Quantum Information (pt. 2)

CSE 490Q: Quantum Computation
• We will look at some differences between bits and qubits

• Copying qubits is *impossible*
  • unless they are classical bits, $|0\rangle$ or $|1\rangle$
  • (in general, any two orthogonal states can be measured or copied)

• Will see two more new behaviors today
$\langle \psi \rangle := a_{10} + b_{11}$

Scenario 1

Ahmed

$\frac{1}{\sqrt{2}}(100 + 111)$

Beatrice
Protocol 1

A B C D E

prep

Separate planes

Ahmed
Beatrice
A  \( |x\rangle \otimes |10\rangle \otimes |10\rangle \)

B  \( (I \otimes H \otimes I) |x\rangle |00\rangle = |x\rangle |0\rangle + \frac{1}{\sqrt{2}} |1\rangle |10\rangle \)

\[ = |x\rangle |0\rangle + \frac{1}{\sqrt{2}} (|10\rangle + |11\rangle) \]

\[ = |x\rangle |0\rangle + \frac{1}{\sqrt{2}} (|10\rangle + |11\rangle) \]

C  \( (I \otimes CNOT) (|x\rangle \otimes \frac{1}{\sqrt{2}} (|10\rangle + |11\rangle)) \)

\[ = |x\rangle 2 \otimes \frac{1}{\sqrt{2}} (|10\rangle + |11\rangle) \]

\[ \text{EPR pair} \]
D \ \ (\text{NOT} \ \otimes \ \Sigma) \ \left(\frac{\alpha}{\sqrt{2}} (1,000) + \frac{\alpha}{\sqrt{2}} (1,011) + \frac{5}{\sqrt{2}} (1,001) + \frac{5}{\sqrt{2}} (1,111)\right) \\
= \left(\frac{\alpha}{\sqrt{2}} (1,000) + \frac{\alpha}{\sqrt{2}} (1,011) + \frac{5}{\sqrt{2}} (1,001) + \frac{5}{\sqrt{2}} (1,111)\right) \\
= \frac{1}{\sqrt{2}} \left(\alpha (1,000) + \alpha (1,011) + 5(1,001) + 5(1,111)\right)

E \ \ (\Sigma \otimes \Sigma) \ [?] \\
= \frac{1}{\sqrt{2 \sqrt{2}}} \left(\alpha (1,0) + \alpha (1,1) \otimes 1,000 + \alpha (1,0) + \alpha (1,1) \otimes 1,11\right) + \\
\left(\alpha (1,0) - 1,11\right) \otimes (1,0) + \left(\alpha (1,0) - 1,11\right) \otimes (1,01)\right)
\[
\frac{1}{2} (a|1000\rangle + a|100\rangle + a|011\rangle + a|111\rangle + b|010\rangle - 5|110\rangle + b|001\rangle - 5|101\rangle)
\]

\[
= \frac{1}{2} |100\rangle \otimes (a|10\rangle + 5|11\rangle) + \frac{1}{2} |101\rangle \otimes (a|11\rangle + b|10\rangle) + \frac{1}{2} |110\rangle \otimes (a|00\rangle - 6|11\rangle) + \frac{1}{2} |111\rangle \otimes (a|11\rangle - b|10\rangle)
\]

\[
= \left( \frac{1}{2} |100\rangle \otimes |1\rangle \langle 1| + \frac{1}{2} |101\rangle \otimes |1\rangle \langle 1| \right) \\
\quad + \frac{1}{2} |110\rangle \otimes |2\rangle \langle 1| + \frac{1}{2} |111\rangle \otimes |2\rangle \langle 1| 
\]
Protocol 1 Summary

• If Ahmed measures $|00\rangle$, then Beatrice has $|x\rangle$
• If Ahmed measures $|01\rangle$, then Beatrice has $X|x\rangle$
• If Ahmed measures $|10\rangle$, then Beatrice has $Z|x\rangle$
• If Ahmed measures $|11\rangle$, then Beatrice has $ZX|x\rangle$

• Ahmed transmits his measurement outcome to Beatrice
• Beatrice can apply one of $\{I, X, Z, ZX\}$ in order to produce $|x\rangle$
  • if Ahmed measures ab, then Beatrice applies $Z^bX^a$ to her qubit
Quantum Teleportation

• Result is that $|x\rangle$ has been “teleported” from Ahmed to Beatrice

• Note that this requires sending classical information
  • information is not sent faster than the speed of light

• Note that this destroys Ahmed’s copy of $|x\rangle$
  • it does not copy a qubit (that is impossible)

• This “uses up” the EPR pair that they started with
  • EPR pairs are a valuable resource for computing
  • (more examples to come...)
Scenario 2

EPR pair

Ahmed

Beatrice

[(ab)]

rocket

[soo, .. 11]
Protocol 2

\[ 10 \]

\[ \text{prep} \]

\[ x^2 + 2^\alpha \]

\[ \text{Rocket} \]

Ahmed

Beatrice
Simplifying

Since \( x \ket{10} = \ket{15} \)
Simplifying

\[
|10\rangle \rightarrow \frac{H}{4} \rightarrow |11\rangle \rightarrow |12\rangle \\rightarrow |13\rangle \rightarrow |Hx\rangle \rightarrow \text{Result}
\]

Since \( Hx^bH = \frac{\pi}{2} \) \[\Rightarrow Hx^b = \frac{\pi}{2} H \]
Simplifying
Simplifying

\[
\begin{align*}
&= \, \text{(a)} - \text{(b)} - \text{(c)} + \text{(d)} - \text{(e)} - \text{(f)} - \text{(g)} + \text{(h)} + \text{(i)} - \text{(j)} - \text{(k)} \\
&= (a) - 2^g (b) \\
&= 1^c \times (1)^g \times 1^b (b)
\end{align*}
\]
Protocol 2 Summary

- Final state is $\ket{-1^a} \ket{ab}$, which is indistinguishable from $\ket{ab}$
- Measuring the state produces $\ket{ab}$ with certainty
Super-Dense Coding

• Result is that we “transmit” 2 bits (a and b) by sending 1 qubit
  • achieved a 2x data compress

• However, note that an EPR pair was used up in the process
  • really, it is 1 qubit + 1 EPR = 2 bits
  • provably impossible to transmit 2 bits by sending just 1 qubit

• The two protocols are semi-reversals of one another
  • teleportation sends two bits to transmit a qubit
  • super-dense coding sends a qubit to transmit two bits