

**CSE 484 / CSE M 584:  
Finish Hash Functions + MACs;  
Start Asymmetric Crypto**

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# Announcements

- Lab 1B due **Wednesday**
- Homework 2 due **next Friday**
  - Individual assignment
  - You should be able to do much of it at this point
- Reminder: my OH are on **Wednesday** this week

# Hash Functions Review

- Map large domain to small range (e.g., range of all 160- or 256-bit values)
- Properties of cryptographically secure hash functions:
  - **One-wayness:** Given a point in the range (that was computed as the hash of a random domain element), hard to find a preimage
  - **Collision Resistance:** Hard to find two distinct inputs that map to same output
  - **Weak Collision Resistance:** Given a point in the domain and its hash in the range, hard to find a new domain element that maps to the same range element

# Common Hash Functions

- **SHA-2: SHA-256, SHA-512, SHA-224, SHA-384**
- **SHA-3: standard released by NIST in August 2015**
- **MD5 – Don't use for security!**
  - 128-bit output
  - Designed by Ron Rivest, used very widely
  - Collision-resistance broken (summer of 2004)
- **SHA-1 (Secure Hash Algorithm) – Don't use for security!**
  - 160-bit output
  - US government (NIST) standard as of 1993-95
  - Theoretically broken 2005; practical attack 2017!

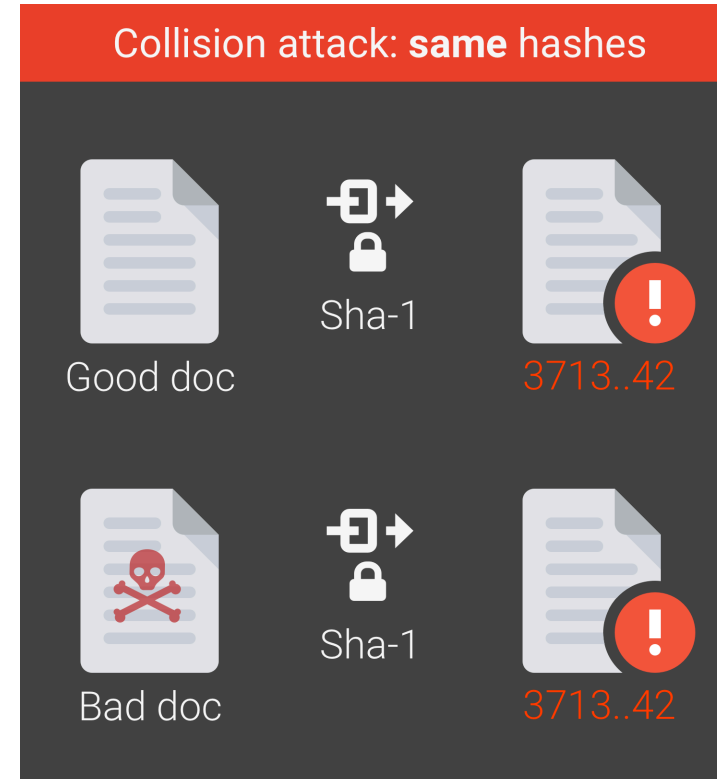
# SHA-1 Broken in Practice (2017)

**Google just cracked one of the building blocks of web encryption (but don't worry)**

*It's all over for SHA-1*

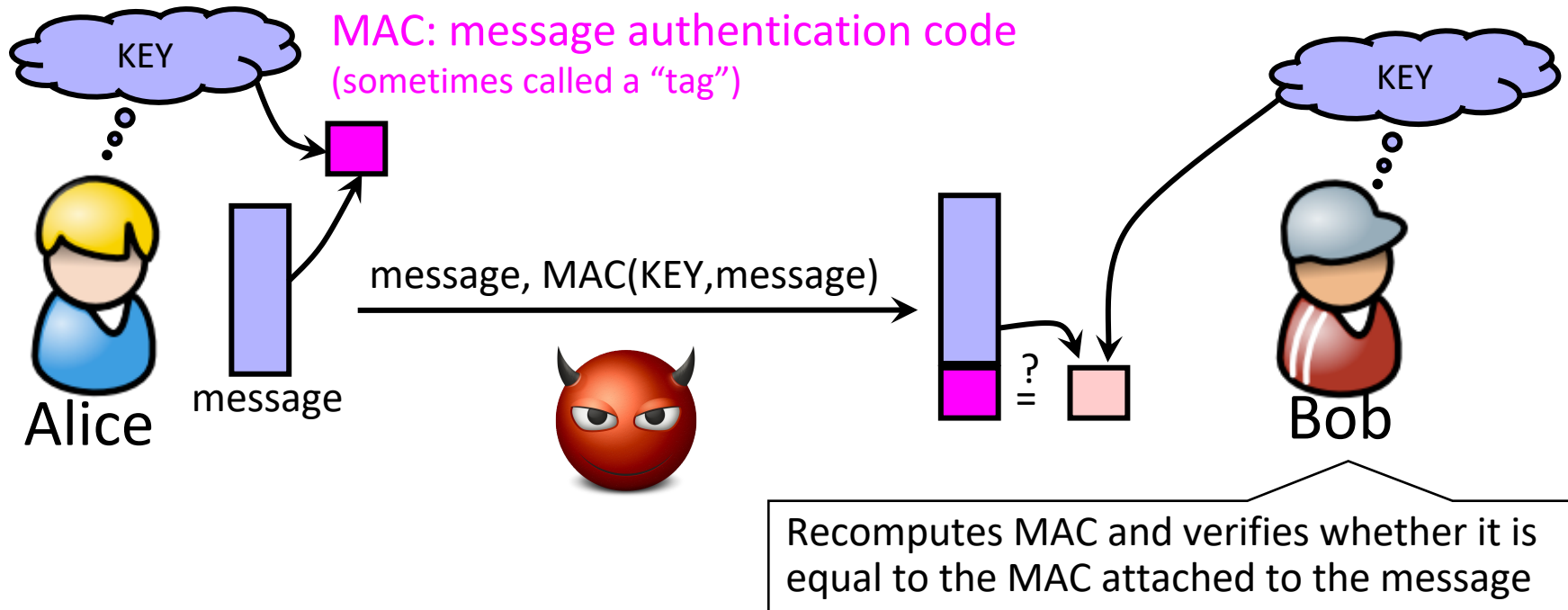
by [Russell Brandom](#) | [@russellbrandom](#) | Feb 23, 2017, 11:49am EST

<https://shattered.io>



# Recall: Achieving Integrity

Message authentication schemes: A tool for protecting integrity.



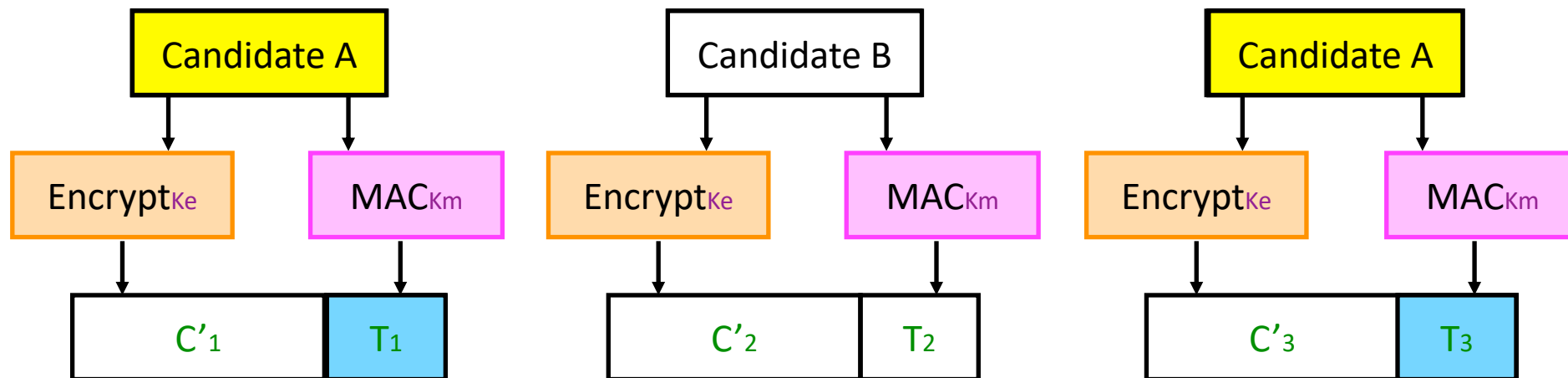
**Integrity and authentication:** only someone who knows KEY can compute correct MAC for a given message.

# MAC with SHA3

- $\text{SHA3}(\text{Key} \parallel \text{Message})$
- Nice and simple 😊
- FYI: Previous hash functions couldn't quite be used in this way (see: length extension attack, HMAC construction)
- Why not encryption? (Historical reasons)
  - Hashing is faster than block ciphers in software
  - Can easily replace one hash function with another
  - There used to be US export restrictions on encryption

# Authenticated Encryption

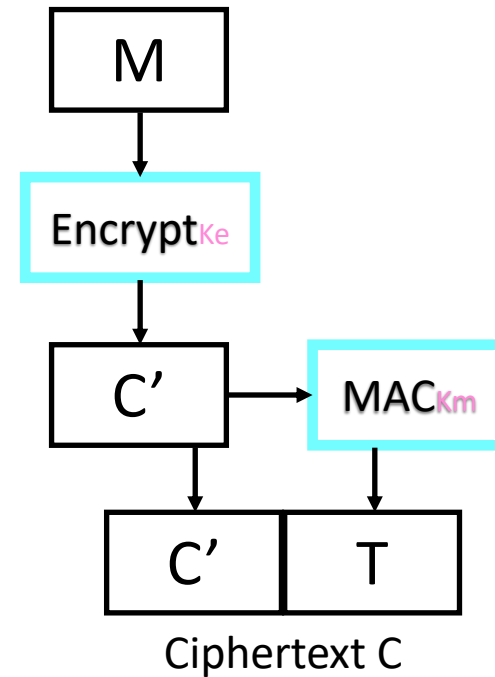
- What if we want both privacy and integrity?
- Natural approach: combine **encryption scheme** and a **MAC**.
- **But be careful!**
  - Obvious approach: Encrypt-and-MAC
  - Problem: MAC is deterministic! same plaintext  $\rightarrow$  same MAC





# Authenticated Encryption

- Instead:  
*Encrypt then MAC.*
- (Not as good:  
MAC-then-Encrypt)



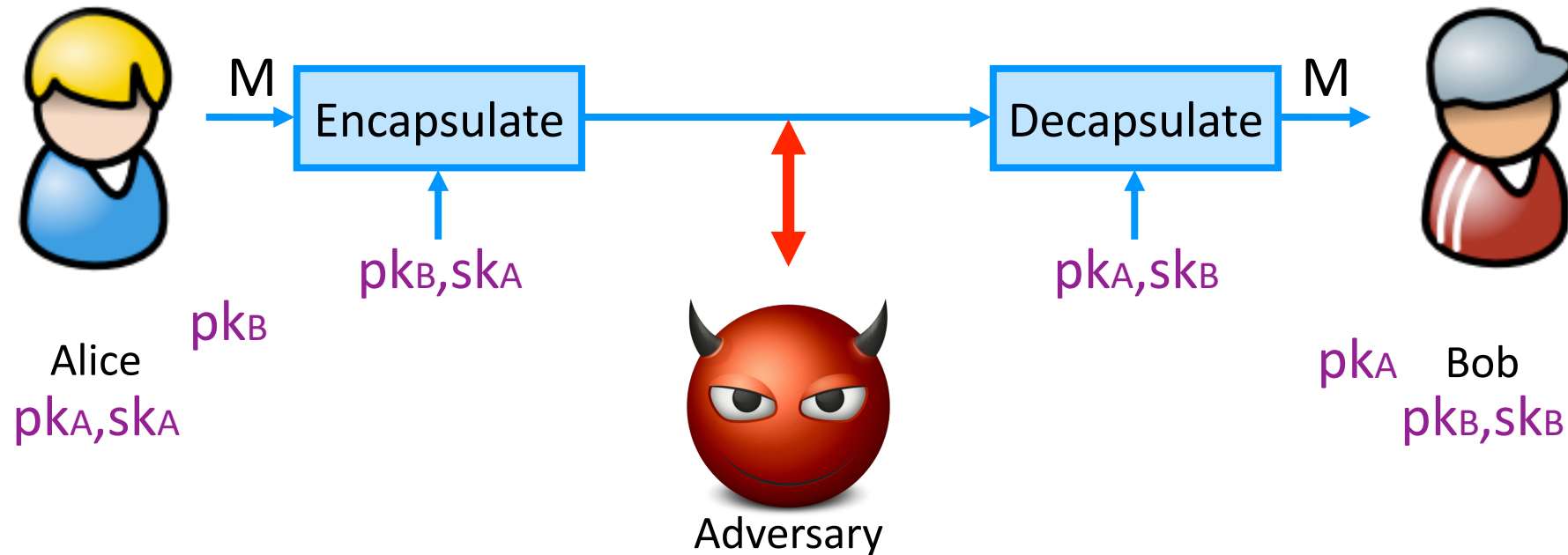
**Encrypt-then-MAC**

# Flavors of Cryptography

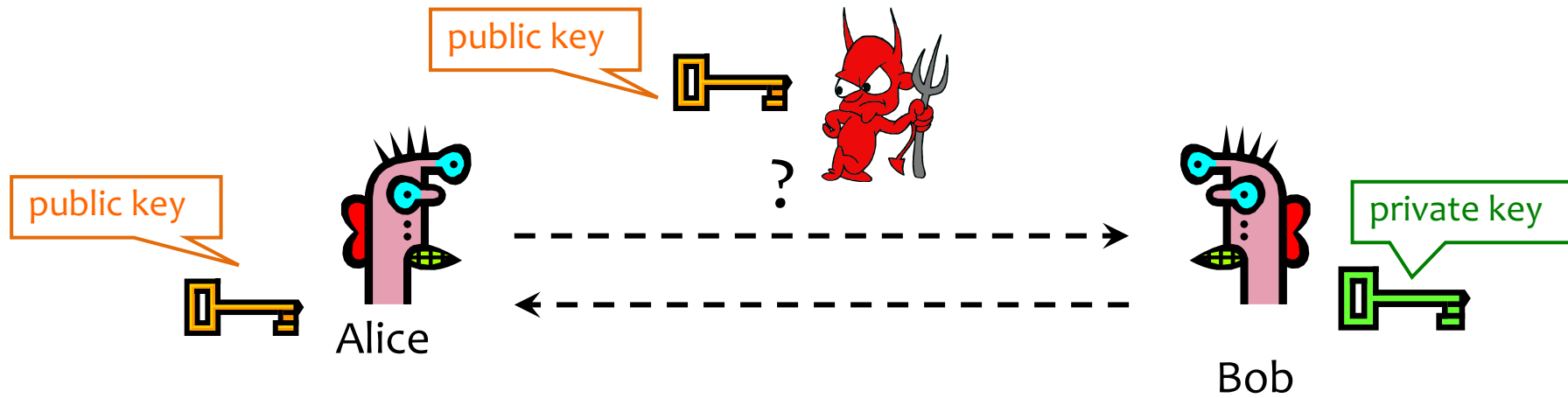
- Symmetric cryptography
  - Both communicating parties have access to a **shared random string  $K$** , called the **key**.
- Asymmetric cryptography
  - Each party creates a public key  **$pk$**  and a secret key  **$sk$** .

# Asymmetric Setting

Each party creates a public key  $pk$  and a secret key  $sk$ .



# Public Key Crypto: Basic Problem



Given: Everybody knows Bob's **public key**  
Only Bob knows the corresponding **private key**

Ignore for now: How do we know it's REALLY Bob's??

Goals: 1. Alice wants to send a secret message to Bob  
2. Bob wants to authenticate themselves

# Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can “sign” a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

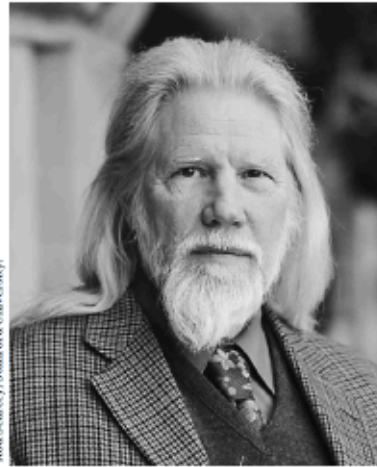
# Next: Session Key Establishment

# Modular Arithmetic

- Given  $g$  and prime  $p$ , compute:  $g^1 \bmod p, g^2 \bmod p, \dots, g^{100} \bmod p$ 
  - For  $p=11, g=10$ 
    - $10^1 \bmod 11 = 10, 10^2 \bmod 11 = 1, 10^3 \bmod 11 = 10, \dots$
    - Produces cyclic group  $\{10, 1\}$  (order=2)
  - For  $p=11, g=7$ 
    - $7^1 \bmod 11 = 7, 7^2 \bmod 11 = 5, 7^3 \bmod 11 = 2, \dots$
    - Produces cyclic group  $\{7, 5, 2, 3, 10, 4, 6, 9, 8, 1\}$  (order = 10)
      - Numbers “wrap around” after they reach  $p$
    - $g=7$  is a “generator” of  $Z_{11}^*$

# Diffie-Hellman Protocol (1976)

## Diffie and Hellman Receive 2015 Turing Award



Rod Searcy/Stanford University.

**Whitfield Diffie**



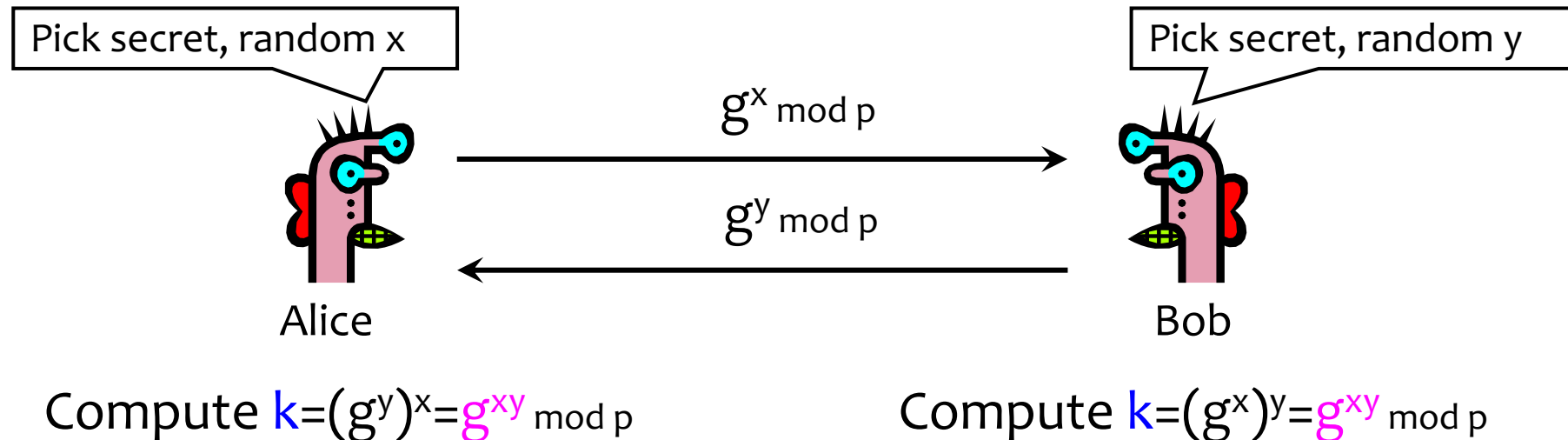
Linda A. Cicero/Stanford News Service.

**Martin E. Hellman**



# Diffie-Hellman Protocol (1976)

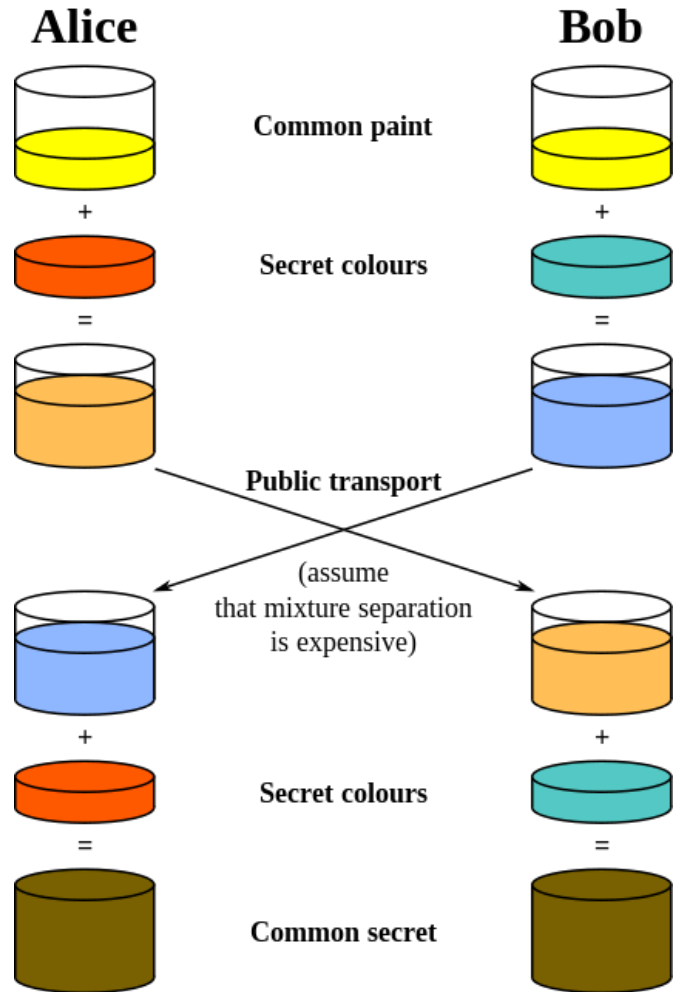
- Alice and Bob never met and share no secrets
- Public info:  $p$  and  $g$ 
  - $p$  is a large prime,  $g$  is a **generator** of  $Z_p^*$ 
    - $Z_p^* = \{1, 2 \dots p-1\}$ ;  $a$  is in  $Z_p^*$  if there is an  $i$  such that  $a = g^i \pmod p$



# Example Diffie Hellman Computation

- PUBLIC
  - $p = 11$
  - $g = 2$
  - ( $g$  is a generator for group mod  $p$ )
- Alice:  $x=9$ , sends 6 ( $g^x \text{ mod } p = 2^9 \text{ mod } 11 = 6$ )
- Bob:  $y=4$ , send 5 ( $g^y \text{ mod } p = 2^4 \text{ mod } 11 = 5$ )
- A compute:  $5^x \text{ mod } 11$  ( $5^9 \text{ mod } 11 = 9$ )
- B compute  $6^y \text{ mod } 11$  ( $6^4 \text{ mod } 11 = 9$ )
- Both get 9
- All computations modulo 11

# Diffie-Hellman: Conceptually



Common paint:  $p$  and  $g$

Secret colors:  $x$  and  $y$

Send over public transport:

$g^x \bmod p$

$g^y \bmod p$

Common secret:  $g^{xy} \bmod p$

[from Wikipedia]

# Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:  
given  $g^x \bmod p$ , it's hard to extract  $x$ 
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:  
given  $g^x$  and  $g^y$ , it's hard to compute  $g^{xy} \bmod p$ 
  - ... unless you know  $x$  or  $y$ , in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:  
given  $g^x$  and  $g^y$ , it's hard to tell the difference between  $g^{xy} \bmod p$  and  $g^r \bmod p$  where  $r$  is random

# Diffie-Hellman Caveats (1)

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose  $p=2q+1$ , where  $q$  is also a large prime
    - Choose  $g$  that generates a subgroup of order  $q$  in  $Z_p^*$
    - DDH is hard in this group
  - Eavesdropper can't tell the difference between the established key and a random value
  - In practice, often hash  $g^{xy} \bmod p$ , and use the hash as the key
  - Can use the new key for symmetric cryptography

# Diffie-Hellman Caveats (2)

- Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
  - Person in the middle attack (aka “man in the middle attack”)

# Diffie-Hellman Key Exchange Today

- **Important Note:**
  - We have discussed discrete logs modulo integers
  - Significant advantages in using **elliptic curve groups**
    - Groups with some similar mathematical properties (i.e., are “groups”) but have better security and performance (size) properties
    - Today’s de-facto standard

# Stepping Back: Asymmetric Crypto

- We've just seen **session key establishment**
  - Can then use shared key for symmetric crypto
- Next: **public key encryption**
  - For confidentiality
- Then: **digital signatures**
  - For authenticity