CSE 484 / CSE M 584: Asymmetric Cryptography

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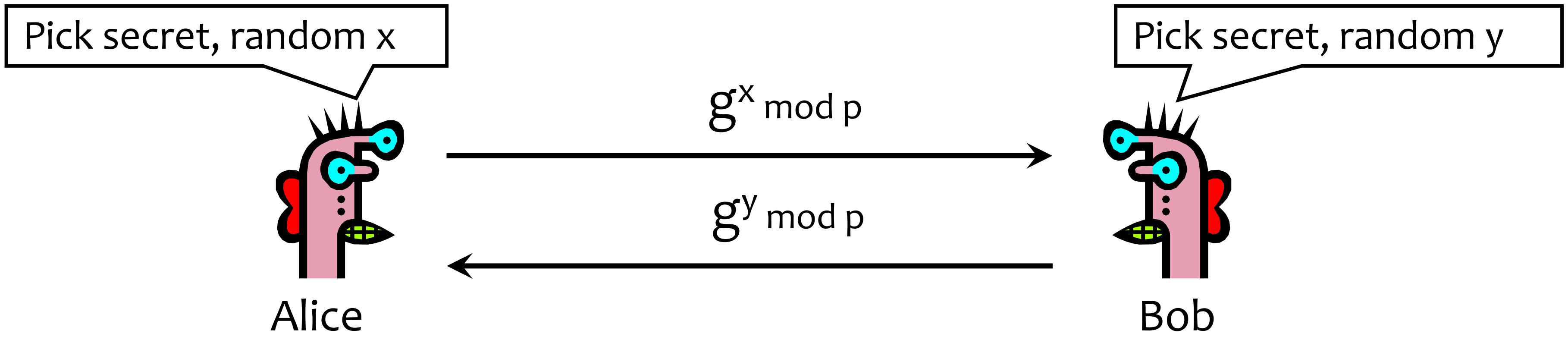
Announcements

- Things due
  - Lab 1: today!
  - Homework 2: Next Friday
  - CSE 584M: Don’t forget about weekly research readings
- Roadmap
  - Today: Finish asymmetric crypto
  - Friday: Crypto in practice (on the web)
  - Next week: Web security

Reminder: Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- <u>Public</u> info: p and g
  - p is a large prime, g is a **generator** of Z_p^*
    - $Z_p^*=\{1, 2 \ldots p-1\}$; a is in Z_p^* if there is an i such that $a=g^i \bmod p$



Compute $k=(g^y)^x=g^{xy} \bmod p$

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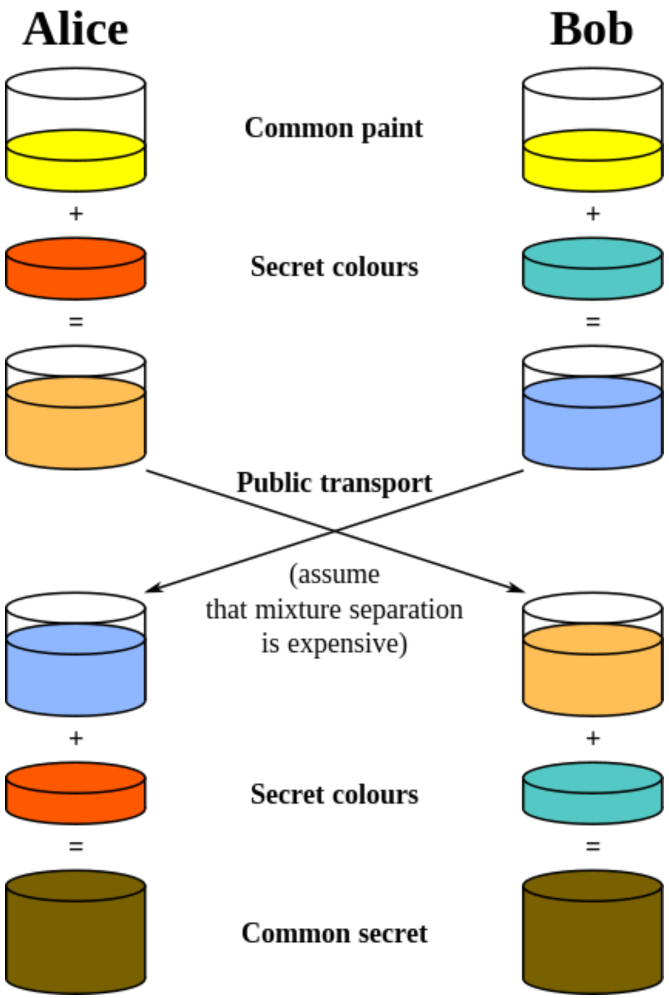
Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:
g^x mod p
g^y mod p

Common secret: g^{xy} mod p

[from Wikipedia]

Why is Diffie-Hellman Secure?

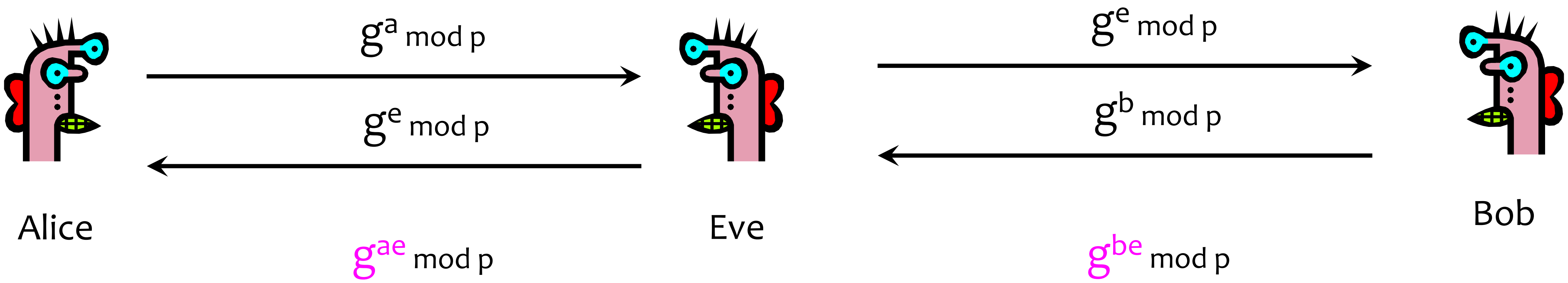
- Discrete Logarithm (DL) problem:
  given g^x *mod p*, it's hard to extract x
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:
  given g^x and g^y, it's hard to compute g^{xy} *mod p*
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:
  given g^x and g^y, it's hard to tell the difference between
  g^{xy} *mod p* and g^r *mod p* where r is random

Diffie-Hellman Caveats (1)

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against <u>passive</u> attackers
  - Common recommendation:
    - Choose p=2q+1, where q is also a large prime
    - Choose g that generates a subgroup of order q in Z_p*
    - DDH is hard in this group
  - Eavesdropper can't tell the difference between the established key and a random value
  - In practice, often hash $g^{xy} \bmod p$, and use the hash as the key
  - Can use the new key for symmetric cryptography

Diffie-Hellman Caveats (2)

- Diffie-Hellman protocol (by itself) does not provide authentication (against <u>active</u> attackers)
  – Person in the middle attack (aka "man in the middle attack")



Diffie-Hellman Key Exchange Today

- Important Note:
  - We have discussed discrete logs modulo integers
  - Significant advantages in using **elliptic curve groups**
    - Groups with some similar mathematical properties (i.e., are "groups") but have better security and performance (size) properties
    - Today's de-facto standard

Stepping Back: Asymmetric Crypto

- We've just seen session key establishment
  - Can then use shared key for symmetric crypto
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For authenticity

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function φ(n) (n≥1) is the number of integers in the [1,n] interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
    - Example: **34** (factors: 1, 2, 17, 34) and **35** (1, 5, 7, 35)
  - Easy to compute for primes: φ(p) = p-1
  - Note that φ(ab) = φ(a) φ(b) if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 2048 bits each (need primality testing, too)
  - Compute **n**=pq and φ(**n**)=(p-1)(q-1)
  - Choose small **e**, relatively prime to φ(n)
    - Typically, **e=3** or **e=2^{16}+1=65537**
  - Compute unique **d** such that ed ≡ 1 mod φ(n)
    - Modular inverse: d ≡ e^{-1} mod φ(n)
  - Public key = (e,n); private key = (d,n)
- Encryption of m: c = m^e mod n
- Decryption of c: c^d mod n = $(m^e)^d$ mod n = m

A 512-bit prime:
```
9238392041438079003450838646
6608111904604104720064333111
8274222861101608716534554412
4307595017420038487576191853
6796640686377031035140080035
82827766514729
```

How to compute?

- Extended Euclidian algorithm
- Wolfram Alpha ☺
- Brute force for small values

Why is RSA Secure?

- RSA problem: given c, n=pq, and e such that gcd(e, φ(n))=1, find m such that $m^e=c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this *without* knowing p and q
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n=p_1^{e_1}p_2^{e_2}\ldots p_k^{e_k}$

- If factoring is easy, then RSA problem is easy
(knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n – but if it is, we don't know how

RSA Encryption Caveats

- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow.
- Plain RSA also does <u>not</u> provide integrity
  – Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt

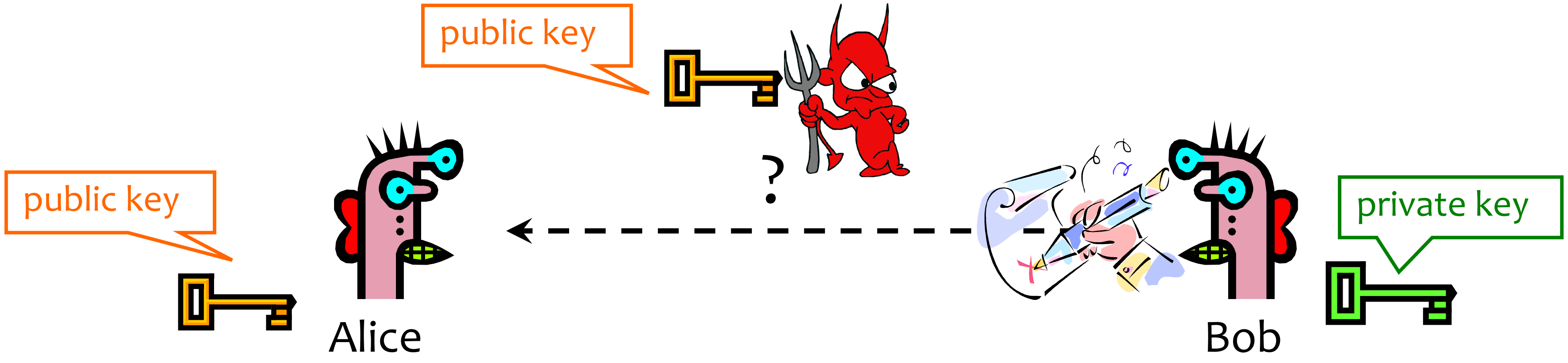
$M \oplus G(r) \,||\, r \oplus H(M \oplus G(r))$

– r is random and fresh, G and H are hash functions

Stepping Back: Asymmetric Crypto

- Last time we saw session key establishment (Diffie-Hellman)
  - Can then use shared key for symmetric crypto
- We just saw: public key encryption
  - For confidentiality
- Finally, now: digital signatures
  - For authenticity

Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: $s = m^d \bmod n$
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m: verify that $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Without padding and hashing: Consider multiplying two signatures together
  - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

- Digital Signature Standard (DSS)
  - U.S. government standard (1991, most recent rev. 2013)
- Public key: $(p, q, g, y=g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from g^x mod p (public key)
- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Post-Quantum Cryptography

- If quantum computers become a reality
  - It becomes much more efficient to break conventional asymmetric encryption schemes (e.g., factoring becomes “easy”)
- There exists efforts to make quantum-resilient asymmetric encryption schemes

Cryptography Summary

- Goal: Privacy
  - Symmetric keys:
    - One-time pad, Stream ciphers
    - Block ciphers (e.g., DES, AES) → modes: ECB, CBC, CTR
  - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
  - MACs, often using hash functions (e.g, SHA-256)
- Goal: Privacy and Integrity ("authenticated encryption")
  - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
  - Digital signatures (e.g., RSA, DSS)