Homework 1
CSE 490q

Due: Fri, Oct 9th by 11pm PST

Note: for problems 2 and later, when writing vectors or matrices, assume that the basis vectors are ordered in the usual way: $|0,0\rangle, |0,1\rangle, |1,0\rangle, |1,1\rangle$.

1. Suppose we flip one weighted coin, with 60% probability of tails, and then a second coin with heads on both sides. Write the resulting state of the system at that point as a vector. (Use $|H\rangle$ and $|T\rangle$ for heads and tails.)
2. Write a matrix that describes the following operation, taking two bits to one bit: If the first bit is 0, then throw it away and just keep the second bit. If the first bit is 1, then output the XOR of the two bits.
3. Write the matrix above as a sum of rank-one matrices. (E.g., an operation that takes state $|0,0\rangle$ to state $|1\rangle$ could be written as $|1\rangle\langle 0,0|$.
4. Describe, both as a matrix and a sum of rank-one matrices, the following operation taking two bits to two bits: XOR the two bits to get the first output bit, and flip a fair coin to get the second bit. (Verify that its columns each sum to one.)
5. If an operation is described by a matrix that has only a single 1 in each column, what does that mean? And what does it mean when that is not the case?
6. Write out the 4 x 4 matrix for $F_1 \otimes G$, with these matrices as defined in lecture.
7. Suppose that $|x\rangle := a|0\rangle + b|1\rangle$ and $|y\rangle := c|0\rangle + d|1\rangle$ are valid probabilistic states (i.e., each coefficient is non-negative and they all sum to one).
   (a) Write out $|x\rangle \otimes |y\rangle$ as a vector?
   (b) Show that this is also a valid probabilistic state.
8. Explain why, say, the entries in the second column of the Kronecker product $F \otimes G$ as defined in lecture is always sums to 1, as long as F and G are themselves stochastic.
9. Consider the following operation. The input is any Java program with no more than 1,000 characters (i.e, 8,000 bits), and the the output is 1 if the program halts when given its own code as input and 0 otherwise.
   (a) Can that operation be described as a stochastic matrix?
   (b) How about if we allowed 1 million characters? 1 billion characters?
   (c) Explain why that fact (that these can or cannot be written as matrices) is problematic.