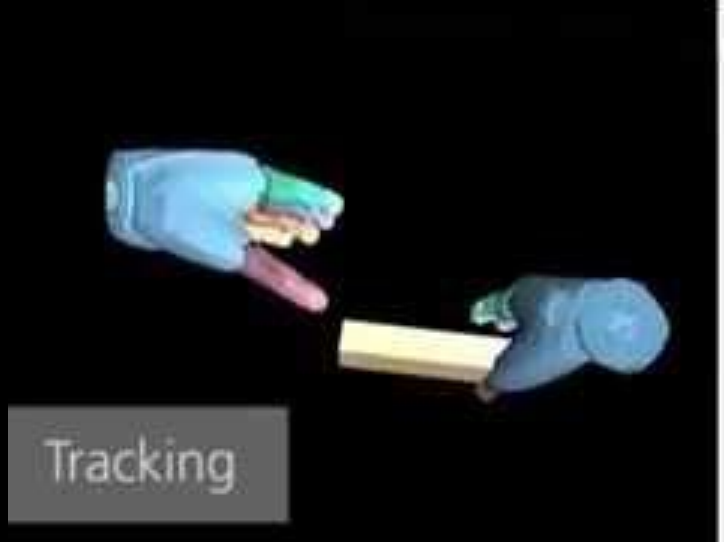
Obs. depth | Model to obs. DA | Obs. to model error | Model depth | Model to obs. DA | Model to obs. error


- The full objective function is given by:

$$\hat{\theta} = \arg\min_{\theta} \sum_{u \in \Omega} \mathrm{SDF}_{\mathrm{mod}}\left(\mathbf{x}_{\mathbf{u}}; \theta\right)^2 + \lambda \sum_{u \in \Omega} \mathrm{SDF}_{\mathrm{obs}}\left(\hat{\mathbf{x}}_{\mathbf{u}}(\theta); D\right)^2$$




Kinect RGB


Tracking


Grasp Planner

Overview

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Live Input Depth Map

Live Model Output

Live RGB Image (unused)

Canonical Model Reconstruction

Warped Model