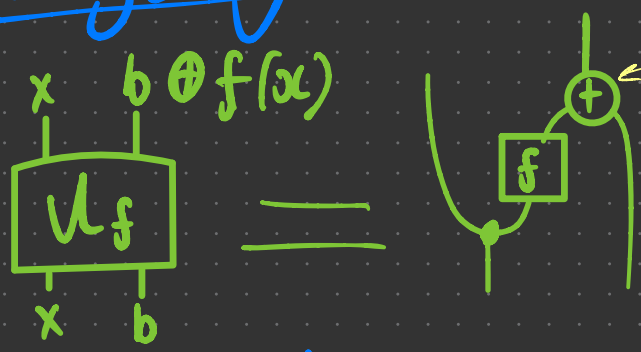
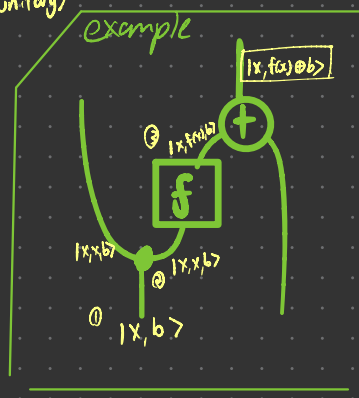
Diagrammatic Reasoning (pt. 2)

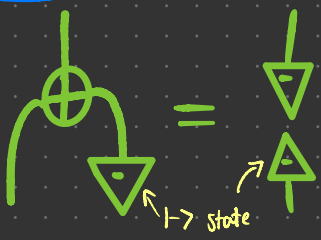
- not same thing as circuits, cannot be directly implemented
- but have useful mathematical properties

Analyzing an Oracle

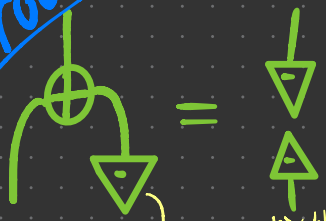




Plus Operation ⊕



Proof where input is $a|0\rangle + b|1\rangle$



left hand side

$a|0\rangle + b|1\rangle$

$(a|0\rangle + b|1\rangle) \otimes (|0\rangle - |1\rangle)$

$= a|00\rangle - a|01\rangle + b|10\rangle - b|11\rangle$

$= a|0\rangle - a|1\rangle + b|1\rangle - b|0\rangle = (a-b)|0\rangle + (-a+b)|1\rangle$

$= (a-b)(|0\rangle - |1\rangle)$

$= (a-b)(|0\rangle - |1\rangle)$

$= (a-b)|-\rangle$

right hand side

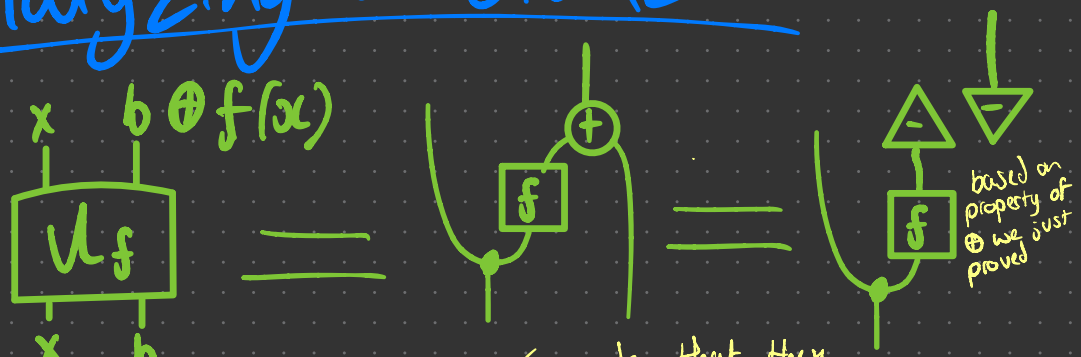
$\langle -| (a|0\rangle + b|1\rangle)|-\rangle$

$= (\langle 0| - \langle 1|)(a|0\rangle + b|1\rangle)|-\rangle$

$= (a-b)|-\rangle$

are the same ∴ equivalent

Analyzing an Oracle



(reminder that these are not computable!)

Deutsch - Josza

Problem: determine whether f is constant or balanced.

circuit:

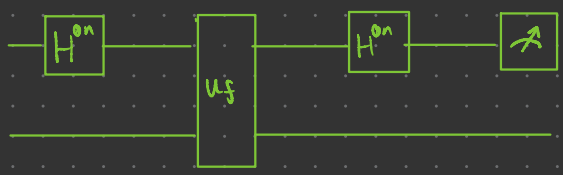
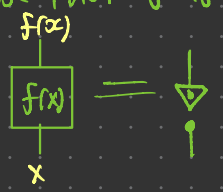


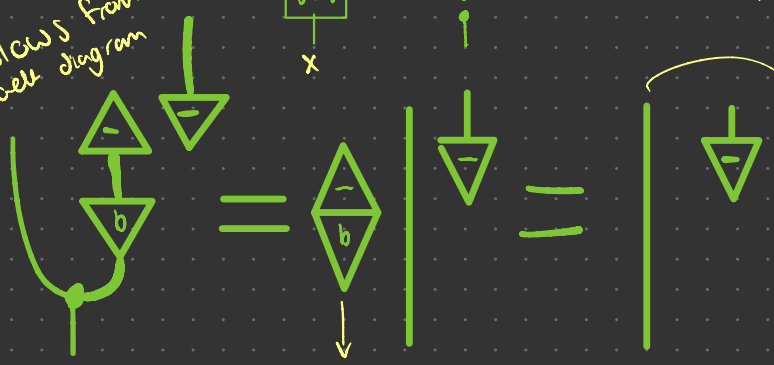
diagram:

suppose that f is constant $f(x)=b$ for all x, where $b=0$ or 1



much more obvious that it erases input & replaces with b

follows from oracle diagram



this is the top line from circuit which we now see does nothing when f is constant

$\langle -|b\rangle = (\langle 0| - \langle 1|)|b\rangle = \pm 1$ which is just a global phase which has no effect

Deutsch – Josza (contd.)

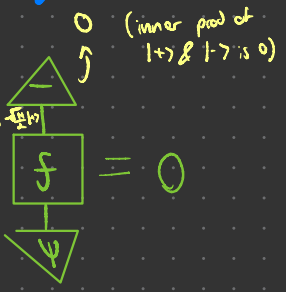
Now lets consider when f is balanced

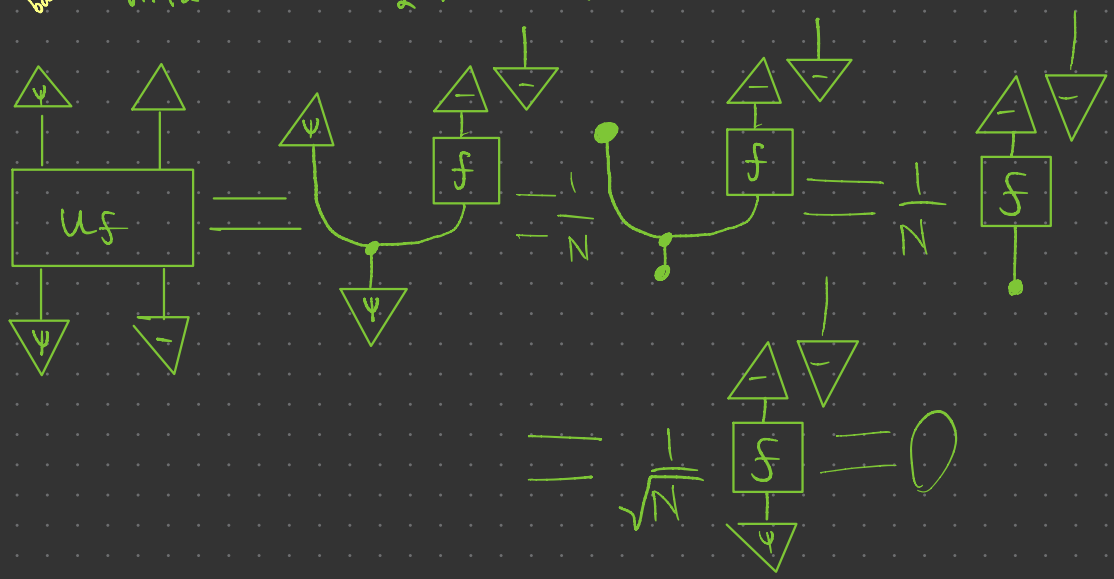
f operates on ψ which is an equal superposition of multiple states

$$f\left(\frac{1}{\sqrt{N}}\sum_{z=0}^{2^n}|z\rangle\right)$$

$$= \frac{1}{\sqrt{N}}\sum_{z=0}^{2^n}|f(z)\rangle$$

because balanced $= \frac{1}{\sqrt{N}}\left(\frac{N}{2}|0\rangle + \frac{N}{2}|1\rangle\right) = \frac{\sqrt{N}}{2}\left(|0\rangle + |1\rangle\right) = \frac{\sqrt{N}}{2}|+\rangle$





(notes continue)

Grover Search

we will consider single iteration case

This occurs when 1/4 inputs map to 1

This diagram describes a measurement result of S