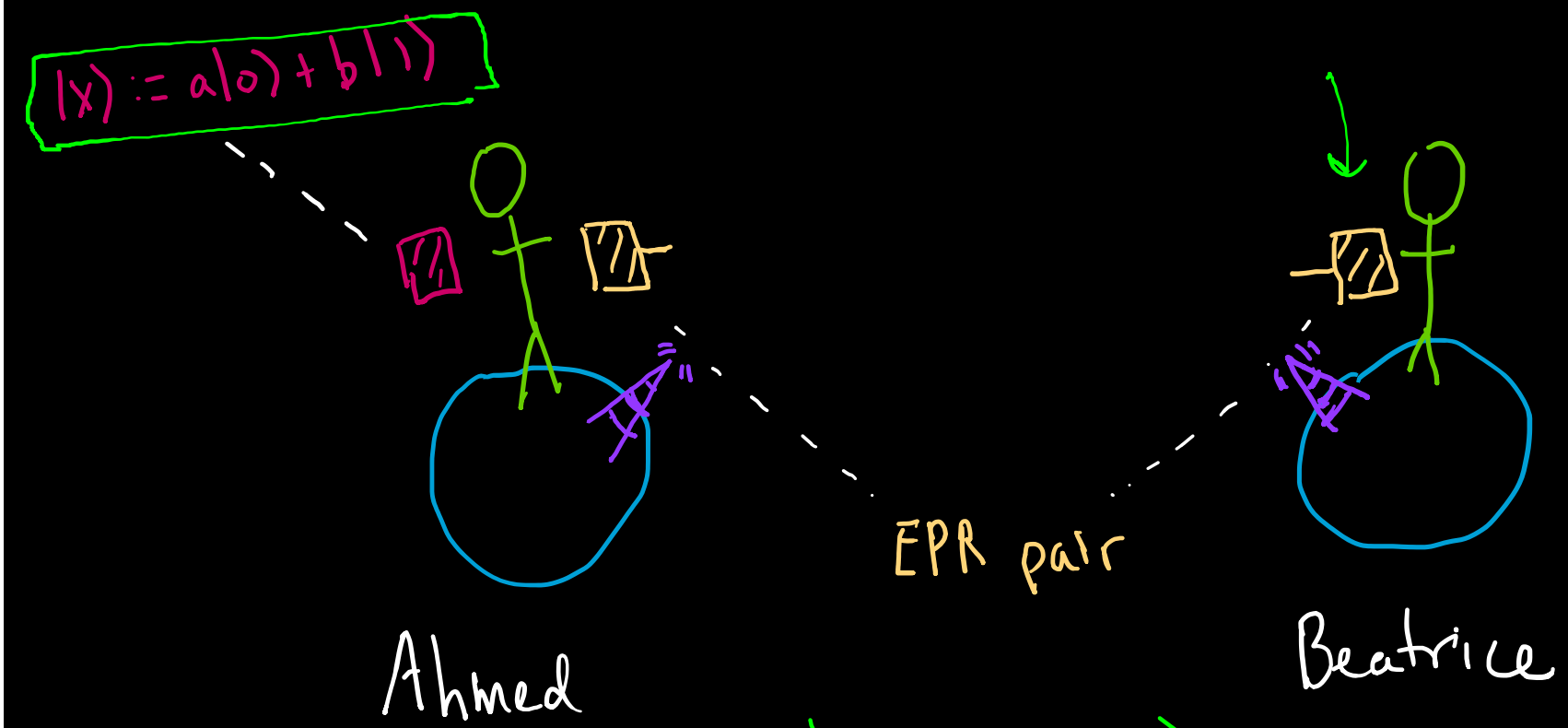
Quantum Information (pt. 2)

CSE 490Q: Quantum Computation

Quantum Information

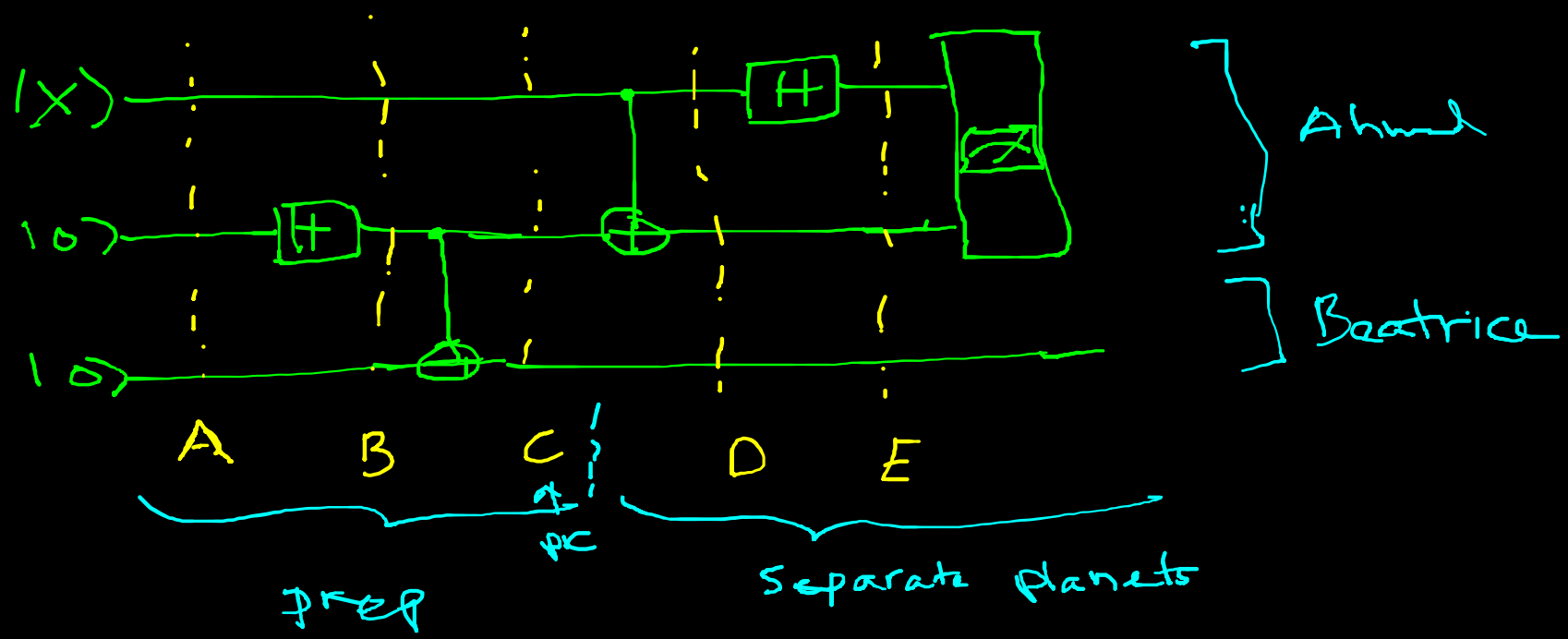
- We will look at some differences between bits and qubits
- Copying qubits is *impossible*
  - unless they are classical bits, |0> or |1>
  - (in general, any two orthogonal states can be measured or copied)
- Will see two more new behaviors today

Scenario 1



$$\frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$

Protocol 1



Prep

A $|x\rangle \otimes |0\rangle \otimes |0\rangle$

B $(I \otimes H \otimes I)|x00\rangle = |x\rangle \otimes |+\rangle \otimes |0\rangle$

$$= |x\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle$$

$$= |x\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$$

C $(I \otimes CNOT)(|x\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle))$

$$= |x\rangle \otimes \underbrace{\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)}_{\text{EPR pair}}$$

Step 1

D $(CNOT \otimes I)\left((\underline{a|0\rangle} + b|1\rangle) \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)$

$= (CNOT \otimes I)\left(\frac{a}{\sqrt{2}}|\underline{0}00\rangle + \frac{a}{\sqrt{2}}|\underline{0}11\rangle + \frac{b}{\sqrt{2}}|\underline{1}00\rangle + \frac{b}{\sqrt{2}}|\underline{1}11\rangle\right)$

$= \frac{1}{\sqrt{2}}\left(a|000\rangle + a|011\rangle + b|110\rangle + b|101\rangle\right)$

E $(H \otimes I \otimes I)\,[\;]$

$= \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\Big(a(|0\rangle + |1\rangle) \otimes |00\rangle + a(|0\rangle + |1\rangle) \otimes |11\rangle +$

$b(|0\rangle - |1\rangle) \otimes |10\rangle + b(|0\rangle - |1\rangle) \otimes |01\rangle\Big)$

Step 2

$$\frac{1}{2}\Big(a|000\rangle + a|100\rangle + a|011\rangle + a|111\rangle +$$
$$b|010\rangle - b|110\rangle + b|001\rangle - b|101\rangle\Big)$$

$$= \frac{1}{2}|00\rangle\otimes(a|0\rangle + b|1\rangle) + \frac{1}{2}|01\rangle\otimes(a|1\rangle + b|0\rangle) +$$
$$\frac{1}{2}|10\rangle\otimes(a|0\rangle - b|1\rangle) + \frac{1}{2}|11\rangle\otimes(a|1\rangle - b|0\rangle)$$

$$= \frac{1}{2}|00\rangle\otimes|x\rangle + \frac{1}{2}|01\rangle\otimes X|x\rangle$$
$$+ \frac{1}{2}|10\rangle\otimes Z|x\rangle + \frac{1}{2}|11\rangle\otimes XZ|x\rangle$$

E

Protocol 1 Summary

- If Ahmed measures |00>, then Beatrice has |x>
- If Ahmed measures |01>, then Beatrice has X|x>
- If Ahmed measures |10>, then Beatrice has Z|x>
- If Ahmed measures |11>, then Beatrice has ZX|x>

ab

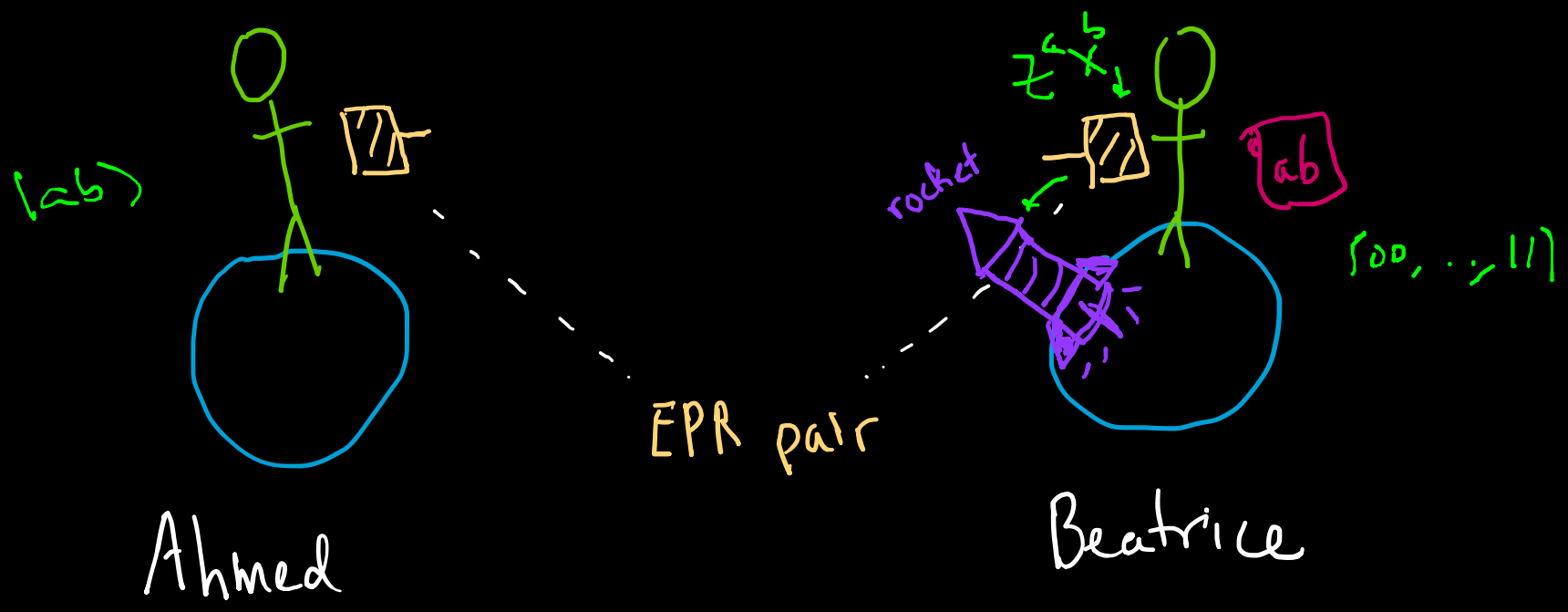
- Ahmed transmits his measurement outcome to Beatrice
- Beatrice can apply one of {I, X, Z, ZX} in order to produce |x>
  - if Ahmed measures ab, then Beatrice applies Z^bX^a to her qubit

Quantum Teleportation

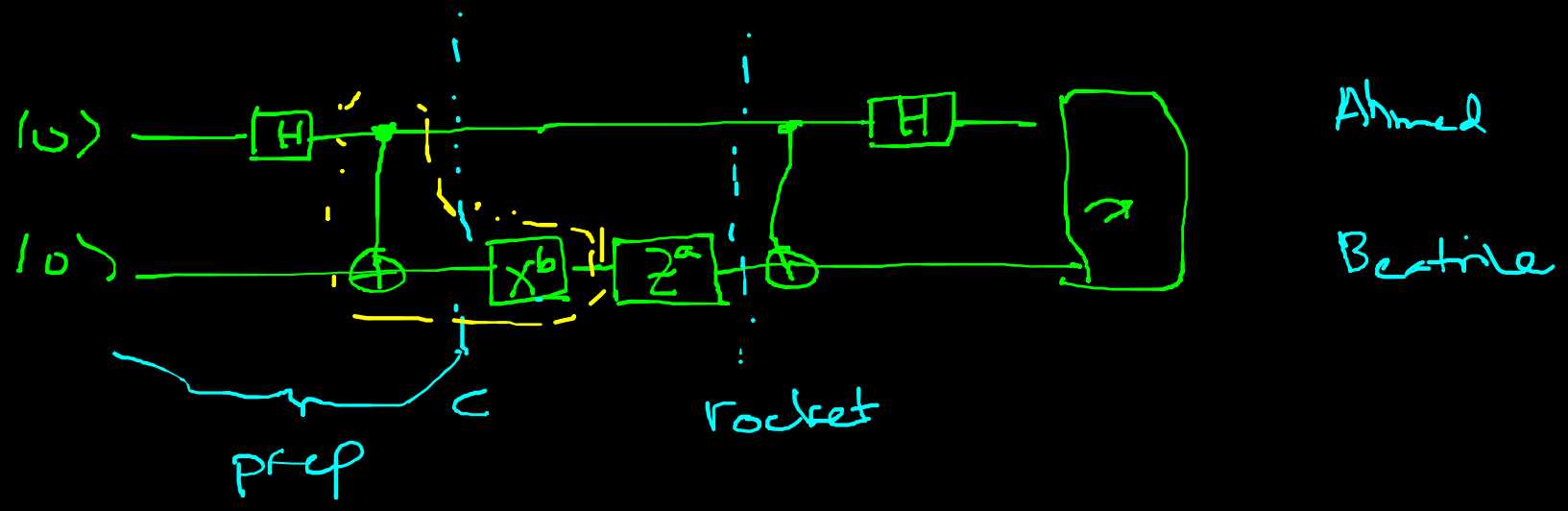
- Result is that |x> has been "teleported" from Ahmed to Beatrice
- Note that this requires sending classical information
  - information is not sent faster than the speed of light
- Note that this destroys Ahmed's copy of |x>
  - it does not copy a qubit (that is impossible)
- This "uses up" the EPR pair that they started with
  - EPR pairs are a valuable resource for computing
  - (more examples to come…)

$|ab\rangle \otimes |x\rangle$

Scenario 2



Protocol 2



Simplifying

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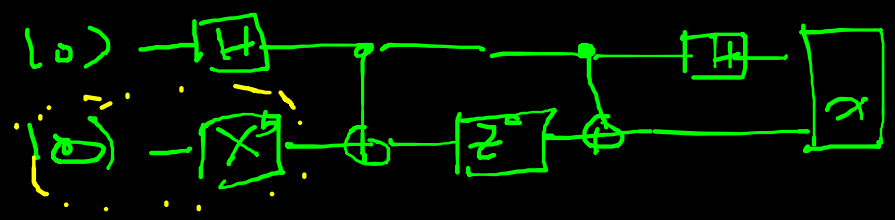

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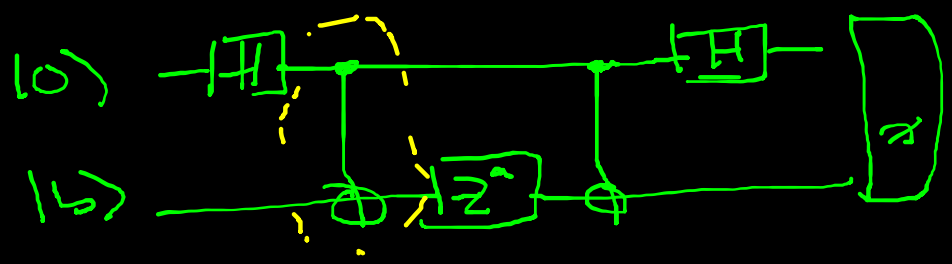
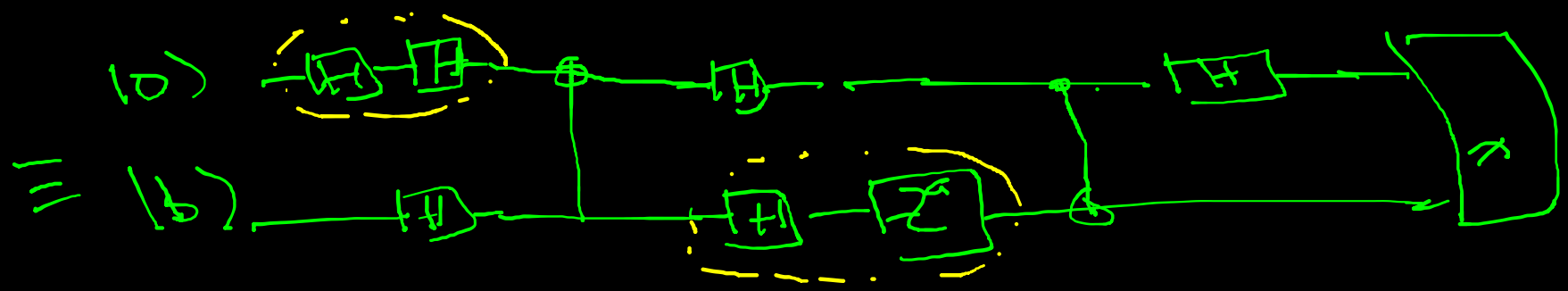


since $X^b|0\rangle = |b\rangle$

Simplifying



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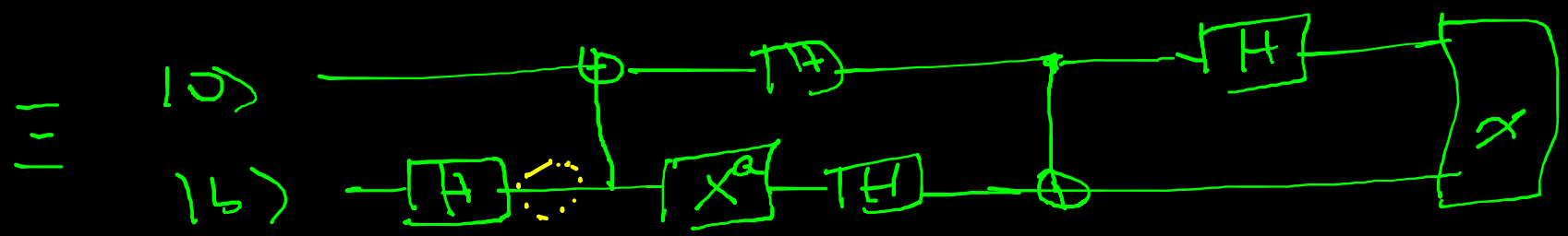
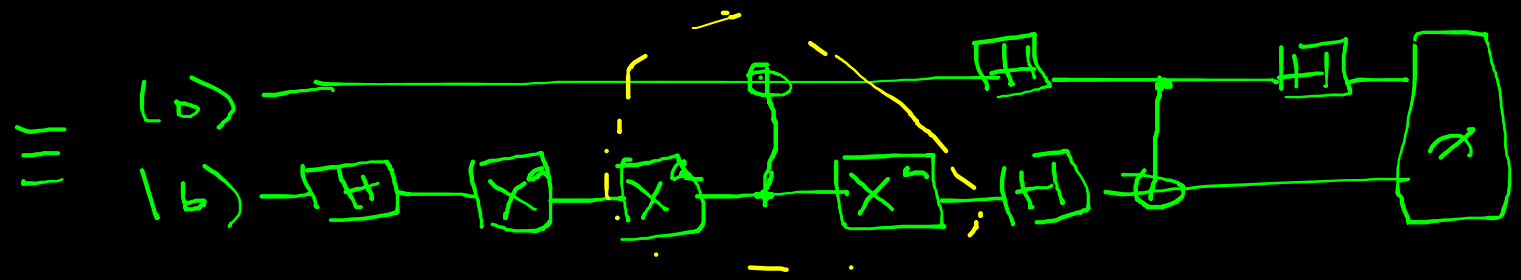
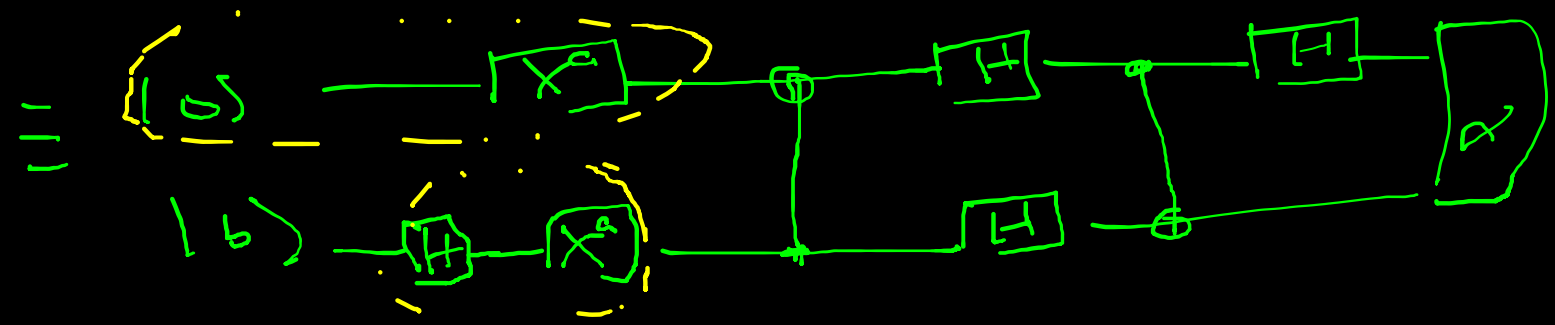


Since $\quad HX^bH = Z^b$

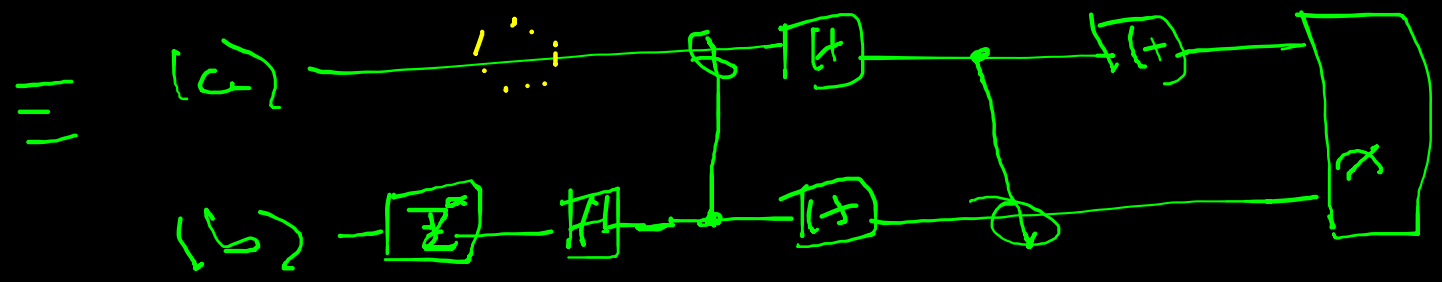
$\Leftrightarrow \quad HX^b = Z^bH$

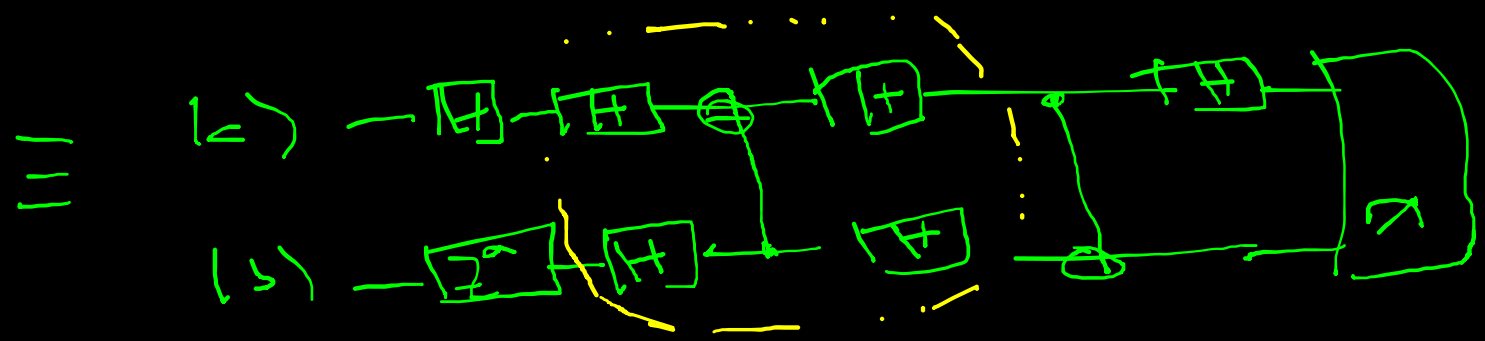
Simplifying





Simplifying





Simplifying

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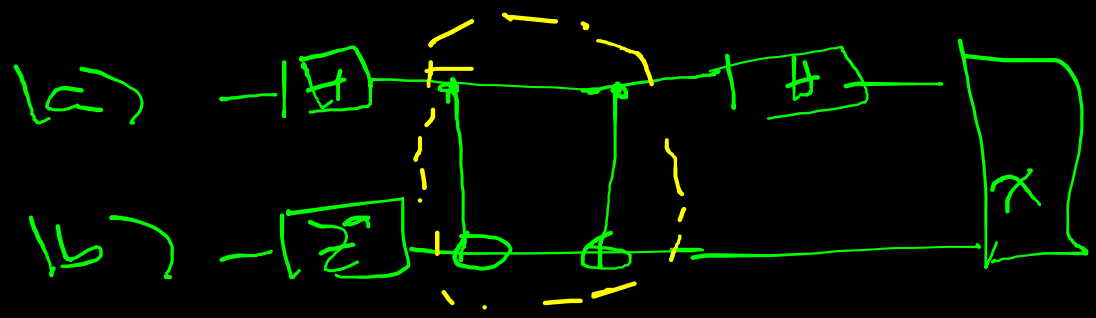

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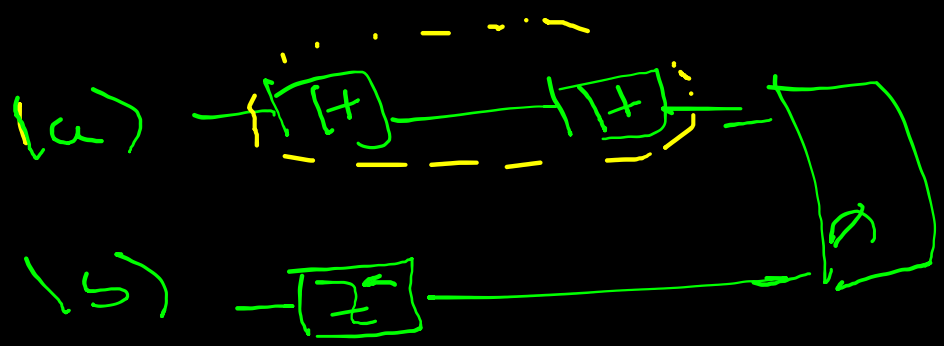

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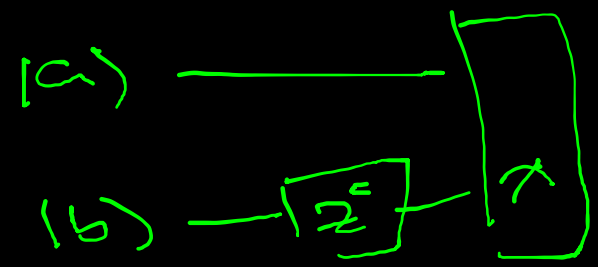

$$\boxed{(-1)^{ab}\,|ab\rangle}$$

$$|a\rangle \otimes Z^a |b\rangle$$

$$= |a\rangle \otimes (-1)^{ab} |b\rangle$$

Protocol 2 Summary

- Final state is $-1^{a}|ab\rangle$, which is indistinguishable from $|ab\rangle$
- Measuring the state produces $|ab\rangle$ with certainty

Super-Dense Coding

- Result is that we "transmit" 2 bits (a and b) by sending 1 qubit
  - achieved a 2x data compress

- However, note that an EPR pair was used up in the process
  - really, it is 1 qubit + 1 EPR = 2 bits
  - provably impossible to transmit 2 bits by sending just 1 qubit

- The two protocols are semi-reversals of one another
  - teleportation sends two bits to transmit a qubit
  - super-dense coding sends a qubit to transmit two bits

+ EPR.