Review from last time

- States are vectors of complex numbers with magnitude 1
- Operations are described as unitary matrices
- Measurement forces a qubit into a classical state

Product States

- States that are the tensor product of two states are called Product States
- Some 2-qubit states are not the tensor product of 2 1-qubit states
    - $\frac{1}{\sqrt{2}}|00>+\frac{1}{\sqrt{2}}|11>$ is an example
    - Called Entangled States/"EPR pairs"
    - Must be described together
- The Bell states are "maximally entangled" (Above state is an example)

Product Operations

- Some 2-qubit operations are not writable as the product of two 1-qubit operations
- CNOT (Flip the second bit iff the first bit is 1) cannot be written as a product

Adding/removing bits

- Can always add or "adjoin" a new bit
- Can throw away bits if they are not entangled
    - Otherwise, removing a bit removes information
- Removing entangled qubits
    - Measuring the bit will move the bits into a product state

Marginals

- In a probabilistic state, valid to throw away a bit
    - Called computing a marginal distribution
    - $p|00>+q|01>+r|10>+s|11>$ becomes $(p+q)|0>+(r+s)|1>$
- Does not work for quantum states

Throwing Away A Bit™

- What if someone throws one end of an EPR pair into a black hole/the sun/molten lava?
    - I believe "Man, they're gone" is referring to this
- Nature measures the bit we threw away turning the EPR pair into a product state
- This must happen, otherwise would allow faster than light communication

    - Imagine Adrian goes to the black hole to dispose of his qubit but instead measures it
    - If it acted differently Bianca would be able to tell instantly even though they are very far apart
- Each part of an entangled state behaves as if the other part has been measured
    - But the bits don't actually change

Measurement of Entangled pairs

- Parts of entangled states behave like a probabilistic state
- Measurement is not something happening to the state, you're just seeing part of it
- Measurement can be thought of as entangling the bit with the environment
    - Corollary of this is quantum computers must be shielded from the environment