CSE 490 GZ
Introduction to Data Compression
Winter 2002

Golomb Codes
Tunstall Codes

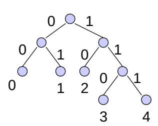
Run-Length Coding

- Lots of 0's and not too many 1's.
  - Fax of letters
  - Graphics
- Simple run-length code
  - 00000010000000001000000000010001001.....
  - 6 9 10 3 2 ...
  - Code the bits as a sequence of integers

Golomb Code of Order m

- Let n = qm + r where $0 \leq r < m$.
  - Divide m into n to get the quotient q and remainder r.
- Code for n has two parts:
  1. q is coded in unary
  2. r is coded as a fixed prefix code

Example: m = 5



code for r

Example

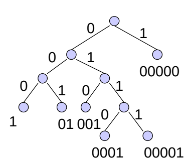
- n = qm + r is represented by:

$$\overbrace{11\cdots1}^{q}0\hat{r}$$

  - where $\hat{r}$ is the fixed prefix code for r
- Example:

2	6	9	10	27
010	1001	10111	11000	11111010

Alternative Explanation Golomb Code



input	output
00000	1
00001	0111
0001	0110
001	010
01	001
1	000

Run Length Example: m = 5

$\boxed{00000}$01000000000100000000000010001001.....
1
00000$\boxed{01}$000000000100000000000010001001.....
001
0000001$\boxed{00000}$0000100000000000010001001.....
1
000000100000$\boxed{00001}$00000000000010001001.....
0111

In this example we coded 17 bit in only 9 bits.

Choosing m

- Suppose that 0 has the probability p and 1 has probability 1-p.
- The probability of 0^n1 is $p^n(1-p)$. The Golomb code of order
$$m = \left\lceil \frac{-1}{\log_2 p} \right\rceil$$
is optimal.
- Example: p = 127/128.
$$m = \left\lceil \frac{-1}{\log_2 (127/128)} \right\rceil = 89$$

Average Bit Rate for Golomb Code

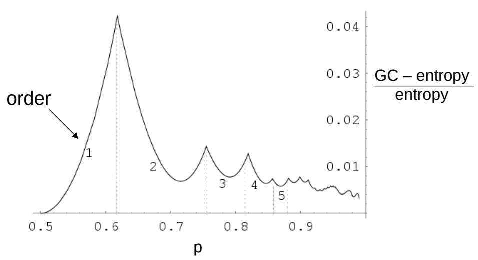
$$\text{Average Bit Rate} = \frac{\text{Average output code length}}{\text{Average input code length}}$$

- m = 4 as an example. With p as the probability of 0.

$$ABR = \frac{p^4 + 3(1-p^4)}{4p^4 + 4p^3(1\text{-}p) + 3p^2(1\text{-}p) + 2p(1-p) + (1-p)}$$

ouput	1	011	010	001	000
input	0000	0001	001	01	1
probability	p^4	$p^3(1\text{-}p)$	$p^2(1\text{-}p)$	$p(1\text{-}p)$	1-p

Comparison of GC with Entropy



Notes on Golomb codes

- Useful for binary compression when one symbol is much more likely than another.
  - binary images
  - fax documents
  - bit planes for wavelet image compression
- Need a parameter (the order)
  - training
  - adaptively learn the right parameter
- Variable-to-variable length code
- Last symbol needs to be a 1
  - coder always adds a 1
  - decoder always removes a 1

Tunstall Codes

- Variable-to-fixed length code
- Example

input	output
a	000
b	001
ca	010
cb	011
cca	100
ccb	101
ccc	110

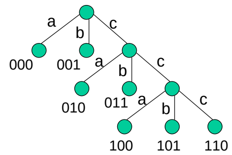
a b cca cb ccc ...
000 001 110 011 110 ...

Tunstall code Properties

1. No input code is a prefix of another to assure unique encodability.
2. Minimize the number of bits per symbol.

Prefix Code Property

a	000
b	001
ca	010
cb	011
cca	100
ccb	101
ccc	110



Unused output code is 111.

Use for unused code

- Consider the string "cc". It does not have a code.
- Send the unused code and some fixed code for the cc.
- Generally, if there are k internal nodes in the prefix tree then there is a need for k-1 fixed codes.

Designing a Tunstall Code

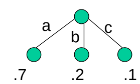
- Suppose there are m initial symbols.
- Choose a target output length n where $2^n > m$.

1. Form a tree with a root and m children with edges labeled with the symbols.
2. If the number of leaves is $> 2^n - m$ then halt.*
3. Find the leaf with highest probability and expand it to have m children.** Go to 2.

* In the next step we will add m-1 more leaves.
** The probability is the product of the probabilities of the symbols on the root to leaf path.

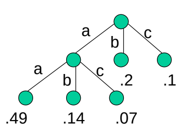
Example

- P(a) = .7, P(b) = .2, P(c) = .1
- n = 3



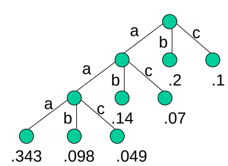
Example

- P(a) = .7, P(b) = .2, P(c) = .1
- n = 3



Example

- P(a) = .7, P(b) = .2, P(c) = .1
- n = 3

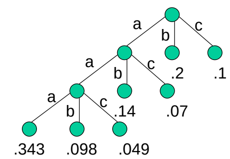


Bit Rate of Tunstall

- The length of the output code divided by the average length of the input code.
- Let p_i be the probability of and r_i the length of input code i ($1 \leq i \leq s$) and let n be the length of the output code.

$$\text{Average bit rate} = \frac{n}{\sum_{i=1}^{s} p_i r_i}$$

Example



ABR = 3/[3 (.343 + .098 + .049) + 2 (.14 + .07) + .2 + .1]
= 1.37 bits per symbol
Entropy = 1.16 bits per symbol

Notes on Tunstall Codes

- Variable-to-fixed length code
- Error resilient
  - A flipped bit will introduce just one error in the output
  - Huffman is not error resilient. A single bit flip can destroy the code.