CSE 484/M584: Computer Security (and Privacy)

Spring 2025

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Admin

- HW1 due Wednesday
- Lab 2 (Cryptolab) next Wednesday
  - Note there was an update to the short answer CTR question (there is an oracle you can interact with now!)
  - Start now if you haven't!
- Lab 1a/b Exploits
  - Check partner handin status ASAP!
  - We will file CSSC cases shortly

How Can a Cipher Be Attacked?

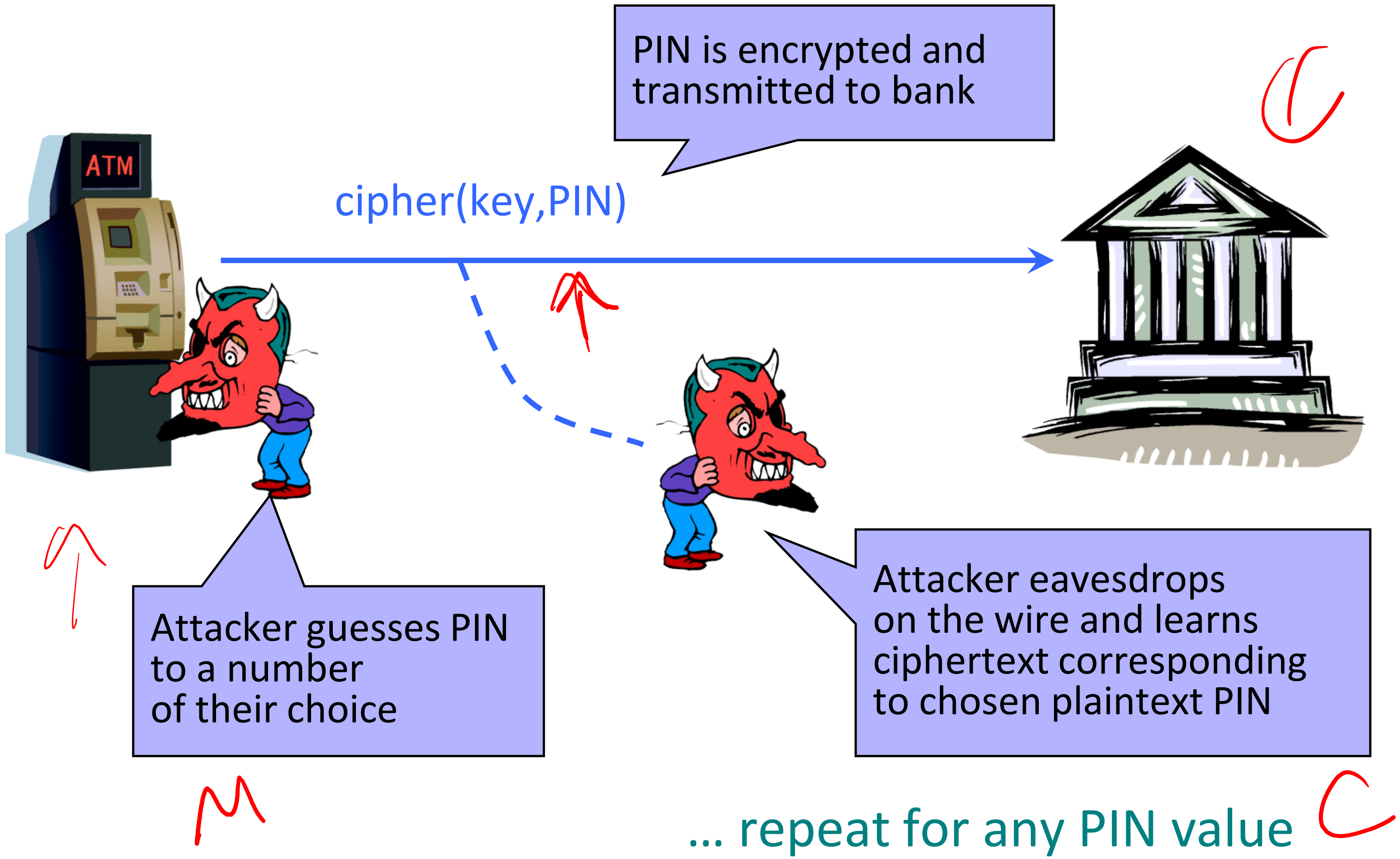
- Attackers knows ciphertext and encryption algorithm
  - What else does the attacker know? Depends on the application in which the cipher is used!
- Ciphertext-only attack
- KPA: Known-plaintext attack (stronger)
  - Knows some plaintext-ciphertext pairs
- CPA: Chosen-plaintext attack (even stronger)
  - Can obtain ciphertext for any plaintext of his choice

M_0, C_0
M_1, C_1

M_0, M_1, M_2
$C_0\ C_1\ C_2$

Chosen Plaintext Attack

CPA



… repeat for any PIN value C

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- CPA: Chosen-plaintext attack (even stronger)
  - Can obtain ciphertext for any plaintext of his choice
- CCA: Chosen-ciphertext attack (very strong)
  - Can decrypt any ciphertext <u>except</u> the target

C_x

$C_0 \; C_1$

$C_0 \; C_1 \; C_2$

$M_0 \; M_1 \; M_2$

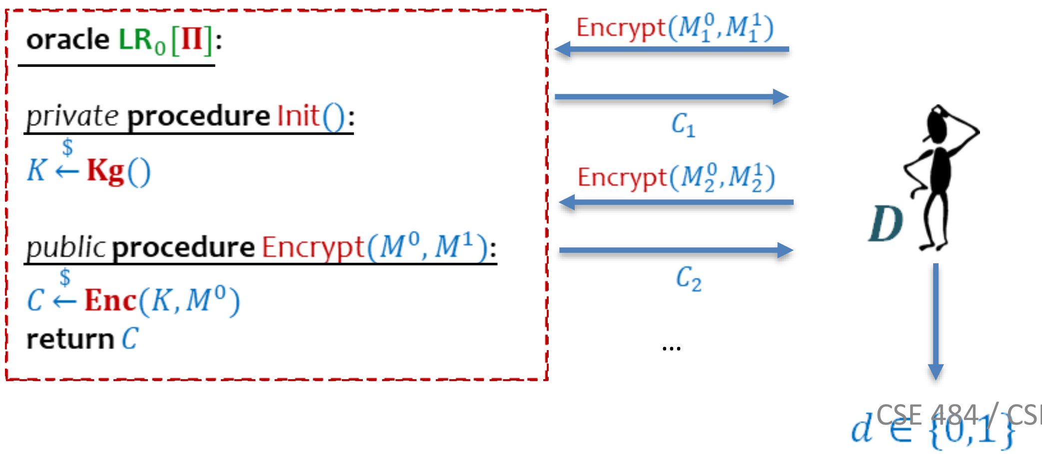
Very Informal Intuition

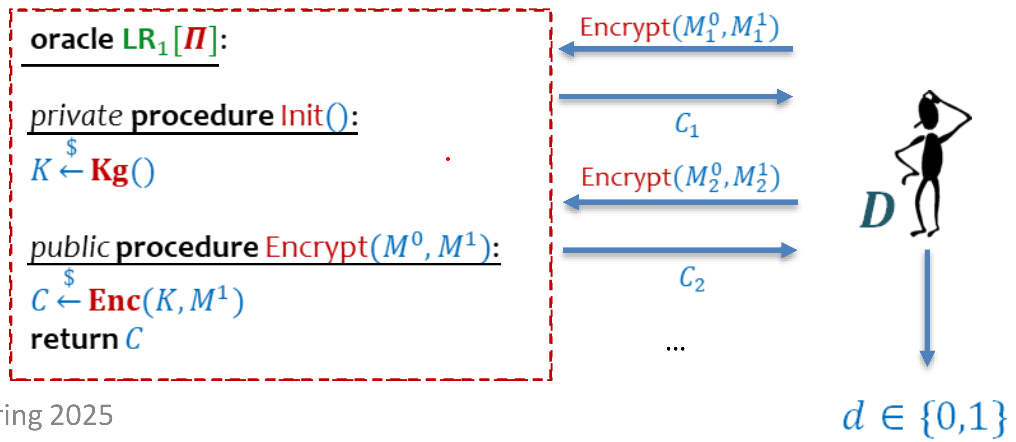
Minimum security requirement for a modern encryption scheme

- Security against chosen-plaintext attack (CPA)
  - Ciphertext leaks no information about the plaintext
  - Even if the attacker correctly guesses the plaintext, they cannot verify their guess
  - Every ciphertext is unique, encrypting same message twice produces completely different ciphertexts
    - Implication: encryption must be randomized or stateful

The Shape of the Formal Approach

- INDistinguishability under Chosen Plaintext Attack ("IND-CPA")
- Formalized cryptographic game
  - Adversary submits pairs of plaintexts (M_0, M_1)
  - Gets back ONE of the ciphertexts (C_b)
  - Adversary must **guess** which ciphertext this is (C_0 or C_1)
  - If they can do better than 50/50, they win





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- Security against chosen-ciphertext attack (CCA)
  - Integrity protection – it is not possible to change the plaintext by modifying the ciphertext

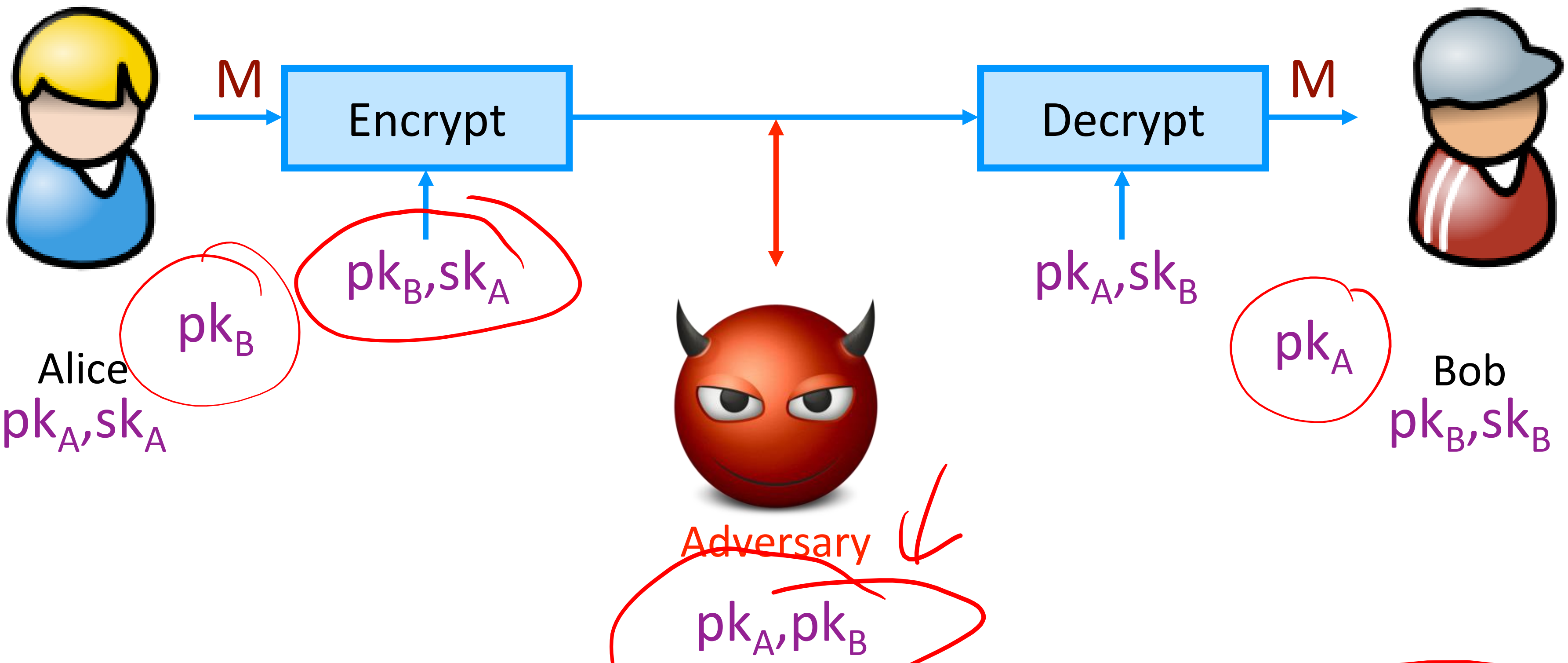
Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string K, called the key.

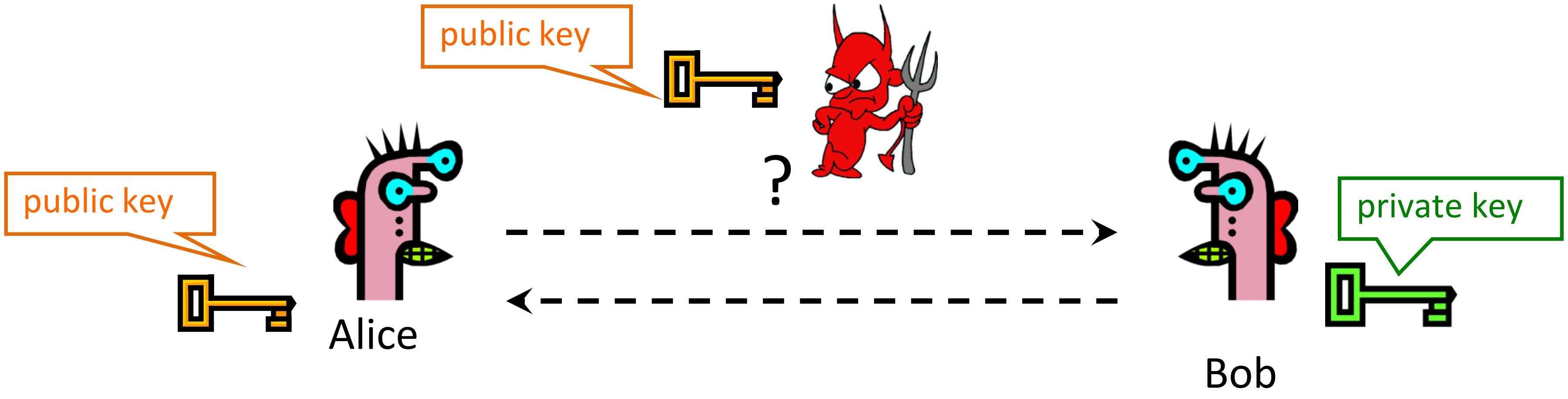
- Asymmetric cryptography
  - Each party creates a public key pk and a secret key sk.

Asymmetric Setting for Encryption

Each party creates a public key pk and a secret key sk



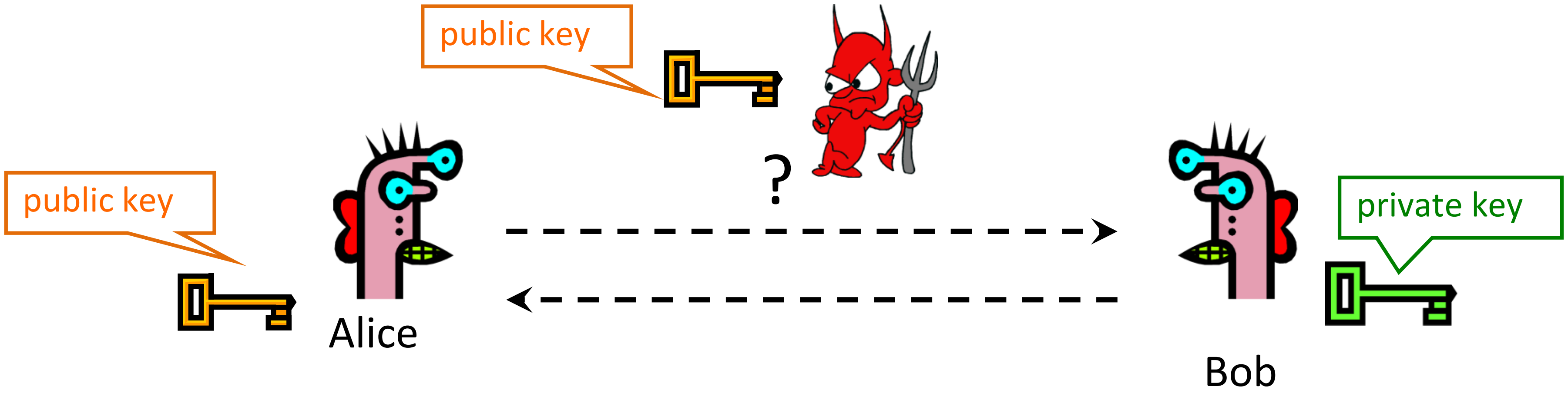
Public Key Crypto: Basic Problem



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate a message

Public Key Crypto: Basic Problem



Ignore for now: How do we know it's REALLY Bob's??

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Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for integrity
  - Can "sign" a message with your private key

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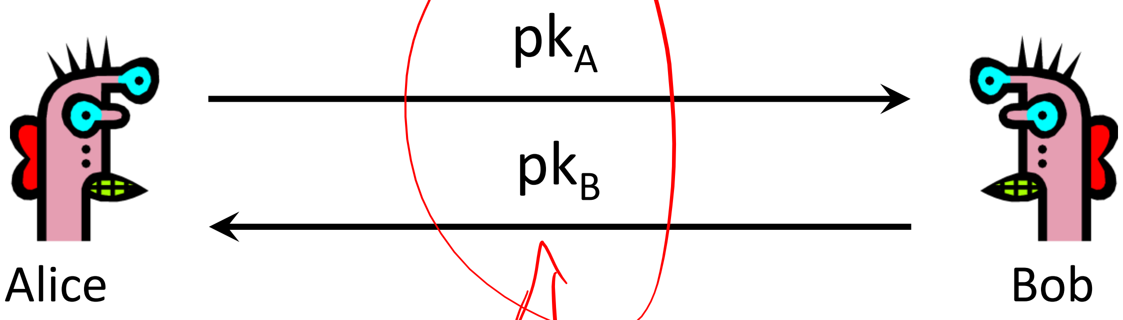
- Digital signatures for integrity
  - Can "sign" a message with your private key
- Session key establishment / "Key exchange"
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

K

Key Exchange

sk_A
pk_A



sk_B
pk_B

Compute shared secret $k = \text{KEx}(sk_A, pk_B)$

Compute shared secret $k = \text{KEx}(sk_B, pk_A)$

Groups

- Group: A set G of elements and an operation $\oplus$ such that:
  - Associative: $(a \oplus b) \oplus c = a \oplus (b \oplus c)$
  - Identity: $a \oplus I = a$
  - Inverse: $a \oplus a^{-1} = I$

Notation: $a^2 = a \oplus a$, $a^3 = a \oplus a \oplus a$, …

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  - Order: Number of elements in group
  - Optional useful property: Cyclic: $\{g, g^2, g^3, \ldots, g^{order}\} = G$ for "generator" g

Notation: $a^2 = a \oplus a$, $a^3 = a \oplus a \oplus a$, …

$g, g \oplus g, g \oplus g \oplus g$

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n-1

- Example Group 1: Additive Group of Integers Modulo n (Z_n or Z/nZ)
  - Special case: $n = p$ where p is a prime (Z_p)
  - $G = \{0, 1, \ldots, p-1\}$
  - $\oplus = +$ mod p

Z_n

p mod p

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- Example Group 2: Multiplicative Group of Integers Modulo n (Z_n*or (Z/nZ)*)
  - Special case: $n = p$ where p is a prime
  - $G = \{1, 2, \ldots, p\text{-}1\}$
  - $\oplus = *$ mod p

Notation: $a^2 = a \oplus a$, $a^3 = a \oplus a \oplus a$, …

Z_p^*

Common Groups Under Modular Arithmetic

- Additive Group of Integers Modulo prime p (Z_p)
  - Example: p=11
  - Can we find a generator?

$+ \bmod 11$

$g = 10$

$10 \bmod 11 = 10$

$g^2 \quad (10+10) \bmod 11 = 9$

$g^3 \quad (10+10+10) \bmod 11 = 8$

$g^4 \quad 7$

g^5

Common Groups Under Modular Arithmetic

- Additive Group of Integers Modulo prime p (Z_p)
  - Example: $p=11$
  - Can we find a generator?
  - ALL non-identity elements are generators for prime-order groups!

Common Groups Under Modular Arithmetic

G = { 1 ... p-1 }

- Multiplicative Group of Integers Modulo prime p (Z_p*)
  - Example: p=11
  - Can we find a generator?

a ⊕ b =

a * b mod p

g ??

g¹

g²

g³ ...

2, 4, 8, 16,

2 4 8, 5, 10, 9, 7, 3, 6

7, 5, 2, 3, 10, 4, 6, 8, 1

7 49

7, 10

Common Groups Under Modular Arithmetic

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  - Example: p=11
  - Can we find a generator?

gradescope!

Hardness Assumptions Over Groups

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- Discrete Logarithm (DL) problem over G for random generator g:
  - Pick random $x \leftarrow \{1, 2, \ldots, \text{order}\}$
  - Compute $X = g^x$
  - Problem: Given g and X, compute x

$X = g^x$

$X = 9$

$g = 7$

$g^x = 9 \bmod 11$

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- Computational Diffie-Hellman (CDH) problem:
  - Pick random $x, y \leftarrow \{1, 2, \ldots, \text{order}\}$
  - Compute $X = g^x$ and $Y = g^y$
  - Problem: Given g, X, and Y, compute g^{xy}

x Y g

Key Generation

$\mathbb{Z}_p^*$

- Public info on group G: order p and generator g

Alice

Pick secret key sk ← {1, 2, …, p}

Set public key pk ← g^{sk}

Diffie-Hellman Protocol

- Alice and Bob never met and share no secrets
- <u>Public</u> info on group G: order p and generator g

$sk_A \leftarrow x$
$pk_A \leftarrow g^x$

Alice

Bob

$sk_B \leftarrow y$
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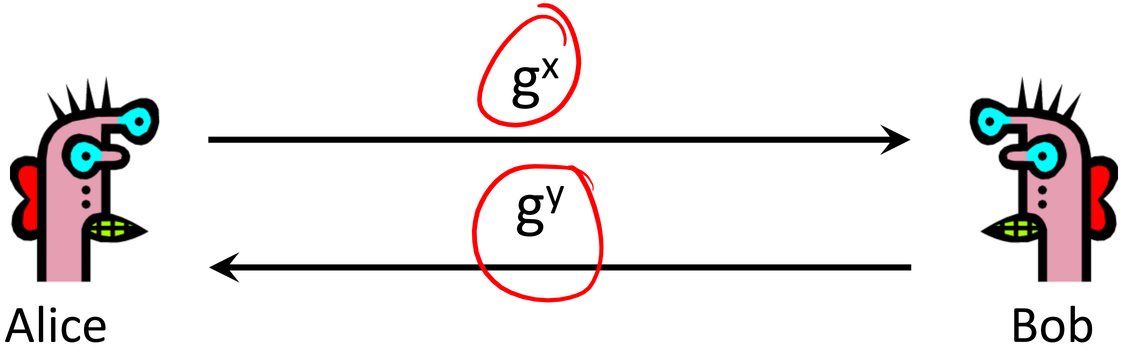

$sk_B \leftarrow y$
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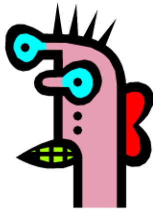
Alice

$\xrightarrow{g^x}$

$\xleftarrow{g^y}$


Bob

$sk_B \leftarrow y$
$pk_B \leftarrow g^y$

Compute shared secret $k=(g^y)^x=g^{xy}$

$Kex(P_B, S_A)$

Compute shared secret $k=(g^x)^y=g^{xy}$

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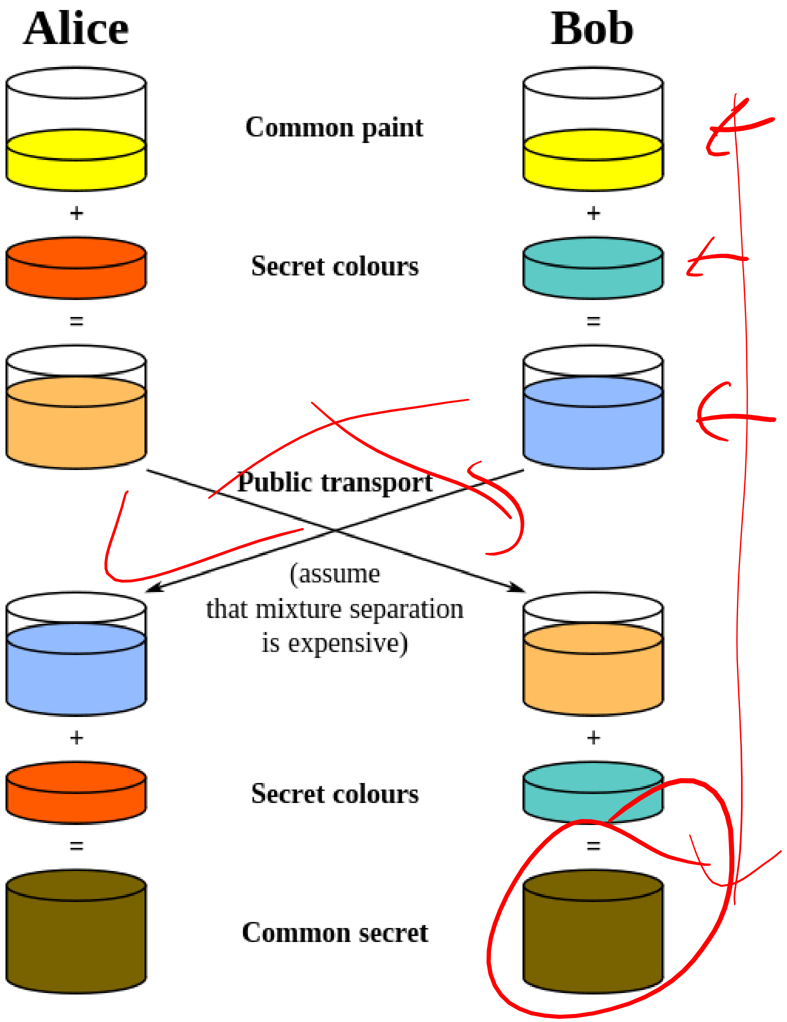
Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:

g^x

g^y

Common secret: g^{xy}

[from Wikipedia]

Why is Diffie-Hellman Secure?

- Alice and Bob never met and share no secrets
- Public info on group G: order p and generator g

$sk_A \leftarrow x$
$pk_A \leftarrow g^x$

Alice

$g^x \longrightarrow$

$\longleftarrow g^y$

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Exactly the CDH problem!

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- Caveat: Assumption doesn't hold or holds differently for different groups!
  - For ~128 bits of security:
  - Z_p: Not secure! Discrete log just corresponds to modular division!
  - Z_p*: 2048-4096 bit prime SAFE p = 2q+1 for prime q, use generator for subgroup of size q

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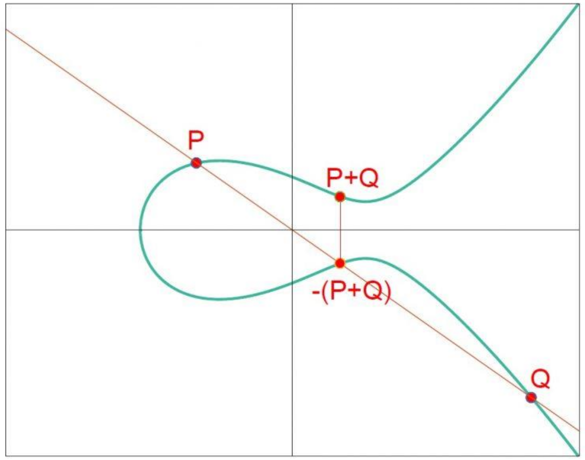

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  - Elliptic curves: (x,y) coordinates in Z_p for 256 bit prime p

Person-in-the-Middle Attacks

- Diffie-Hellman protocol (by itself) does not provide integrity (against <u>active</u> attackers)

$sk_A \leftarrow x$
$pk_A \leftarrow g^x$

$sk_B \leftarrow y$
$pk_B \leftarrow g^y$

Stepping Back: Asymmetric Crypto

- We've just seen session key establishment
  - Can then use shared key for symmetric crypto
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For integrity