CSE 484 / CSE M 584: Applied Cryptography

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UW Instruction Team: David Kohlbrenner, Yoshi Kohno, Franziska Roesner. Thanks to Dan Boneh, Dieter Gollmann, Dan Halperin, John Manferdelli, John Mitchell, Vitaly Shmatikov, Bennet Yee, and many others for sample slides and materials ...

Announcements / Plan

- Through Wednesday (2/8): Applied Crypto
- Friday (2/10): Guest Lecture: Prof. Elissa Redmiles (MPI)
- Wednesday (2/22): Zoom
- Friday (2/24): Guest Lecture: Alex Gantman (Qualcomm) (On Zoom)

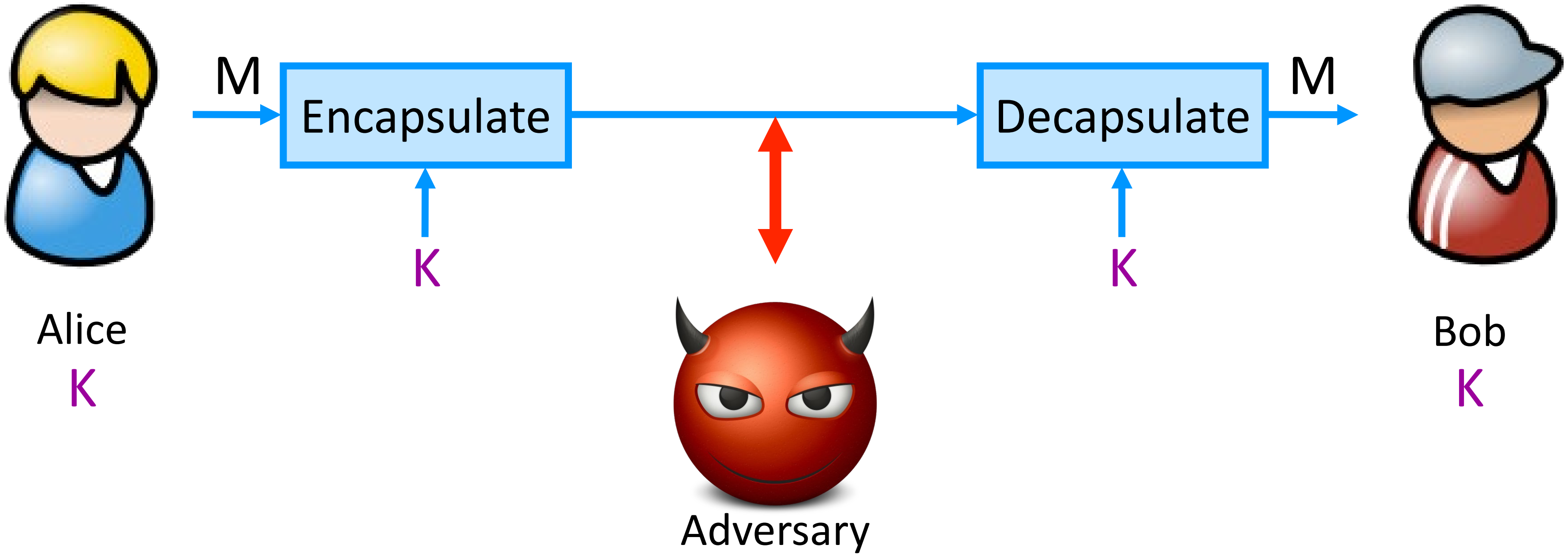
Stepping Back: Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string K, called the key.

- Asymmetric cryptography
  - Each party creates a public key pk and a secret key sk.

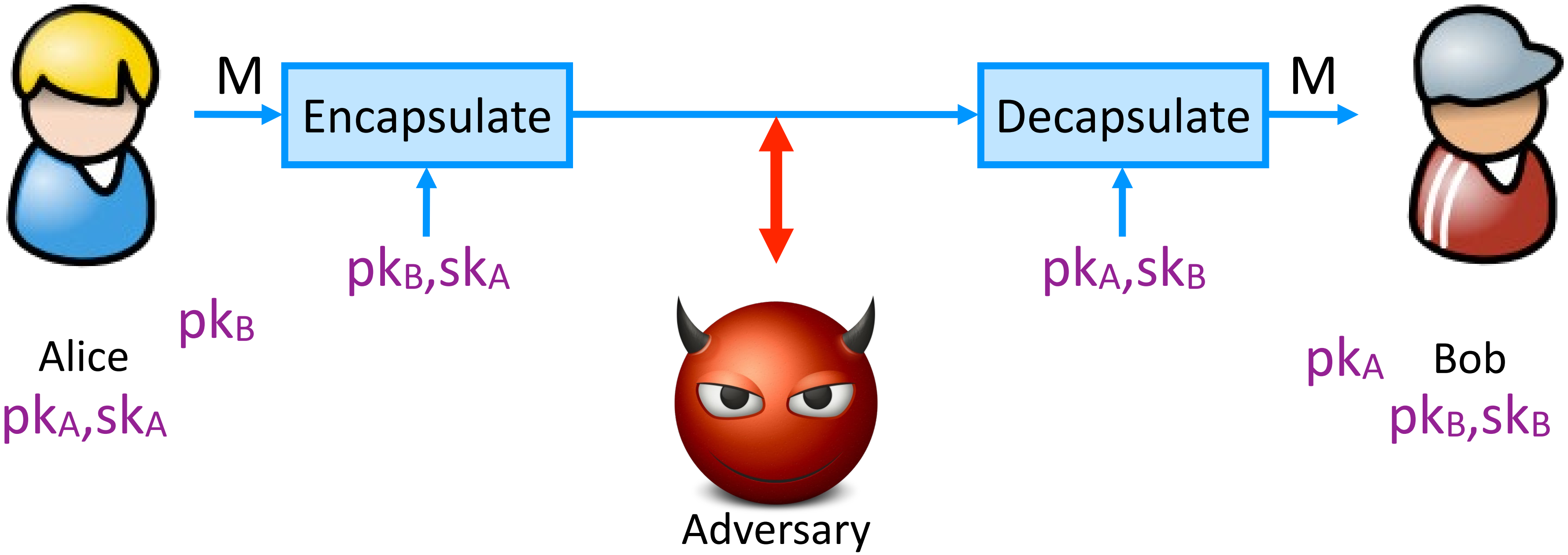
Symmetric Setting

Both communicating parties have access to a shared random string K, called the key.

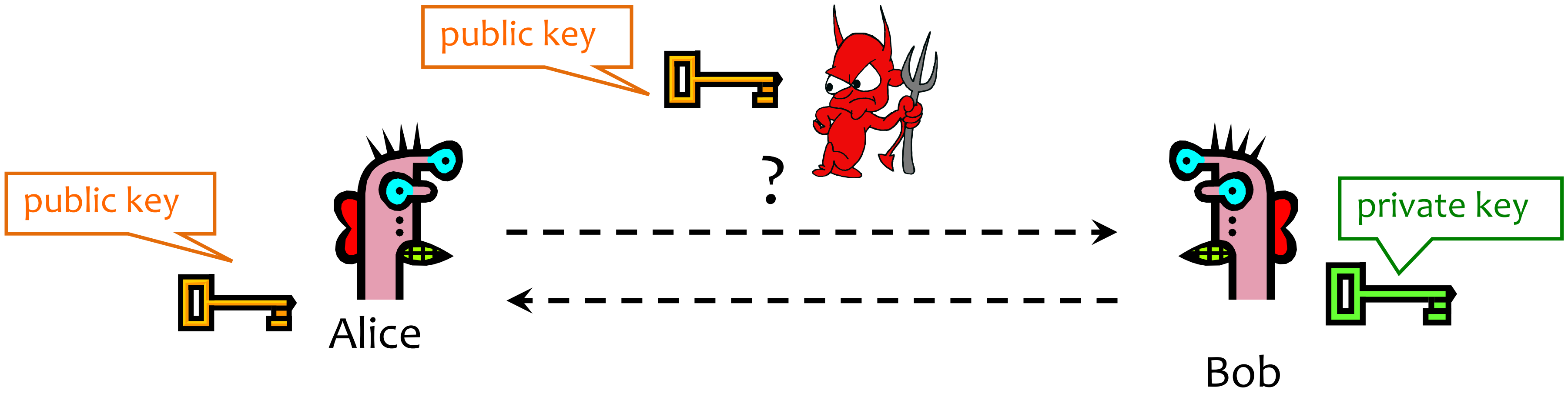


Asymmetric Setting

Each party creates a public key pk and a secret key sk.



Public Key Crypto: Basic Problem



Ignore for now: How do we know it's REALLY Bob's??

Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate themself

Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can "sign" a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

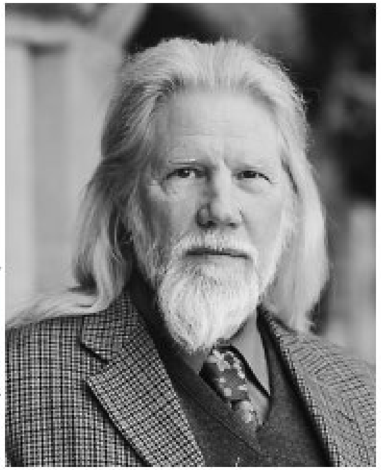
Session Key Establishment

Modular Arithmetic

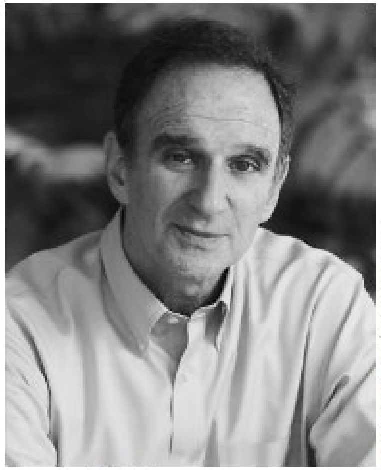
- Given g and prime p, compute: g^1 mod p, g^2 mod p, … g^{100} mod p
  - For p=11, g=10
    - 10^1 mod 11 = 10, 10^2 mod 11 = 1, 10^3 mod 11 = 10, …
    - Produces cyclic group {10, 1} (order=2)
  - For p=11, g=7
    - 7^1 mod 11 = 7, 7^2 mod 11 = 5, 7^3 mod 11 = 2, …
    - Produces cyclic group {7,5,2,3,10,4,6,9,8,1} (order = 10)
      - Numbers "wrap around" after they reach p
    - g=7 is a "generator" of Z_{11}^*

Diffie-Hellman Protocol (1976)

Diffie and Hellman Receive 2015 Turing Award



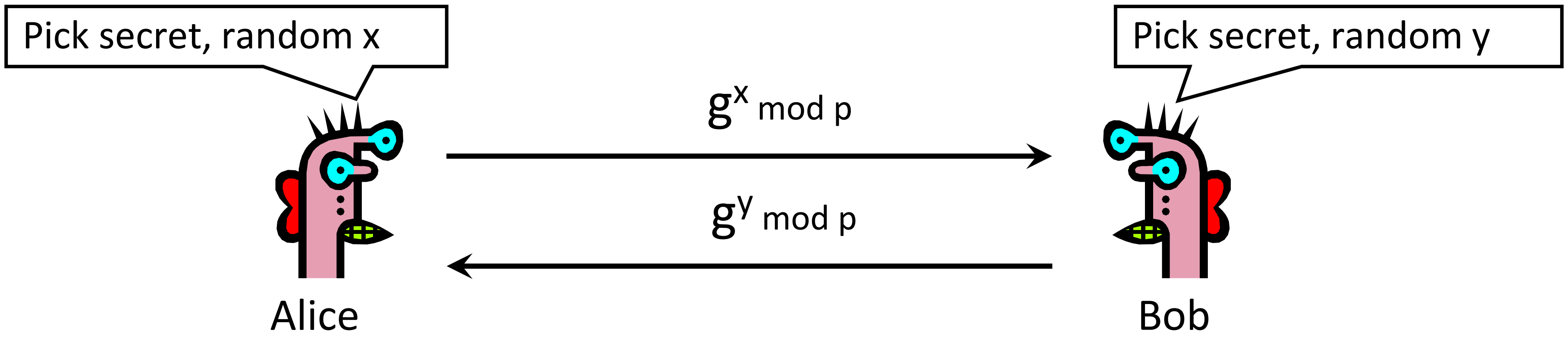
Whitfield Diffie



Martin E. Hellman

Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info: p and g
  - p is a large prime, g is a **generator** of Z_p^*
    - $Z_p^* = \{1, 2 \ldots p-1\}$; a is in Z_p^* if there is an i such that $a = g^i$ mod p



Compute $k = (g^y)^x = g^{xy}$ mod p

Compute $k = (g^x)^y = g^{xy}$ mod p

Example Diffie Hellman Computation

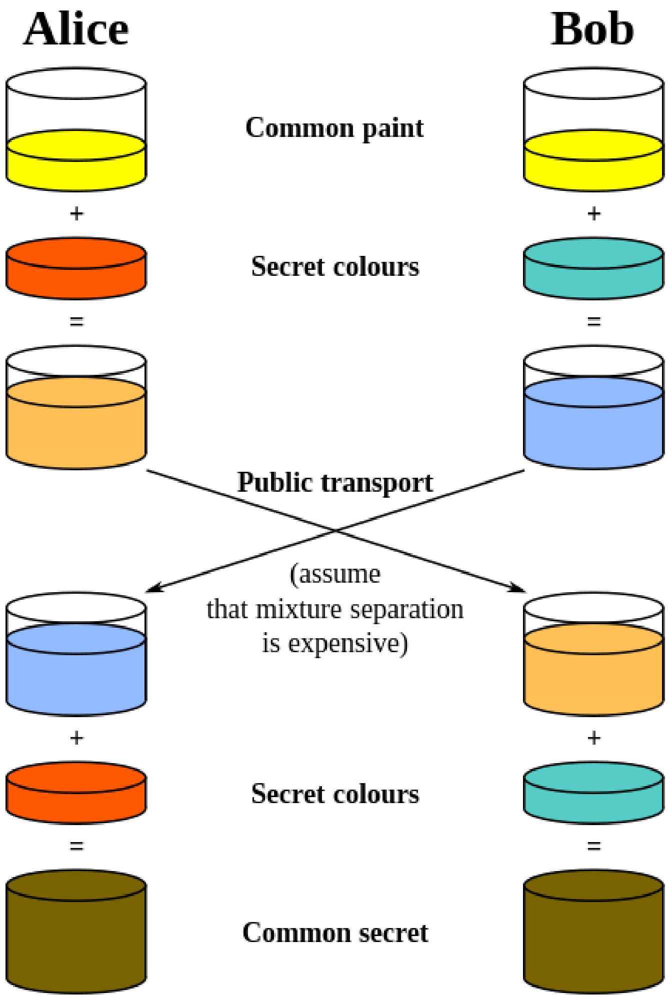
- PUBLIC
  - p = 11
  - g = 2
  - (g is a generator for group mod p)

- Alice: x=9, sends 6 (g^x mod p = 2^9 mod 11 = 6)
- Bob: y=4, send 5 (g^y mod p = 2^4 mod 11 = 5)

- A compute: 5^x mod 11 (5^9 mod 11 = 9)
- B compute: 6^y mod 11 (6^4 mod 11 = 9)
- Both get 9

- All computations modulo 11

Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:
$g^x \bmod p$
$g^y \bmod p$

Common secret: $g^{xy} \bmod p$

[from Wikipedia]

Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:
  given g^x *mod p*, it's hard to extract x
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:
  given g^x and g^y, it's hard to compute g^{xy} *mod p*
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:
  given g^x and g^y, it's hard to tell the difference between
  g^{xy} *mod p* and g^r *mod p* where r is random

Diffie-Hellman Caveats (1)

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose p=2q+1, where q is also a large prime
    - Choose g that generates a subgroup of order q in Z_p*
    - DDH is hard in this group
  - Eavesdropper can't tell the difference between the established key and a random value
  - In practice, often hash $g^{xy} \bmod p$, and use the hash as the key
  - Can use the new key for symmetric cryptography

Example from Earlier

- Given g and prime p, compute: g^1 mod p, g^2 mod p, … g^{100} mod p
  - For p=11, g=10
    - 10^1 mod 11 = 10, 10^2 mod 11 = 1, 10^3 mod 11 = 10, …
    - Produces cyclic group {10, 1} (order=2)
  - For p=11, g=7
    - 7^1 mod 11 = 7, 7^2 mod 11 = 5, 7^3 mod 11 = 2, …
    - Produces cyclic group {7,5,2,3,10,4,6,9,8,1} (order = 10)
    - g=7 is a "generator" of Z_{11}*
  - For p=11, g=3
    - 3^1 mod 11 = 3, 3^2 mod 11 = 9, 3^3 mod 11 = 5, …
    - Produces cyclic group {3,9,5,4,1} (order = 5) (5 is a prime)
    - g=3 generates a group of prime order

Diffie-Hellman Caveats (2)

- Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
  - Person in the middle attack (aka "man in the middle attack")

Diffie-Hellman Key Exchange Today

- Important Note:
  - We have discussed discrete logs modulo integers
  - Significant advantages in using **elliptic curve groups**
    - Groups with some similar mathematical properties (i.e., are "groups") but have better security and performance (size) properties
    - Today's de-facto standard

Stepping Back: Asymmetric Crypto

- We've just seen session key establishment
  - Can then use shared key for symmetric crypto
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For authenticity

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function $\varphi(n)$ $(n \geq 1)$ is the number of integers in the $[1,n]$ interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: $\varphi(p) = p-1$
  - Note that $\varphi(ab) = \varphi(a)\,\varphi(b)$ if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 2048 bits each (need primality testing, too)
  - Compute **n**=pq and $\varphi(\mathbf{n})=(p-1)(q-1)$
  - Choose small **e,** relatively prime to $\varphi(n)$
    - Typically, **e=3** or **e=2^{16}+1=65537**
  - Compute unique **d** such that $ed \equiv 1 \bmod \varphi(n)$
    - Modular inverse: $d \equiv e^{-1} \bmod \varphi(n)$
  - Public key = (e,n); private key = (d,n)
- Encryption of m: $c = m^e \bmod n$
- Decryption of c: $c^d \bmod n = (m^e)^d \bmod n = m$

How to compute?

- Extended Euclidian algorithm
- Wolfram Alpha ☺
- Brute force for small values

Why is RSA Secure?

- RSA problem: given c, $n=pq$, and e such that $\gcd(e, \varphi(n))=1$, find m such that $m^e=c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this *without* knowing p and q
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n=p_1^{e_1}p_2^{e_2}\ldots p_k^{e_k}$

- If factoring is easy, then RSA problem is easy
(knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n – but if it is, we don't know how

RSA Encryption Caveats

- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow.
- Plain RSA also does <u>not</u> provide integrity
  - Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt

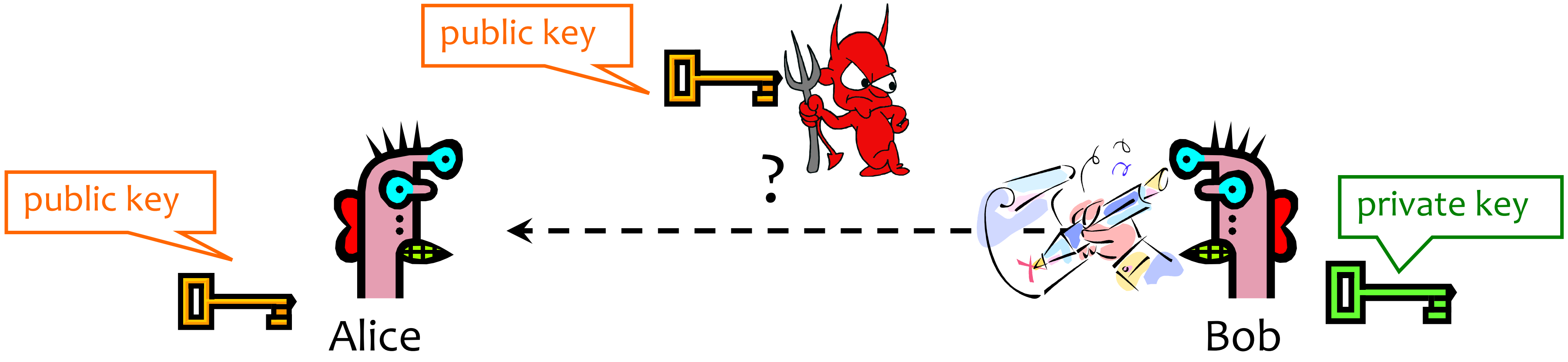
$M \oplus G(r) \,||\, r \oplus H(M \oplus G(r))$

- r is random and fresh, G and H are hash functions

Stepping Back: Asymmetric Crypto

- Last time we saw session key establishment (Diffie-Hellman)
  - Can then use shared key for symmetric crypto
- We just saw: public key encryption
  - For confidentiality
- Finally, now: digital signatures
  - For authenticity

Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: $s = m^d \bmod n$
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m: verify that $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Without padding and hashing: Consider multiplying two signatures together
  - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

- Digital Signature Standard (DSS)
  - U.S. government standard (1991, most recent rev. 2013)
- Public key: $(p, q, g, y=g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from g^x mod p (public key)
- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Post-Quantum Cryptography

- If quantum computers become a reality
  - It becomes much more efficient to break conventional asymmetric encryption schemes (e.g., factoring becomes "easy")
  - For block ciphers (symmetric encryption), use 128-bit keys for 256-bits of security
- There exists efforts to make quantum-resilient asymmetric encryption schemes

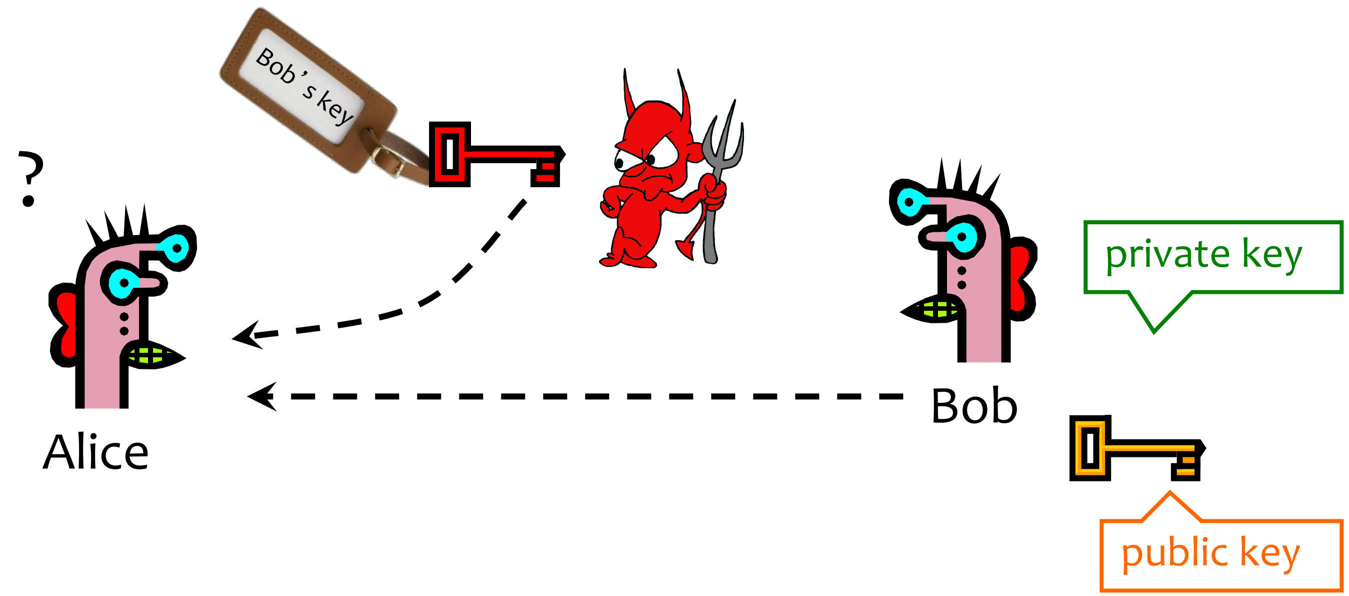
Cryptography Summary

- Goal: Privacy
  - Symmetric keys:
    - One-time pad, Stream ciphers
    - Block ciphers (e.g., DES, AES) → modes: EBC, CBC, CTR
  - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
  - MACs, often using hash functions (e.g, SHA-256)
- Goal: Privacy and Integrity ("authenticated encryption")
  - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
  - Digital signatures (e.g., RSA, DSS)

Want More Crypto?

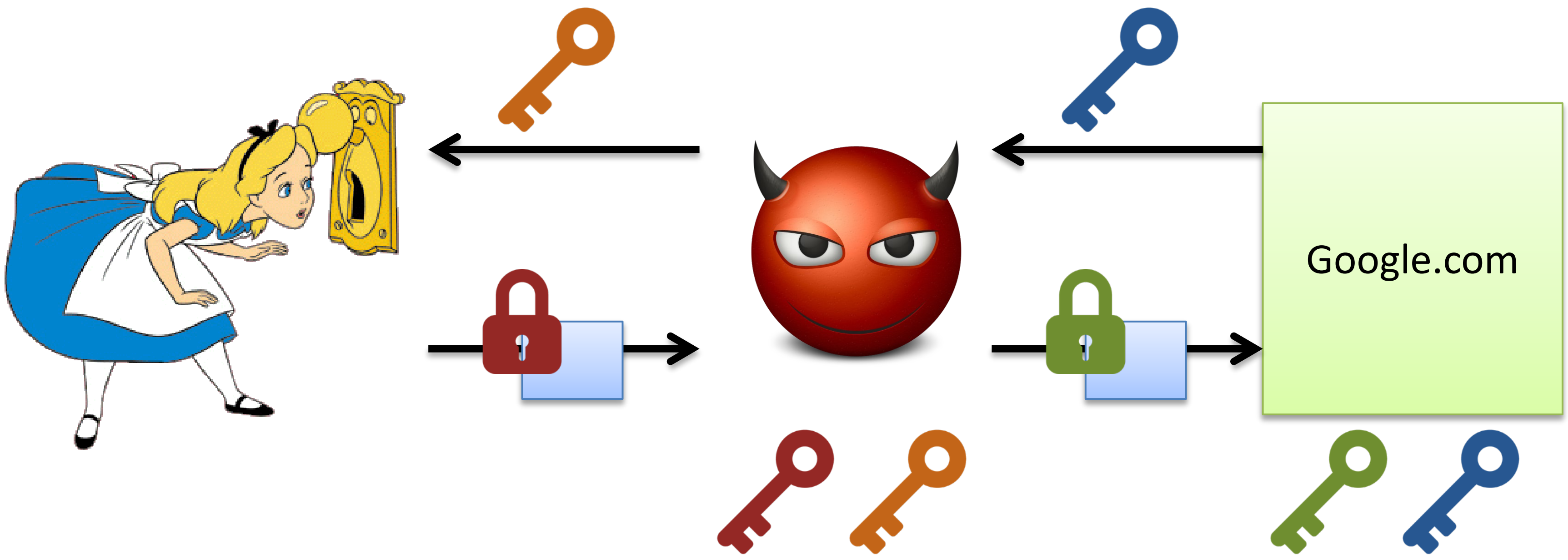
- Some suggestions:
  - Cryptography course
  - Stanford Coursera (Dan Boneh): https://www.coursera.org/learn/crypto

Authenticity of Public Keys



<u>Problem</u>: How does Alice know that the public key she received is really Bob's public key?

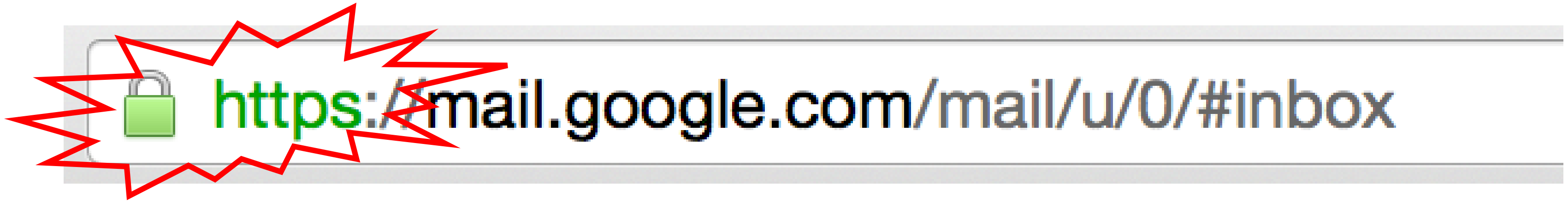
Threat: Person-in-the Middle



Distribution of Public Keys

- Public announcement or public directory
  - Risks: forgery and tampering
- Public-key certificate
  - Signed statement specifying the key and identity
    - sig_{CA}(“Bob”, PK_B)
- Common approach: certificate authority (CA)
  - Single agency responsible for certifying public keys
  - After generating a private/public key pair, user proves his identity and knowledge of the private key to obtain CA’s certificate for the public key (offline)
  - Every computer is pre-configured with CA’s public key

You encounter this every day...



SSL/TLS: Encryption & authentication for connections