1 Quantum Circuit pt. 2

We can draw quantum circuits as graphs:

- Qubits becomes wires.
- Gates becomes boxes with labels on them:

  - $H$
  - $X$
  - $Z$
  - $U$

Since gates are unitary, number of inputs must be equal to number of outputs.

Special gates:

- Measurement: [Diagram]
- CNOT (CX): [Diagram]
- CZ: [Diagram]
- SWAP: [Diagram]
Many algebraic equivalences become obvious in circuits:

- \((B \otimes D)(A \otimes C) = BA \otimes DC:\)

- \(\text{SWAP} \cdot \text{SWAP} = I:\)

- \(\text{SWAP}(U \otimes I)\text{SWAP} = I \otimes U:\)

Some algebraic equivalences are still not intuitive in circuits:

- \((I \otimes H)\text{CNOT}_{1,2}(I \otimes H) = (H \otimes I)\text{CNOT}_{2,1}(H \otimes I):\)
2 Quantum Information pt. 1

Cloning classical bits are easy. Cloning qubits are provably impossible:

Proof. Suppose an unitary operation $U$ can clone a qubit $|x\rangle = a|0\rangle + b|1\rangle$. This means

$$U(|x\rangle \otimes |0\rangle) = (|x\rangle \otimes |x\rangle)$$

$$U(a|00\rangle + b|10\rangle) = a^2 |00\rangle + ab|01\rangle + ab|10\rangle + b^2 |11\rangle$$ (1)

Suppose $U$ can correctly clone qubit $|0\rangle$ and $|1\rangle$, we know that

$$U |00\rangle = |00\rangle$$

$$U |10\rangle = |11\rangle$$

$$U(a|00\rangle + b|10\rangle) = a \cdot U |00\rangle + b \cdot U |10\rangle$$

$$= a |00\rangle + b |11\rangle$$ (2)

By eq 1 and eq 2,

$$a^2 |00\rangle + ab |01\rangle + ab |10\rangle + b^2 |11\rangle = a |00\rangle + b |11\rangle$$

$$a = 1$$

$$b = 0$$

$|x\rangle = |0\rangle$

or

$$a = 0$$

$$b = 1$$

$|x\rangle = |1\rangle$

Therefore, a unitary that correctly clones $|0\rangle$ and $|1\rangle$ can only correctly clone $|0\rangle$ and $|1\rangle$ and not other qubits. $\square$

Similarly, for any orthogonal qubits $|a\rangle$ and $|b\rangle$, we can find a unitary that clones $|a\rangle$ and $|b\rangle$, and that unitary cannot clone other qubits.

For example, CNOT clones $|0\rangle$ and $|1\rangle$; we can make a unitary $U$ that clones $|+\rangle$ and $|−\rangle$ by using $H$:

$$U = (H \otimes I) \cdot \text{CNOT} \cdot (H \otimes I)$$