CSE 484 / CSE M 584: Asymmetric Cryptography

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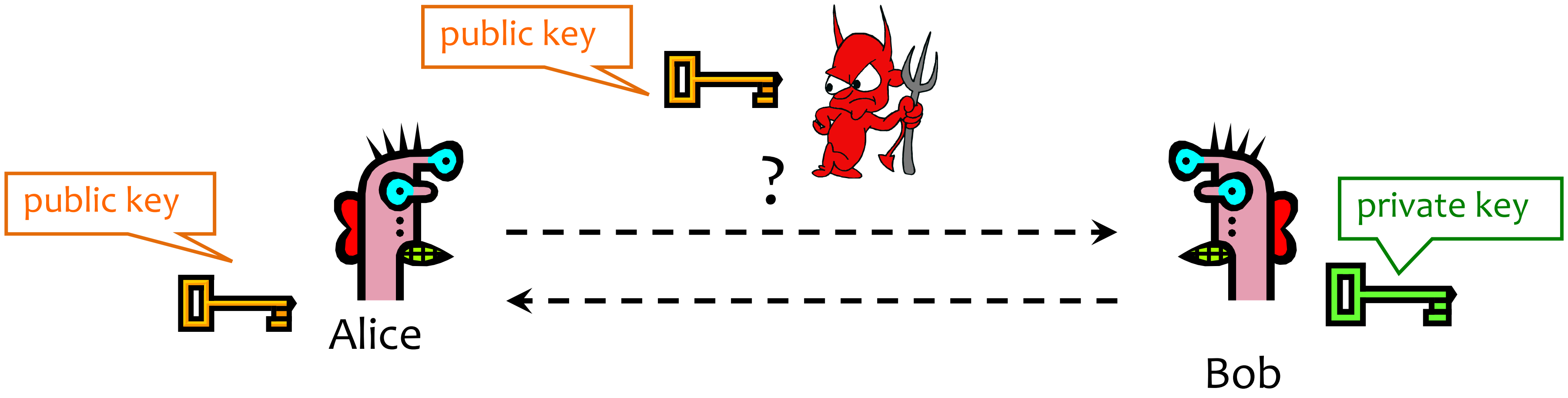
Announcements

- Things due
  - Lab 1: today
    - Extra office hours with Kirsten 10:30am Thursday
  - Homework 2: Next Friday
    - Individual assignment (no groups)
  - CSE 584M: Don't forget about weekly research readings
- Roadmap
  - Today: Asymmetric crypto
  - Friday: Crypto in practice (on the web)
  - Next week: Web security

Public Key Crypto: Basic Problem



Ignore for now: How do we know it's REALLY Bob's??

Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate themself

Some Number Theory Facts

- Euler totient function $\varphi(n)$ $(n \geq 1)$ is the number of integers in the $[1,n]$ interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: $\varphi(p) = p-1$
  - Note that $\varphi(ab) = \varphi(a)\,\varphi(b)$ if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 2048 bits each (need primality testing, too)
  - Compute **n**=pq and φ(**n**)=(p-1)(q-1)
  - Choose small **e**, relatively prime to φ(n)
    - Typically, **e=3** or **$e=2^{16}+1=65537$**
  - Compute unique **d** such that $ed \equiv 1 \bmod \varphi(n)$
    - Modular inverse: $d \equiv e^{-1} \bmod \varphi(n)$
  - Public key = (e,n); private key = (d,n)
- Encryption of m: $c = m^e \bmod n$
- Decryption of c: $c^d \bmod n = (m^e)^d \bmod n = m$

How to compute?

- Extended Euclidian algorithm
- Wolfram Alpha ☺
- Brute force for small values

Why RSA Decryption Works (FYI)

$e \cdot d = 1 \bmod \phi(n)$, thus $e \cdot d = 1 + k \cdot \phi(n)$ for some k

Let m be any integer in Z_n^* (not all of Z_n)

$c^d \bmod n = (m^e)^d \bmod n = m^{1+k \cdot \varphi(n)} \bmod n$

$= (m \bmod n) * (m^{k \cdot \varphi(n)} \bmod n)$

Euler's theorem: if $a \in Z_n^*$, then $a^{\phi(n)} = 1 \bmod n$

$c^d \bmod n = (m \bmod n) * (1 \bmod n)$

$= m \bmod n$

Proof omitted (using Chinese Remainder Theorem):

True for all m in Z_n, not just m in Z_n^*

Why is RSA Secure?

- RSA problem: given c, $n=pq$, and e such that $\gcd(e, \varphi(n))=1$, find m such that $m^e=c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this *without* knowing p and q
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n=p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$

- If factoring is easy, then RSA problem is easy
  (knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n – but if it is, we don't know how

RSA Encryption Caveats (1)

- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow.
- Plain RSA also does not provide integrity
  – Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt

$M \oplus G(r) \,||\, r \oplus H(M \oplus G(r))$

– r is random and fresh, G and H are hash functions

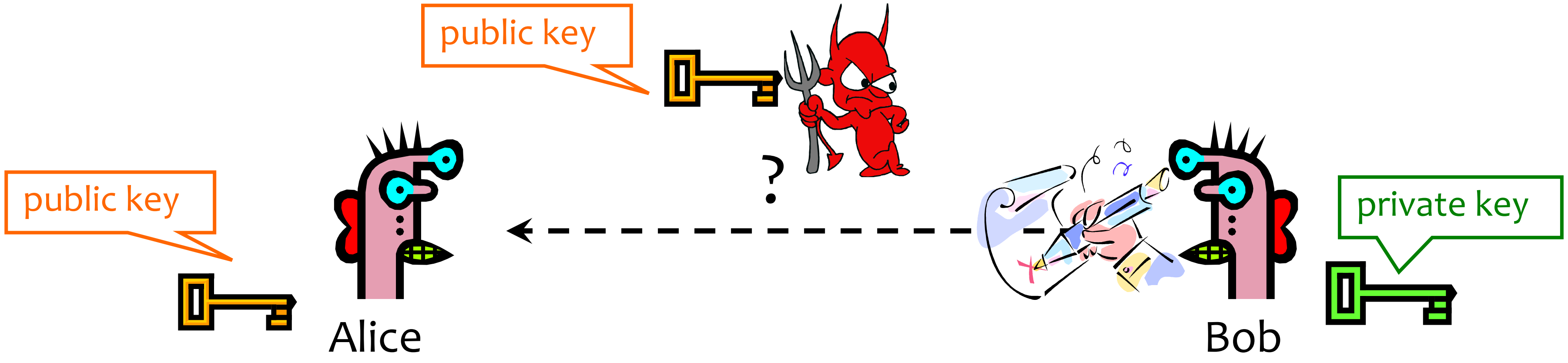
RSA Encryption Caveats (2)

- **There are better options now**
  - E.g., based on elliptic curve cryptography
- Lots of practical issues: https://blog.trailofbits.com/2019/07/08/fuck-rsa/
  - "While it may be theoretically possible to implement RSA correctly, decades of devastating attacks have proven that such a feat may be unachievable in practice."

Stepping Back: Asymmetric Crypto

- Last time we saw session key establishment (Diffie-Hellman)
  - Can then use shared key for symmetric crypto
- We just saw: public key encryption
  - For confidentiality
- Finally, now: digital signatures
  - For authenticity

Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: s = m^d mod n
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m: verify that s^e mod n = $(m^d)^e$ mod n = m
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Without padding and hashing: Consider multiplying two signatures together
  - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

- Digital Signature Standard (DSS)
  - U.S. government standard (1991, most recent rev. 2013)
- Public key: (p, q, g, $y=g^x$ mod p), private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from g^x mod p (public key)
- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Post-Quantum Cryptography

- If quantum computers become a reality
  - It becomes much more efficient to break conventional asymmetric encryption schemes (e.g., factoring becomes "easy")
- There exists efforts to make quantum-resilient asymmetric encryption schemes

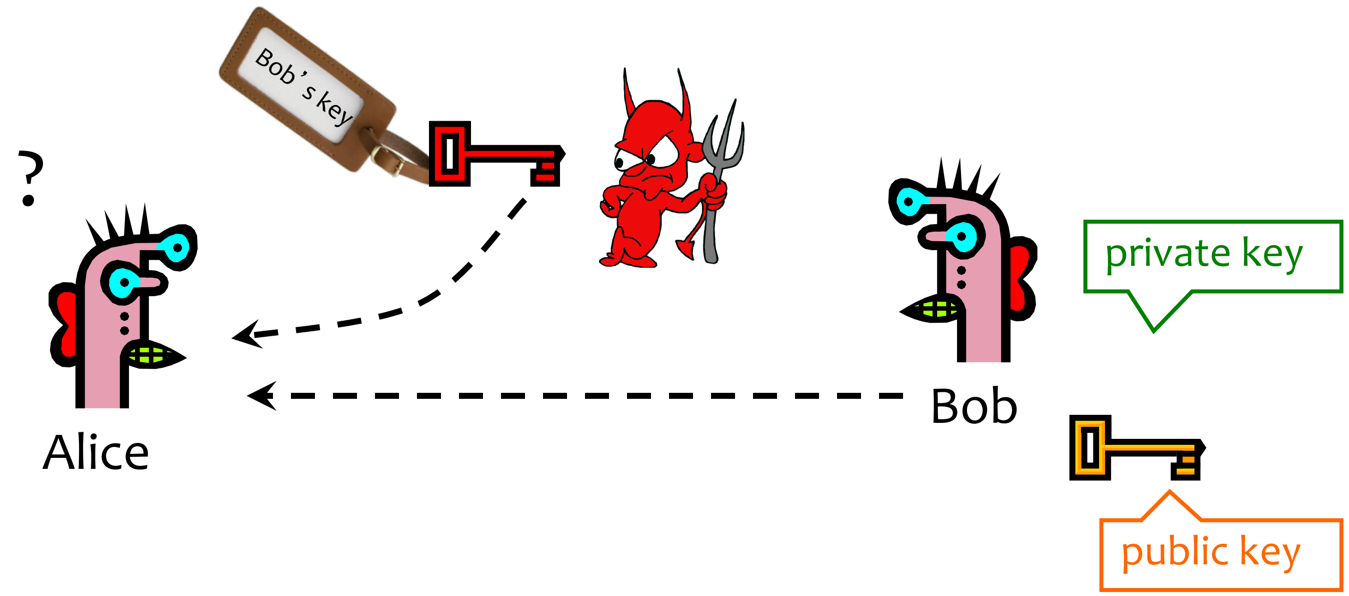
Cryptography Summary

- Goal: Privacy
  - Symmetric keys:
    - One-time pad, Stream ciphers
    - Block ciphers (e.g., DES, AES) → modes: ECB, CBC, CTR
  - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
  - MACs, often using hash functions (e.g, SHA-256)
- Goal: Privacy and Integrity ("authenticated encryption")
  - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
  - Digital signatures (e.g., RSA, DSS)

Want More Crypto?

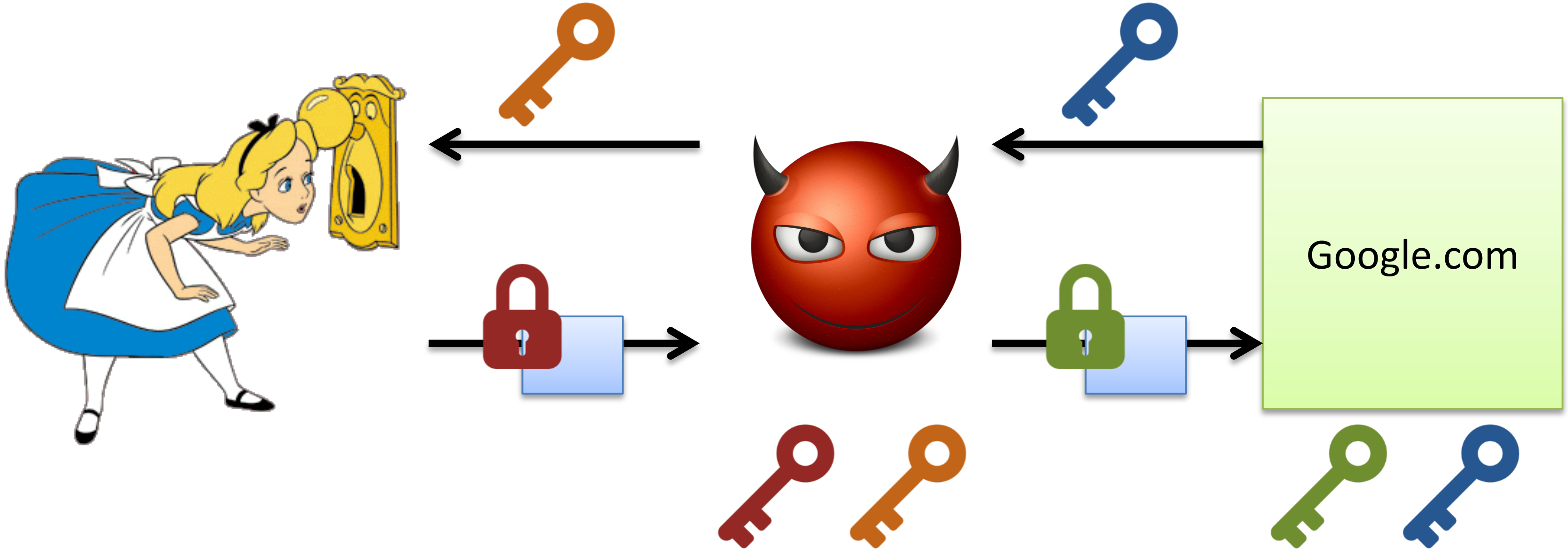
- Some suggestions:
  - CSE 426: Cryptography
  - Stanford Coursera (Dan Boneh): https://www.coursera.org/learn/crypto
- **Mid-quarter check-in:**
  - Questions / thoughts? See Canvas "Quiz" Activity

Authenticity of Public Keys



Problem: How does Alice know that the public key she received is really Bob's public key?

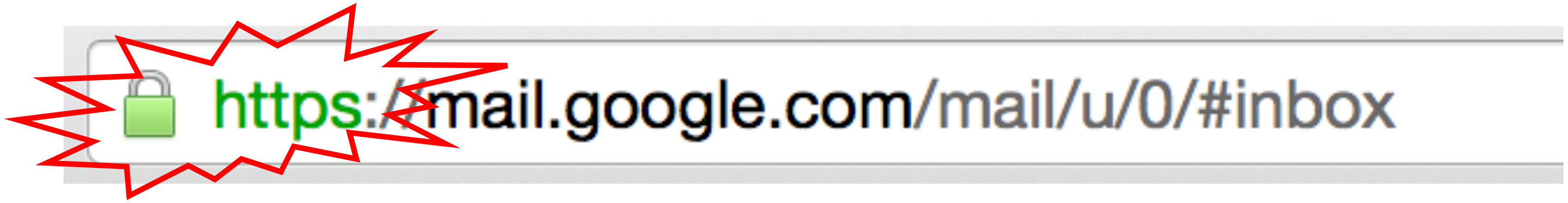
Threat: Person-in-the Middle



Distribution of Public Keys

- Public announcement or public directory
  - Risks: forgery and tampering
- Public-key certificate
  - Signed statement specifying the key and identity
    - sig_{CA}("Bob", PK_B)
- Common approach: certificate authority (CA)
  - Single agency responsible for certifying public keys
  - After generating a private/public key pair, user proves his identity and knowledge of the private key to obtain CA's certificate for the public key (offline)
  - Every computer is <u>pre-configured</u> with CA's public key

You encounter this every day…



SSL/TLS: Encryption & authentication for connections

SSL/TLS High Level

- SSL/TLS consists of two protocols
  - Familiar pattern for key exchange protocols
- Handshake protocol
  - Use public-key cryptography to establish a shared secret key between the client and the server
- Record protocol
  - Use the secret symmetric key established in the handshake protocol to protect communication between the client and the server

Example of a Certificate