Diagrammatic Reasoning (Pt. 2)

- Not the same thing as circuits.
  - Cannot be directly implemented.
- But have useful mathematical properties.

Analyzing an Oracle

\[ a \oplus b \]

**Plus Operation** \( \oplus \)

Proof:

- **Left hand side**: \( a \left| 0 \right> + b \left| 1 \right> \)
  - \( (a \left| 0 \right> + b \left| 1 \right>) \otimes (b \left| 1 \right>) \)
  - \( a \left| 0 \right> \left| 0 \right> + b \left| 1 \right> \left| 1 \right> - b \left| 1 \right> \left| 0 \right> \)
  - \( a \left| 0 \right> \left| 0 \right> + b \left| 0 \right> + b \left| 1 \right> - b \left| 0 \right> - b \left| 1 \right> \)
- **Right hand side**: \( (a-b) \left| 1 \right> \)
  - \( = (a-b) \left| 1 \right> \)

**Note**: These operations are roughly equivalent.
Analyzing an Oracle

Deutsch-Josza

Problem: Determine whether \( f \) is constant or balanced

Circuit:

Diagram:

Suppose that \( f \) is constant \( f(x) = b \) for all \( x \), where \( b = 0 \) or 1

This is the top line from circuit which we now see does nothing when \( f \) is constant.

\[
\langle 1 | b \rangle = \langle 0 | -1 \rangle b = \pm 1 \text{ which is just a global phase which has no effect.}
\]
Now lets consider when $f$ is balanced.

$f$ operates on $\Psi$ which is an equal superposition of multiple states:

$$f\left(\frac{1}{\sqrt{N}} \sum_{z=0}^{n-1} |z\rangle\right)$$

$$= \frac{1}{\sqrt{N}} \sum_{z=0}^{n-1} |f(z)\rangle$$

Because balanced:

$$= \frac{1}{\sqrt{N}} \left( \frac{N}{2} |0\rangle + \frac{N}{2} |1\rangle \right) = \frac{\sqrt{N}}{2} (|0\rangle + |1\rangle) = \frac{\sqrt{N}}{2} |\Psi\rangle$$

(Inner product of $1\langle \Psi | 1 \rangle = 0$)
Grover Search

We will consider single iteration case. This occurs when $\frac{1}{4}$ inputs map to 1.

This diagram describes a measurement result of $S^*$. It shows the Grover iteration and its effect on the probability distribution. The diagram includes labeled nodes and arrows indicating the flow of the algorithm.

Mathematically, the Grover iteration is given by:

$$D = 2|\psi\rangle \langle \psi| - I$$

where $|\psi\rangle$ is the initial state on the diagrams.

The diagram also illustrates the transformation of the state vector due to the Grover iteration, aligning with the mathematical representation.