On Programs and Games

Pick a role from 2 players: E & A.

1. *My dad is richer than your dad*: A goes first, pick a number & show it to E. Then E picks a number. Whoever picks the larger number wins. Who has a winning strategy? The player to move second writes down the strategy as a function of the other player's number. (don't overthink, yes this is a stupid game). If you know first order logic, write down a proposition that's equivalent to this game.

2. *I am not poorer than your dad but not richer than your mom*: A goes first & write down 2 numbers x, y & show them to E. Then E writes down a number z. E wins if $x \leq z \leq y$, otherwise A wins. Again, the second player writes down the strategy as a function of the other player's number(s). (again don't overthink, this is a simple game). If you know first order logic, write down a proposition that's equivalent to this game.

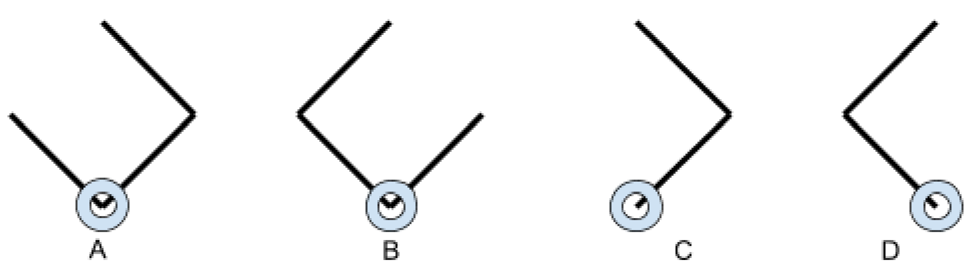
3. *Least upper bound*: LUB of x, y is the least number that is no less than x or y. Come with a strategy to compute LUB from x & y (you can write it down as a function of x & y). Try to write down the sentence in first order logic that says z is LUB of x & y (hint: you need a 4th variable).

On Numbers and Games

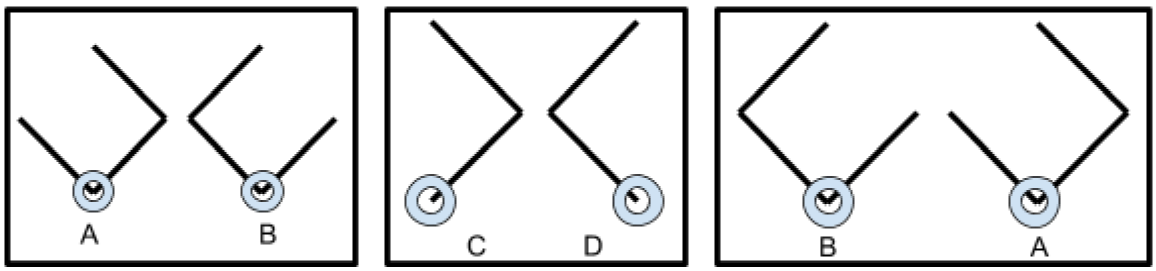
1. *Negating games*: Starting from the bottom, 2 players take turns moving the donut upwards. Left can only move along left branches, right along right branches. A player loses if s/he can no longer move the donut up.

First pick a role (Left / Right) (keep the role for all game plays). Then for each of the 4 games (A, B, C, D), take turns starting first. Once you have figured out a strategy, say whether the game is positive, negative, zero or fuzzy. The rules are:

> *G is positive if Left has a winning strategy,*
> *G is negative if Right has a winning strategy,*
> *G is 0 if 2nd to move has a winning strategy,*
> *G is fuzzy if 1st to move has a winning strategy*

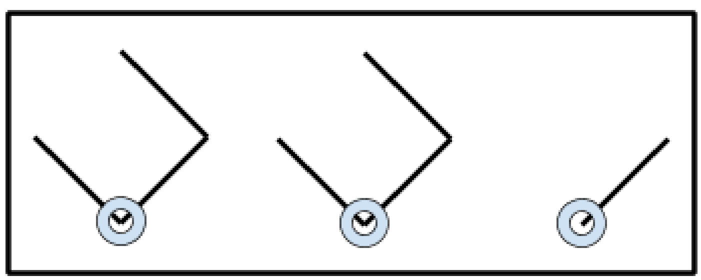


Then play each **pair** of games in the same box simultaneously (move only one donut at each turn). Once you figured out a strategy, think about whether each pair is positive, negative, zero or fuzzy. Finally, discuss how to take the negative of a game.



2. *Sums of games*: Discuss how to add two games x + y. (recall from reading)

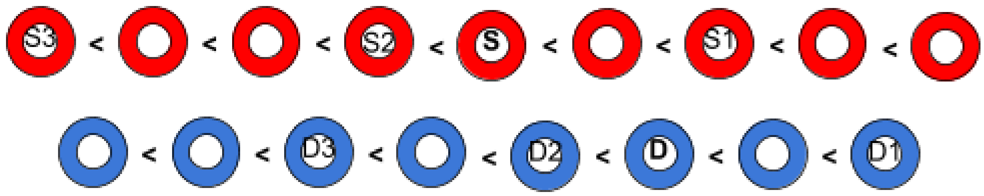
3. *Equational proofs*: Play the following 3 patterns simultaneously. From the previous games, we know this game represents the sum of the patterns. Based on your winning strategy, can you tell what number this sum is? And can you guess what number each pattern represents? (hint: the last pattern is related to 1)



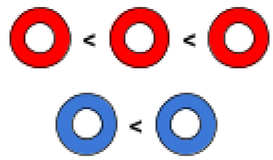
Ehrenfeucht–Fraïssé Games

Pick a role form 2 players: Spoiler (Red) & Duplicator (Blue). Two players takes turn putting down a pebble on their chain. The duplicator must put down the pebbles to match (as explained below) the pattern of the spoiler. If the duplicator can keep matching until pebbles run out, s/he wins. Otherwise the spoiler wins (the duplicator cannot match when there's pebbles remaining).

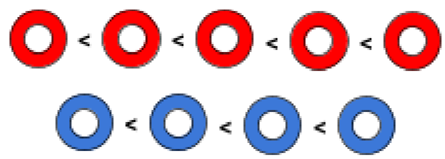
To match a move, the duplicator must put down a pebble in a position relative to the existing pebbles similarly as the spoiler's last move. For example, in the game play below, the spoiler's last move is **S**, which is to the left of (less than) S1 but to the right (greater than) of S3 & S2. The Duplicator can move to **D**, which is to the left of D1 and to the right of D2 & D3.



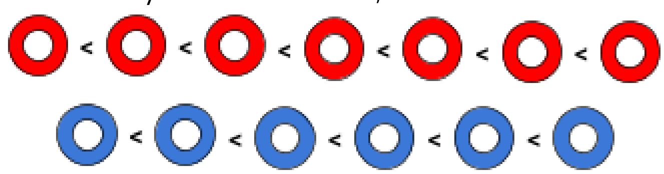
1. *2 & 3 chains w/2 moves*: each of you have 2 pebbles, and the chain looks like this:



2. *4 & 5 chains w/2 moves*: each of you have 2 moves, and the chain looks like:



3. *6&7 chains w/3 moves*: each of you have 3 moves, and the chain looks like:



We will try to work out the strategy together, and find out connections to logic.