CSE 490 GZ
Introduction to Data Compression
Winter 2002

Course Policies
Introduction to Data Compression
Entropy
Prefix Codes

Instructors

- Instructor
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Prerequisites

- CSE 142, 143
- CSE 326 or CSE 373
- Reason for the prerequisites:
  - Data compression has many algorithms
  - Some of the algorithms require complex data structures

Resources

- Text Book
  - Khalid Sayood, Introduction to Data Compression, Second Edition, Morgan Kaufmann Publishers, 2000.
- 490gz Course Web Page
  - http://www.cs.washington.edu/490gz/
- Papers and Sections from Books
- E-mail list
  - Send mail to majordomo to subscribe

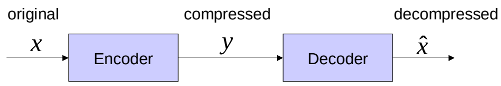
Engagement by Students

- Weekly Assignments
  - Understand compression methodology
  - Due in class on Fridays (except midterm Friday)
  - No late assignments accepted except with prior approval
- Programming Projects
  - Experimental comparison of compression methods
  - Modification of compression methods.
  - Build a decoder from an encoder.

Final Exam and Grading

- Final Exam - 8:30-10:20 a.m. Tuesday, March 19, 2002
- Midterm Exam – Friday, February 8, 2002
- Percentages
  - Weekly assignments (25%)
  - Midterm exam (20%)
  - Projects (15%)
  - Final exam (40%)

Basic Data Compression Concepts



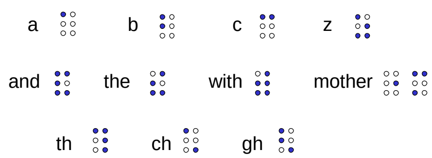
- Lossless compression $x = \hat{x}$
  – Also called entropy coding, reversible coding.
- Lossy compression $x \neq \hat{x}$
  – Also called irreversible coding.
- Compression ratio = $|x|/|y|$
  – $|x|$ is number of bits in x.

Why Compress

- Conserve storage space
- Reduce time for transmission
  – Faster to encode, send, then decode than to send the original
- Progressive transmission
  – Some compression techniques allow us to send the most important bits first so we can get a low resolution version of some data before getting the high fidelity version
- Reduce computation
  – Use less data to achieve an approximate answer

Braille

- System to read text by feeling raised dots on paper (or on electronic displays). Invented in 1820s by Louis Braille, a French blind man.



Braille Example

Clear text:
Call me Ishmael. Some years ago -- never mind how long precisely -- having \\ little or no money in my purse, and nothing particular to interest me on shore, \\ I thought I would sail about a little and see the watery part of the world. (238 characters)

Grade 2 Braille in ASCII.
,call me ,i\%mael4 ,‘‘s ye$>$s ago -- n‘‘e m9d h[l;g precisely -- hav+ \\ ll or no m‘‘oy 9 my purse1 \& no?+ ‘‘picul$>$ 6 9t]e/ me on \%ore1 \\ ,i $?$‘‘$|$,i wd sail ab a ll \& see ! wat]y ‘‘p (! _w4 (203 characters)

Compression ratio = 238/203 = 1.17

Lossless Compression

- Data is not lost - the original is really needed.
  – text compression
  – compression of computer binaries to fit on a floppy
- Compression ratio typically no better than 4:1 for lossless compression on many kinds of files.
- Statistical Techniques
  – Huffman coding
  – Arithmetic coding
  – Golomb coding
- Dictionary techniques
  – LZW, LZ77
  – Sequitur
  – Burrows-Wheeler Method
- Standards - Morse code, Braille, Unix compress, gzip, zip, bzip, GIF, JBIG, Lossless JPEG

Lossy Compression

- Data is lost, but not too much.
  – audio
  – video
  – still images, medical images, photographs
- Compression ratios of 10:1 often yield quite high fidelity results.
- Major techniques include
  – Vector Quantization
  – Wavelets
  – Block transforms
  – Standards - JPEG, MPEG

Why is Data Compression Possible

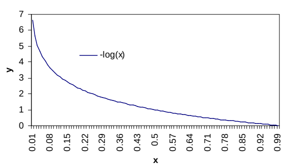
- Most data from nature has redundancy
  - There is more data than the actual information contained in the data.
  - Squeezing out the excess data amounts to compression.
  - However, unsqeezing out is necessary to be able to figure out what the data means.
- Information theory is needed to understand the limits of compression and give clues on how to compress well.

Information Theory

- Developed by Shannon in the 1940's and 50's
- Attempts to explain the limits of communication using probability theory.
- Example: Suppose English text is being sent
  - Suppose a "t" is received. Given English, the next symbol being a "z" has very low probability, the next symbol being a "h" has much higher probability. Receiving a "z" has much more information in it than receiving a "h". We already knew it was more likely we would receive an "h".

First-order Information

- Suppose we are given symbols $\{a_1, a_2, \ldots, a_m\}$.
- $P(a_i)$ = probability of symbol a_i occurring in the absence of any other information.
  - $P(a_1) + P(a_2) + \ldots + P(a_m) = 1$
- $inf(a_i) = -\log_2 P(a_i)$ bits is the information of a_i in bits.



Example

- {a, b, c} with P(a) = 1/8, P(b) = 1/4, P(c) = 5/8
  - $inf(a) = -\log_2(1/8) = 3$
  - $inf(b) = -\log_2(1/4) = 2$
  - $inf(c) = -\log_2(5/8) = .678$
- Receiving an "a" has more information than receiving a "b" or "c".

First Order Entropy

- The first order entropy is defined for a probability distribution over symbols $\{a_1, a_2, \ldots, a_m\}$.

$$H = -\sum_{i=1}^{m} P(a_i)\log_2(P(a_i))$$

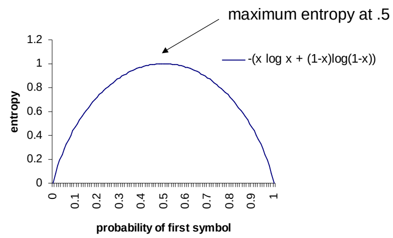
- H is the average number of bits required to code up a symbol, given all we know is the probability distribution of the symbols.
- H is the Shannon lower bound on the average number of bits to code a symbol in this "source model".
- Stronger models of entropy include context. We'll talk about this later.

Entropy Examples

- {a, b, c} with a 1/8, b 1/4, c 5/8.
  - H = 1/8 *3 + 1/4 *2 + 5/8* .678 = 1.3 bits/symbol
- {a, b, c} with a 1/3, b 1/3, c 1/3. (worst case)
  - $H = -3^* (1/3)^*\log_2(1/3) = 1.6$ bits/symbol
- {a, b, c} with a 1, b 0, c 0 (best case)
  - $H = -1^*\log_2(1) = 0$
- Note that the standard coding of 3 symbols takes 2 bits.

Entropy Curve

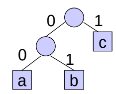
- Suppose we have two symbols with probabilities x and 1-x, respectively.



A Simple Prefix Code

- {a, b, c} with a 1/8, b 1/4, c 5/8.
- A prefix code is defined by a binary tree
- Prefix code property
  - no output is a prefix of another

tree

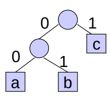


code

input	output
a	00
b	01
c	1

ccabccbccc
1 1 00 01 1 1 01 1 1 1

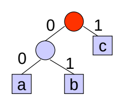
Decoding a Prefix Code




```
repeat
start at root of tree
  repeat
    if read bit = 1 then go right
    else go left
  until node is a leaf
  report leaf
until end of the code
```

11000111100

Decoding a Prefix Code



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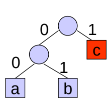
Decoding a Prefix Code



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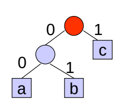
c

Decoding a Prefix Code



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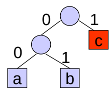
c

Decoding a Prefix Code



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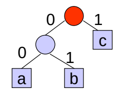
Decoding a Prefix Code



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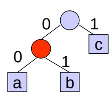

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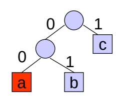
cc

Decoding a Prefix Code



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cca

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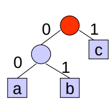

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cca

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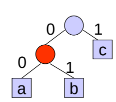

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cca

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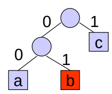

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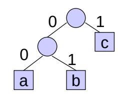
ccab

Decoding a Prefix Code



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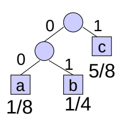
ccabccca

How Good is the Code



bit rate = (1/8)2 + (1/4)2 + (5/8)1 = 11/8 = 1.375 bps
Entropy = 1.3 bps
Standard code = 2 bps

(bps = bits per symbol)