CSE 484/M584: Computer Security (and Privacy)

Spring 2025

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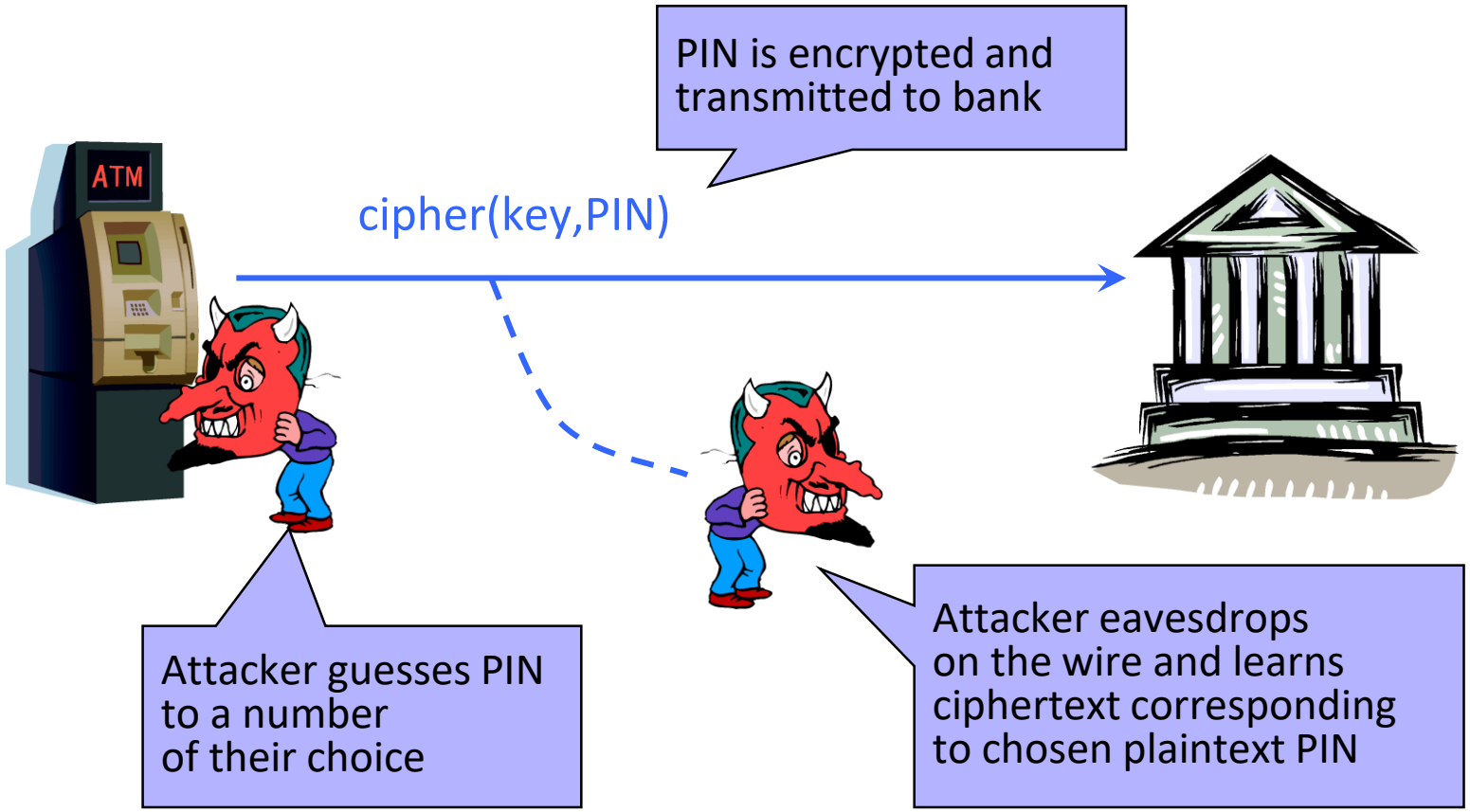
Admin

- HW1 due Wednesday
- Lab 2 (Cryptolab) next Wednesday
  - Note there was an update to the short answer CTR question (there is an oracle you can interact with now!)
  - Start now if you haven't!
- Lab 1a/b Exploits
  - Check partner handin status ASAP!
  - We will file CSSC cases shortly

How Can a Cipher Be Attacked?

- Attackers knows ciphertext and encryption algorithm
  - What else does the attacker know? Depends on the application in which the cipher is used!
- Ciphertext-only attack
- KPA: Known-plaintext attack (stronger)
  - Knows some plaintext-ciphertext pairs
- CPA: Chosen-plaintext attack (even stronger)
  - Can obtain ciphertext for any plaintext of his choice

Chosen Plaintext Attack



… repeat for any PIN value

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- CPA: Chosen-plaintext attack (even stronger)
  - Can obtain ciphertext for any plaintext of his choice
- CCA: Chosen-ciphertext attack (very strong)
  - Can decrypt any ciphertext except the target

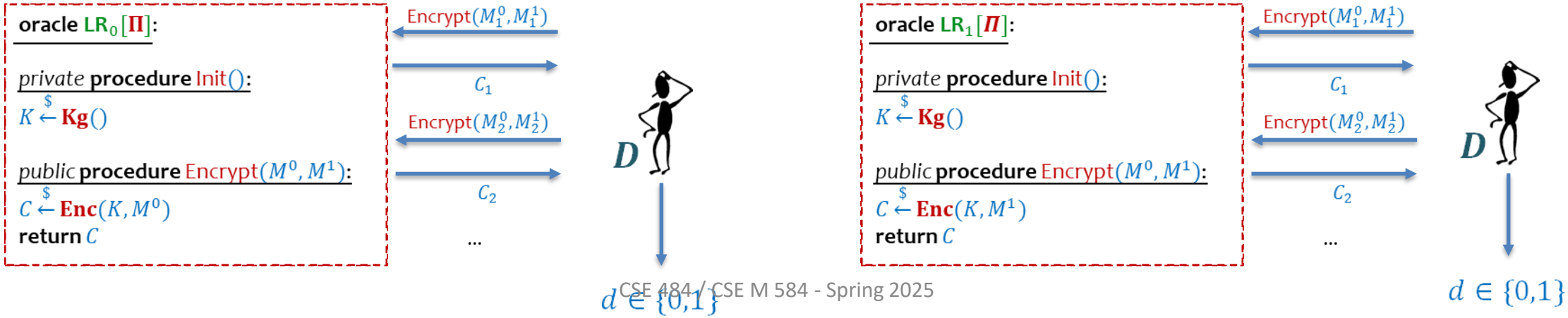
Very Informal Intuition

Minimum security requirement for a modern encryption scheme

- Security against chosen-plaintext attack (CPA)
  - Ciphertext leaks no information about the plaintext
  - Even if the attacker correctly guesses the plaintext, they cannot verify their guess
  - Every ciphertext is unique, encrypting same message twice produces completely different ciphertexts
    - Implication: encryption must be randomized or stateful

The Shape of the Formal Approach

- INDistinguishability under Chosen Plaintext Attack ("IND-CPA")
- Formalized cryptographic game
  - Adversary submits pairs of plaintexts (M_0, M_1)
  - Gets back ONE of the ciphertexts (C_b)
  - Adversary must **guess** which ciphertext this is (C_0 or C_1)
  - If they can do better than 50/50, they win



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- Security against chosen-ciphertext attack (CCA)
  - Integrity protection – it is not possible to change the plaintext by modifying the ciphertext

Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string K, called the key.

- Asymmetric cryptography
  - Each party creates a public key pk and a secret key sk.

Asymmetric Setting for Encryption

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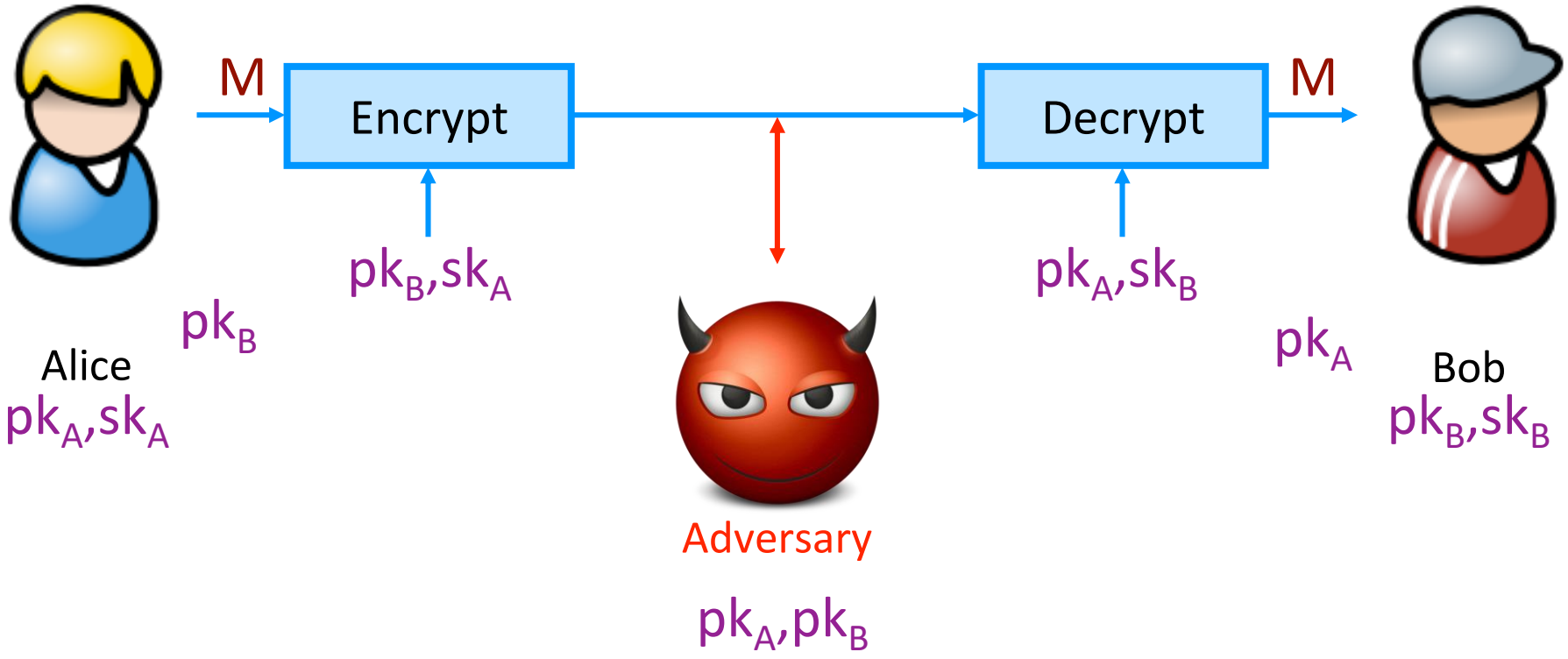
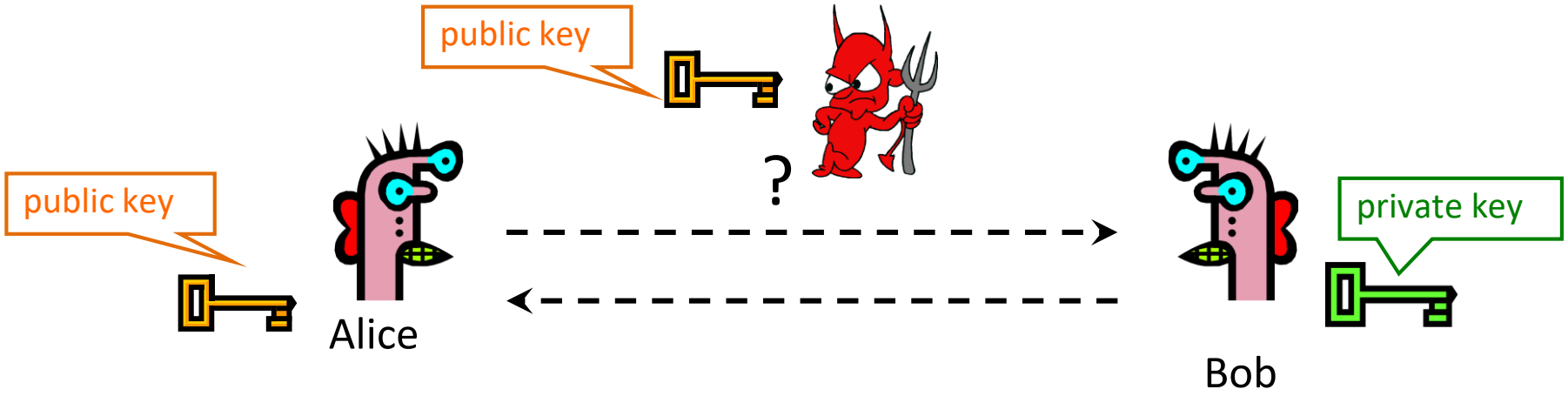


Public Key Crypto: Basic Problem



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate a message

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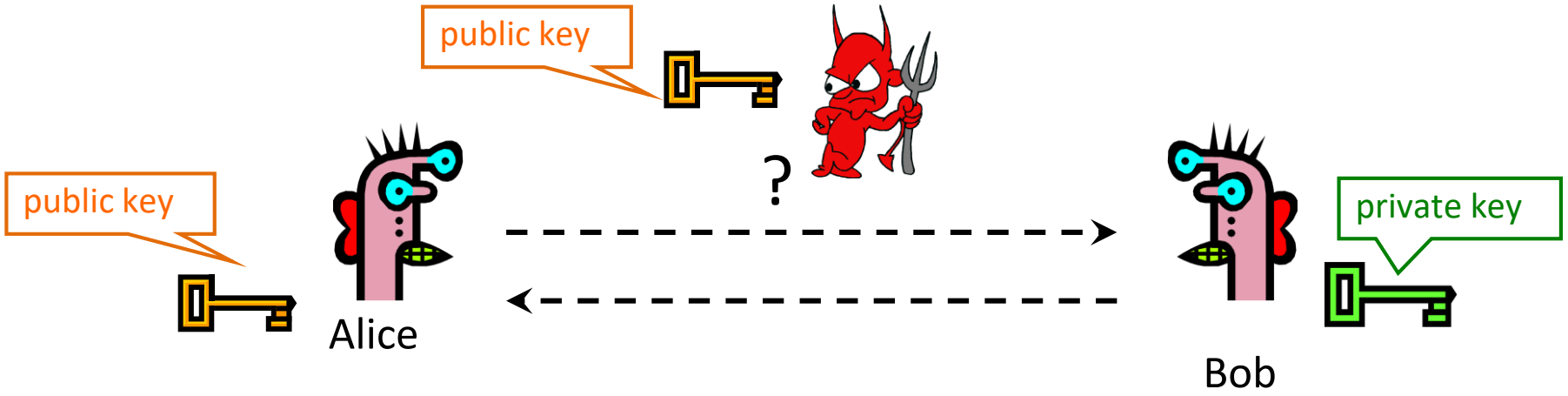

Ignore for now: How do we know it's REALLY Bob's??

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Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for integrity
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- Digital signatures for integrity
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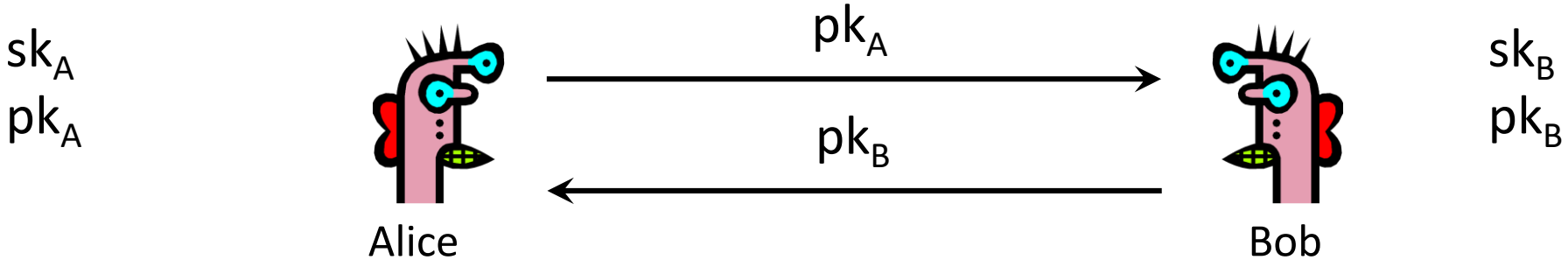
- Session key establishment / "Key exchange"
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

Key Exchange



Compute shared secret k= KEx(sk_A , pk_B)

Compute shared secret k= KEx(sk_B , pk_A)

Groups

- Group: A set G of elements and an operation $\oplus$ such that:
  - Associative: $(a \oplus b) \oplus c = a \oplus (b \oplus c)$
  - Identity: $a \oplus I = a$
  - Inverse: $a \oplus a^{-1} = I$

Notation: $a^2 = a \oplus a$, $a^3 = a \oplus a \oplus a$, …

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- Example Group 1: Additive Group of Integers Modulo n (Z_n or Z/nZ)
  - Special case: $n = p$ where p is a prime (Z_p)
  - $G = \{0, 1, \ldots, p\text{-}1\}$
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- Example Group 2: Multiplicative Group of Integers Modulo n (Z_n^*or $(Z/nZ)^*$)
  - Special case: n = p where p is a prime
  - $G = \{1, 2, \ldots, p\text{-}1\}$
  - $\oplus = *$ mod p

Common Groups Under Modular Arithmetic

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  - Can we find a generator?

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  - Can we find a generator?
  - ALL non-identity elements are generators for prime-order groups!

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gradescope!

Hardness Assumptions Over Groups

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  - Compute $X = g^x$
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Key Generation

- Public info on group G: order p and generator g

Alice

Pick secret key sk ← {1, 2, …, p}

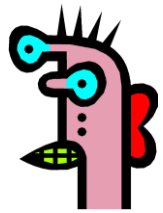
Set public key pk ← g^{sk}

Diffie-Hellman Protocol

- Alice and Bob never met and share no secrets
- Public info on group G: order p and generator g

$sk_A \leftarrow x$
$pk_A \leftarrow g^x$

Alice



Bob

$sk_B \leftarrow y$
$pk_B \leftarrow g^y$

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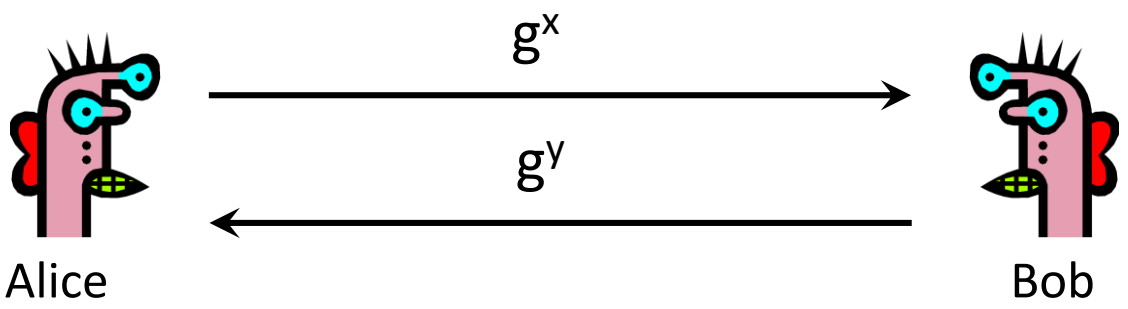

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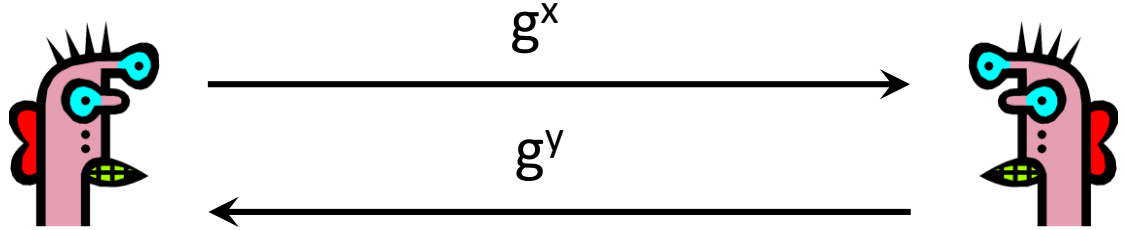

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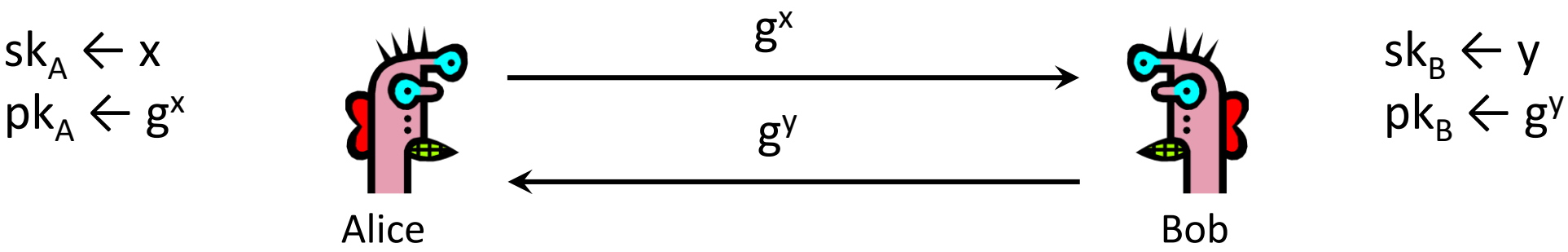

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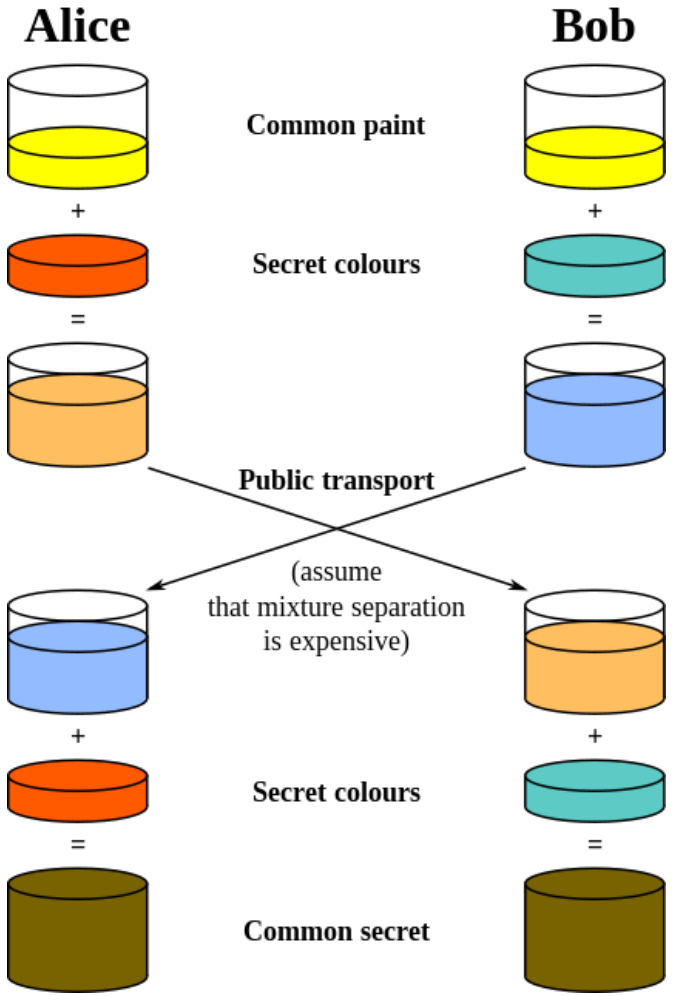
Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:

g^x

g^y

Common secret: g^{xy}

[from Wikipedia]

Why is Diffie-Hellman Secure?

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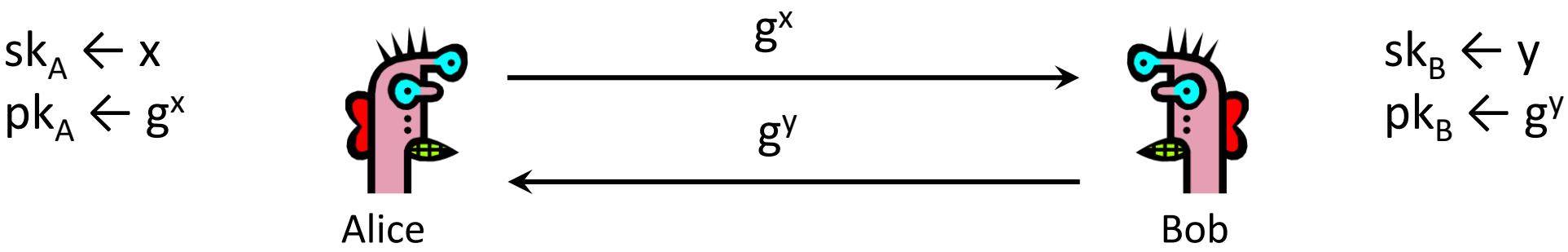

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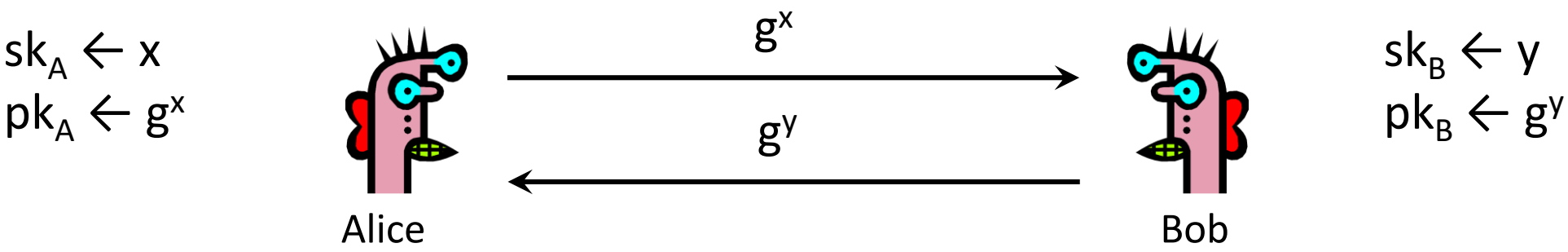

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- Caveat: Assumption doesn't hold or holds differently for different groups!
  - For ~128 bits of security:
  - Z_p: Not secure! Discrete log just corresponds to modular division!
  - Z_p*: 2048-4096 bit prime SAFE p = 2q+1 for prime q, use generator for subgroup of size q

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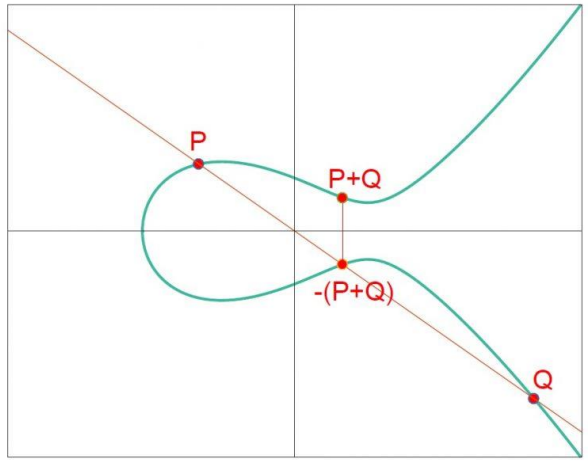

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  - Elliptic curves: (x,y) coordinates in Z_p for 256 bit prime p

Person-in-the-Middle Attacks

- Diffie-Hellman protocol (by itself) does not provide integrity (against <u>active</u> attackers)

$sk_A \leftarrow x$
$pk_A \leftarrow g^x$

$sk_B \leftarrow y$
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Stepping Back: Asymmetric Crypto

- We've just seen session key establishment
  - Can then use shared key for symmetric crypto
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For integrity