

Section 3: Symmetric Encryption

How can two parties use a common secret to communicate privately in the presence of eavesdroppers?

Slides by current and former TAs



Administrivia

HW 1 (Threat Modeling) due 4/23 @ 11:59 pm

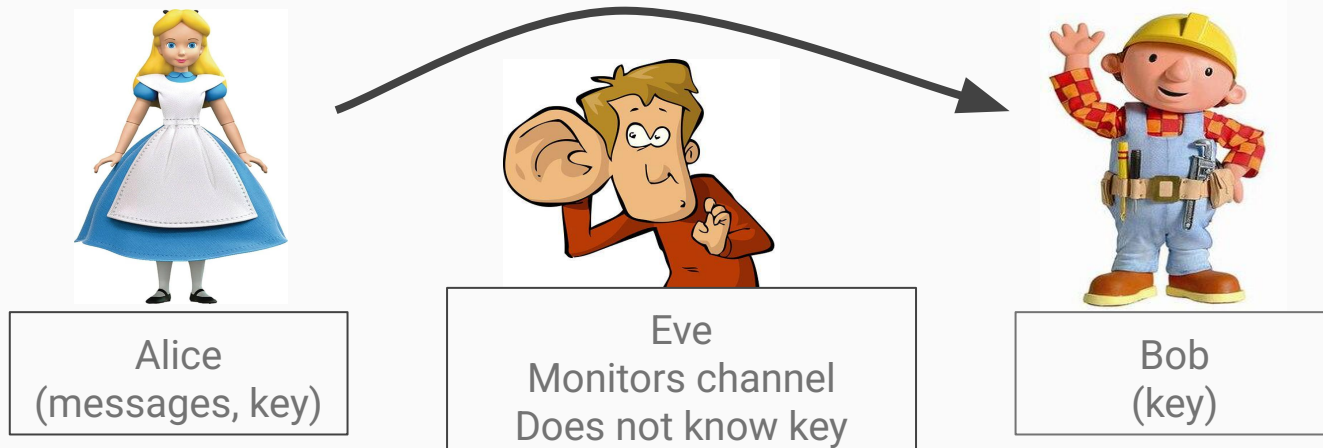
Lab 2 out!

- 3 required programming problems + 1 extra credit (not much harder)
- Wait until you learn about RSA for goldmine-of-ps-and-qs
- Individual write-ups
- Individual written problems (separate from the lab)

It is a new lab, please be patient with us!

Symmetric Encryption: Goal

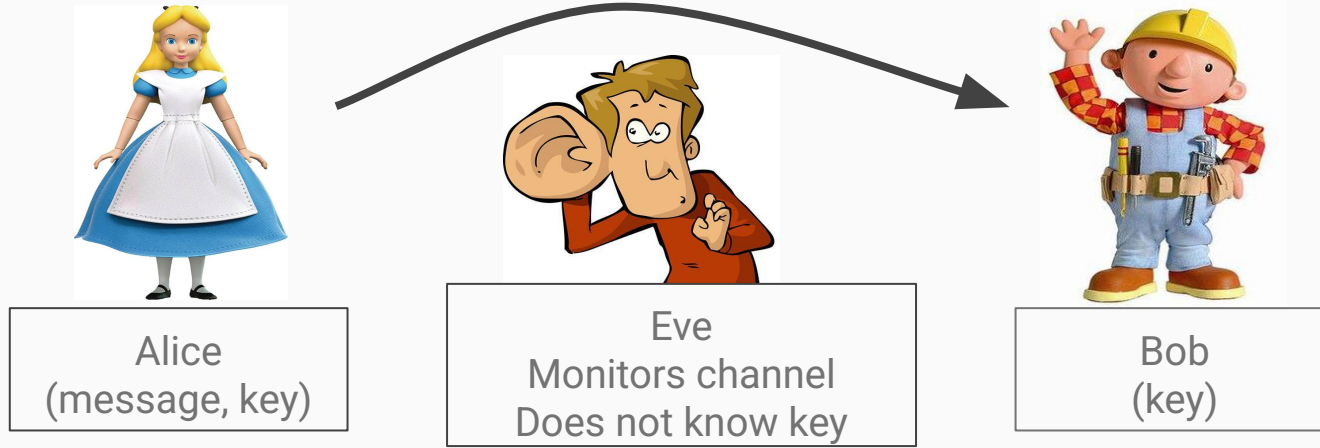
Setting:



Goal: Bob wants to learn Alice's messages.

Security (informally): Eve does not learn anything about the messages, even when seeing many ciphertexts (and seeing ciphertexts for messages of his choice).

Symmetric Encryption: scheme syntax



Key generation():
How Alice and Bob
pick their key.

Usually:

Pick random key k
Return k

Encrypt(k , pt):
How Alice encrypts her
message (pt) using the
key (k).
Returns a ciphertext.

Decrypt(k , ct):
How Bob decrypts
ciphertext (ct) using
the key (k).
Returns a plaintext.

Important Tool: Block Ciphers

“The workhorse of cryptography”

Definition: a function B that takes a key and a single chunk (“block”) of plaintext as input and outputs a “cipher block.”

- Block sizes on the order of 64 bits, 128 bits, 256 bits.
- Key size does not have to equal the block size.
- Each key defines a “random” invertible permutation of inputs to possible outputs.

Security: Without knowledge of the key, $f(x) = B(\text{Key}, x)$ looks like a random permutation on the plaintext blocks.



Important Tool: Block Ciphers

Building a secure block cipher: Not covered in this class

- Not much theoretical understanding of the block ciphers in use
- In Lab2 you break an insecure block cipher (2DES)

This section: building a secure encryption scheme, assuming you have a secure block cipher.

- Related to Lab2 “don’t count on counters” and “confession by compression” problems.

Scheme 1: Block Ciphers Only

Assume we have a secure block-cipher B with block size 128. Consider the following encryption scheme for 128-bit plaintext:

Key generation:

Pick random key k

Return k

Encrypt(k , pt):

$ct = B(k, pt)$

Return ct

Decrypt(k , ct):

Activity 1: How to decrypt?

Scheme 1: Block Ciphers Only

Assume we have a secure block-cipher B with block size 128. Consider the following encryption scheme for 128-bit plaintext:

Key generation:

Pick random key k

Return k

Encrypt(k , pt):

$ct = B(k, pt)$

Return ct

Decrypt(k , ct):

Activity 1: How to decrypt?

$pt = \text{inverse_}B(k, ct)$

return pt

Is this scheme secure?

NO! Encrypting the same plaintext twice results in the same ciphertext.

Other problems:

Can only encrypt one block at a time.

Scheme 1: Conclusion

A secure encryption scheme MUST encrypts the same message to a different ciphertext each time (with high probability).

How?

Include randomness in the encryption algorithm

Scheme 2: Block cipher with randomization

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for 128-bit plaintext:

Key generation:

Pick random key k

Return k

Encrypt(k , pt):

Pick random 128bit IV

$ct = IV || (B(k, IV) \mathbf{XOR} pt)$

Return ct

Decrypt(k , ct):

Activity 2: How to decrypt?

Scheme 2: Block cipher with randomization

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for 128-bit plaintext:

Key generation:

Pick random key k

Return k

Encrypt(k , pt):

Pick random 128bit IV

$ct = IV \parallel (B(k, IV) \mathbf{XOR} pt)$

Return ct

Decrypt(k , ct):

Activity 2: How to decrypt?

Parse ct as $(IV \parallel ct_block)$

$pt = ct_block \mathbf{XOR} B(k, IV)$

return pt

Is this scheme secure?

Yes (can be proved assuming security of B)

Problems:

Can only encrypt one block plaintexts.

Scheme 3: Same thing, more blocks!

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for plaintext blocks $pt_1, \dots, pt_n$:

```
Encrypt( $k, pt_1, \dots, pt_n$ ):  
  For  $i$  from 1 to  $n$ :  
    pick random  $IV_i$   
     $ct_i = IV_i || (B(k, IV_i) \mathbf{XOR} pt_i)$   
   $ct = ct_1 || \dots || ct_n$   
  Return  $ct$ 
```

```
Decrypt( $k, ct$ ):
```

Activity 3: How to decrypt?

Scheme 3: Same thing, more blocks!

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for plaintext blocks $pt_1, \dots, pt_n$:

```
Encrypt(k,  $pt_1, \dots, pt_n$ ):  
  For i from 1 to n:  
    pick random  $IV_i$   
     $ct_i = IV_i \parallel (B(k, IV_i) \mathbf{XOR} pt_i)$   
   $ct = ct_1 \parallel \dots \parallel ct_n$   
  Return ct
```

```
Decrypt(k, ct):  
  Parse ct as  $ct_1 \parallel \dots \parallel ct_n$   
  For i from 1 to n:  
    Parse  $ct_i$  as  $IV_i \parallel ct\_block_i$   
     $pt_i = ct\_block_i \mathbf{XOR} B(k, IV_i)$   
  Return  $pt_1, \dots, pt_n$ 
```

Is this scheme secure?

Yes (can be proved assuming security of B)

Problems:

Ciphertext is twice the length of the plaintext

Scheme 4: Insecure Shortcut

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for plaintext blocks $pt_1, \dots, pt_n$:

```
Encrypt( $k, pt_1, \dots, pt_n$ ):  
  pick random IV  
  For  $i$  from 1 to  $n$ :  
     $ct_i = B(k, IV) \text{ XOR } pt_i$   
   $ct = IV \parallel ct_1 \parallel \dots \parallel ct_n$   
  Return  $ct$ 
```

```
Decrypt( $k, ct$ ):
```

Activity 4: How to decrypt?

Scheme 4: Insecure Shortcut

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for plaintext blocks $pt_1, \dots, pt_n$:

```
Encrypt( $k, pt_1, \dots, pt_n$ ):  
  pick random IV  
  For  $i$  from 1 to  $n$ :  
     $ct_i = B(k, IV) \text{ XOR } pt_i$   
   $ct = IV || ct_1 || \dots || ct_n$   
  Return  $ct$ 
```

```
Decrypt( $k, ct$ ):  
  Parse  $ct$  as  $IV || ct_1 || \dots || ct_n$   
  For  $i$  from 1 to  $n$ :  
     $pt_i = ct_i \text{ XOR } B(k, IV)$   
  Return  $pt_1, \dots, pt_n$ 
```

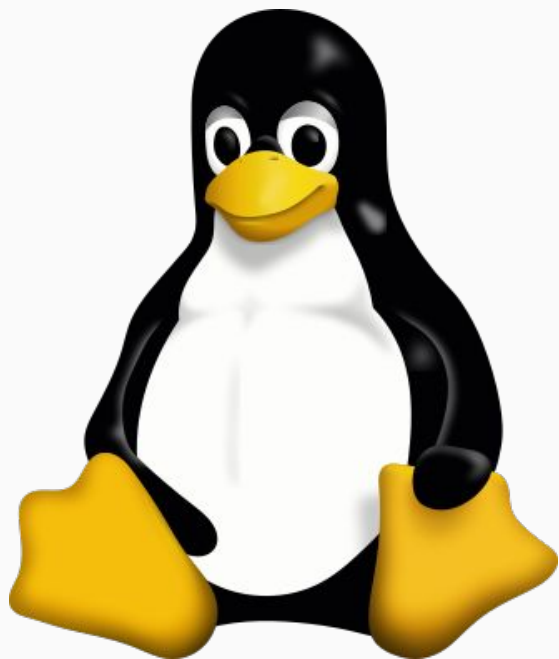
Why is this scheme insecure?

Repeated plaintext blocks
result in repeated ciphertext
blocks

Advantage:

$ct \text{ length} = 128 + (pt \text{ length})$

Scheme 4: Insecure Shortcut



“Encryption”

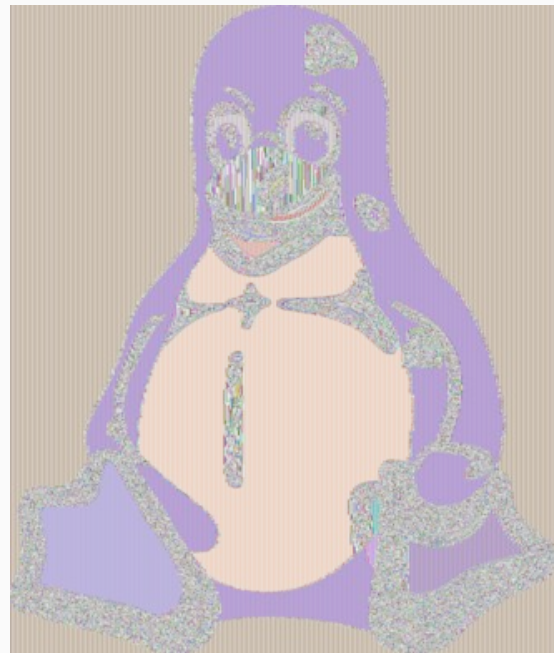
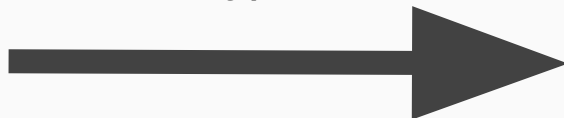


Photo credit: Wikipedia

Scheme 4: Conclusion

A secure encryption scheme MUST encrypt a repeated block to different ciphertext blocks (with high probability).

How?

Scheme 5: Best we got (CTR mode)

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for plaintext blocks $pt_1, \dots, pt_n$:

```
Encrypt(k, pt1, ..., ptn):  
  pick random IV  
  For i from 1 to n:  
    IVi = (IV + i - 1) % 2{128}  
    cti = B(k, IVi) XOR pti  
  ct = IV || ct1 || ... || ctn  
  Return ct
```

```
Decrypt(k, ct):
```

Activity 5: How to decrypt?

Scheme 5: Best we got (CTR mode)

Assume we have a secure block-cipher B with block size 128.

Consider the following encryption scheme for plaintext blocks $pt_1, \dots, pt_n$:

Encrypt($k, pt_1, \dots, pt_n$):

pick random IV

For i from 1 to n :

$IV_i = (IV + i - 1) \% 2^{128}$

$ct_i = B(k, IV_i) \text{ XOR } pt_i$

$ct = IV || ct_1 || \dots || ct_n$

Return ct

Decrypt(k, ct):

Parse ct as $IV || ct_1 || \dots || ct_n$

For i from 1 to n :

$IV_i = (IV + i - 1) \% 2^{128}$

$pt_i = B(k, IV_i) \text{ XOR } ct_i$

$pt = pt_1 || \dots || pt_n$

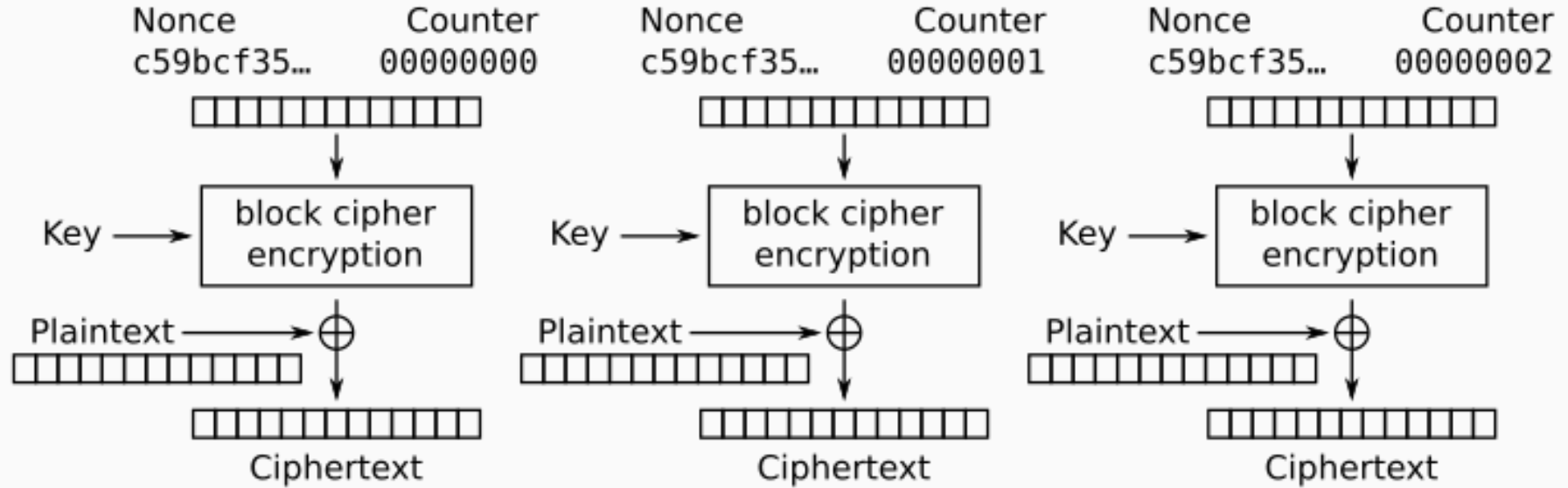
Return pt

Is this scheme secure?

Yes! Can be proven, assuming B is a pseudo-random permutation

Advantage: $ct \text{ length} = 128 + (pt \text{ length})$

Scheme 5 (CTR Mode): Diagram



Counter (CTR) mode encryption

CTR Mode: Considerations

Other good things:

- decryption does not require inverting the block cipher
- plaintext padding often used but is not required
- IV is public, can be generated deterministically (but **CAN'T** use IV twice)

Bad things:

- UNAUTHENTICATED!
 - If I see a ciphertext, I can modify it to decrypt to a different plaintext without knowing the key
- Requires non-colliding IVs
- For provable security guarantees, you have to restart with a new IV often.