Review of Hamiltonian simulation: given Hamiltonian matrix $H$... but how do we get $H$?

Hamiltonian is a sum of terms that act on only a few qubits (usually only a few, not all of them, leaves most unchanged).

That means we can write $H$, it's not too big and nearly $\exp(-iH_t)$ is also only going to act on a few qubits - do we can turn it into executable instructions? Can use Suzuki–Trotter formula to add exponents (or otherwise you can't) v. small error.

Review of Grover search.

Hamiltonians are related to energy - energy should be conserved... $H$'s eigenvectors only change a phase and are thus stable... both Hamiltonians and energy don't really change, which is the intuition for them being connected. Expect to see a state in its lowest energy state.

Read off eigenvalues from using QPE... not obvious this works.

What if the matrix doesn't have eigenvectors? Can write it as a sum of them - will tell you what the state's energy level is.

The closer you are to a state, the higher its probability will be.

Very important! describes electrons physically use a different basis + notation.
Fermions saw, for electrons, Hamiltonian for qubits
Can't quite interchange since electrons are
Indistinguishable from each other.

Jordan-wigner transform lets us go from electrons to qubits.

Fermion or simulation — a guess and check strategy
to find the lowest energy state.
We don't worry about position — leave that to simulator.
All we assume about the Hamiltonian is that it's symmetric and a sum of terms — for more specialists structures you get more properties.

Introducing the \( \frac{1}{4} \text{th essential quantum algorithm} \)

Solves \( Ax = b \) for exponentially large \( A \) and \( x \), \( \mathcal{O}(n \log(n)) \) instead of \( n^3 \).

Caveat: needs \( A \) to be well conditioned.

Sus: algo claims to be faster than \( P \) takes to write the input!

Ok, \( b \) is not a list, \( b \) is a quantum state. Want to get \( x \) by doing \( A^+ \)

Need to renormalize \( b \) to return

More caveats: \( b \) must be able to be prepared efficiently.

In every row of \( A \), there are mostly 0s.

The algorithm is actually really complicated so we won't talk about it in detail.
Step 1: assume A is Hermitian. If it's not, you can make it Hermitian in constant time.
A isn't necessarily unitary, but $e^{-iA}$ is.

Step 2: use Hamiltonian simulation to apply $e^{iA}$ to $|\psi\rangle$.

Step 3: use phase est. to find value of $\lambda$ (an eigenvalue).

Step 4: scale the state by $1/\lambda$.

do these for all eigenvalues simultaneously to get $A^{-1}$.

* This is a harder step. Full details are complicated.
Some authors have a chance of failing, but that's okay because we can fail up to $1/3$ of the time.

HHL can't be solved classically—called BQP complete.
Anything only Hamiltonian is BQP complete.

In principle, HHL is useful, but unclear how it's practically useful. Would need to show a classical algorithm can't come up with the same distribution of outputs.