CSE 484 / CSE M 584: Asymmetric Cryptography

Fall 2023

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Announcements

- Things due
  - Lab 1: Wednesday
  - Homework 2: Next Friday
    - Individual assignment (no groups)
  - CSE 584M: Don’t forget about weekly research readings
- Roadmap
  - Monday & Wednesday: Asymmetric crypto
  - Friday: Crypto in practice (on the web)
  - Next week: Web security

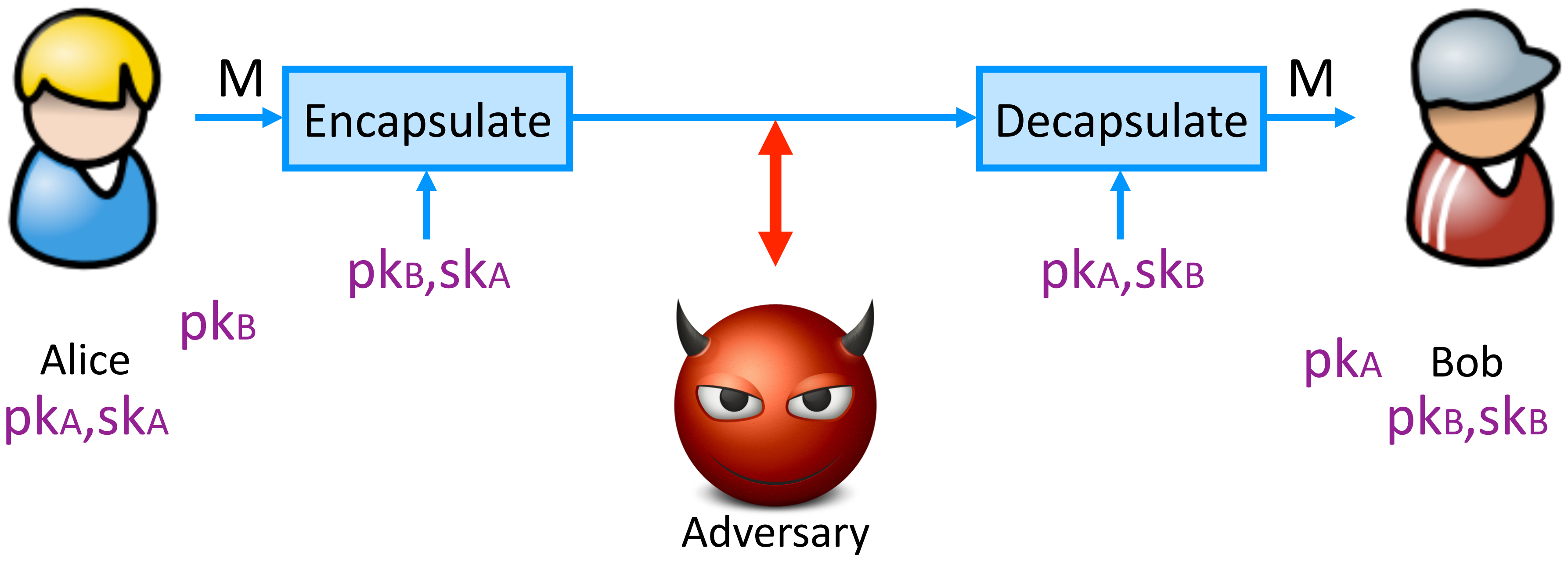
Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string K, called the key.

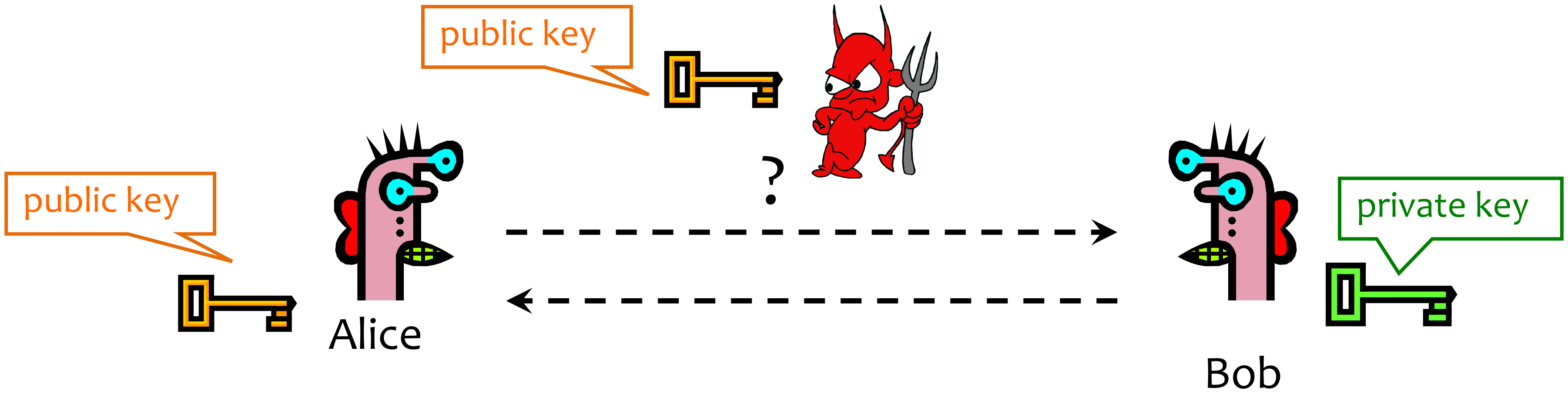
- Asymmetric cryptography
  - Each party creates a public key pk and a secret key sk.

Asymmetric Setting

Each party creates a public key pk and a secret key sk.



Public Key Crypto: Basic Problem



Ignore for now: How do we know it's REALLY Bob's??

Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate themself

Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can "sign" a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

Session Key Establishment

Modular Arithmetic

- Given g and prime p, compute: g^1 mod p, g^2 mod p, … g^{100} mod p
  - For p=11, g=10
    - 10^1 mod 11 = 10, 10^2 mod 11 = 1, 10^3 mod 11 = 10, …
    - Produces cyclic group {10, 1} (order=2)
  - For p=11, g=7
    - 7^1 mod 11 = 7, 7^2 mod 11 = 5, 7^3 mod 11 = 2, …
    - Produces cyclic group {7,5,2,3,10,4,6,9,8,1} (order = 10)
      - Numbers "wrap around" after they reach p
    - g=7 is a "generator" of Z_{11}^*

Diffie-Hellman Protocol (1976)

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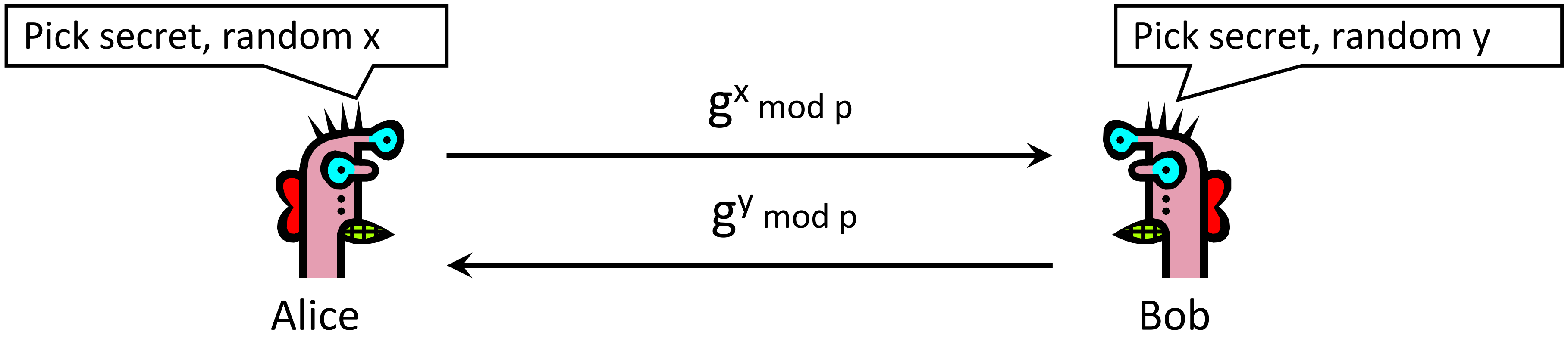
- Alice and Bob never met and share no secrets
- <u>Public</u> info: p and g
  - p is a large prime, g is a **generator** of Z_p^*
    - $Z_p^* = \{1, 2 \dots p-1\}$; a is in Z_p^* if there is an i such that $a = g^i \bmod p$



Compute $k = (g^y)^x = g^{xy} \bmod p$

Compute $k = (g^x)^y = g^{xy} \bmod p$

Example Diffie Hellman Computation

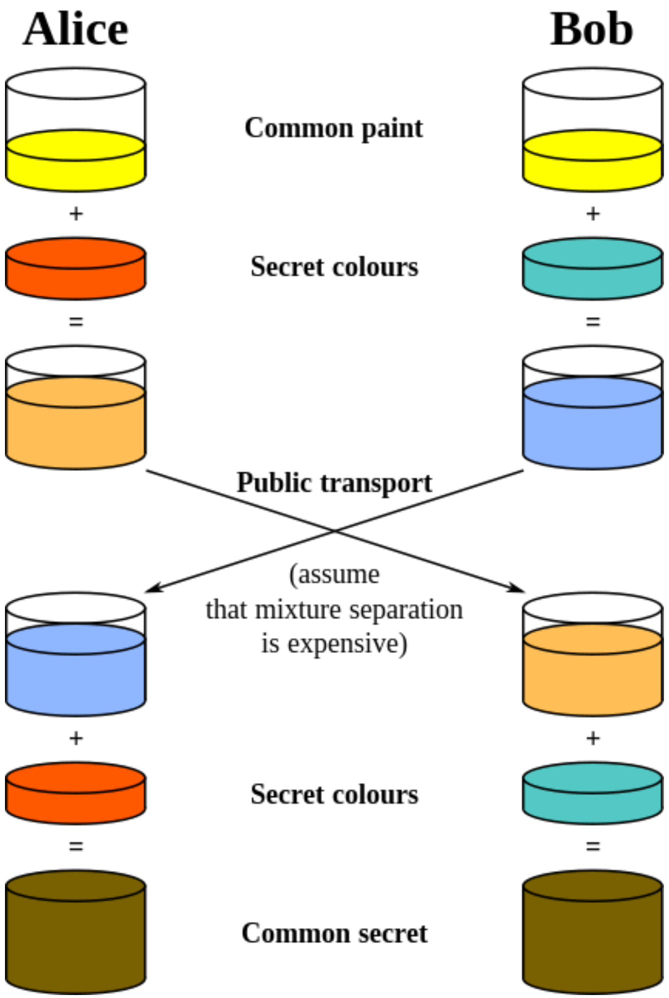
- PUBLIC
  - p = 11
  - g = 2
  - (g is a generator for group mod p)

- Alice: x=9, sends 6 (g^x mod p = 2^9 mod 11 = 6)
- Bob: y=4, send 5 (g^y mod p = 2^4 mod 11 = 5)

- A compute: 5^x mod 11 (5^9 mod 11 = 9)
- B compute 6^y mod 11 (6^4 mod 11 = 9)
- Both get 9

- All computations modulo 11

Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:

g^x mod p

g^y mod p

Common secret: g^{xy} mod p

[from Wikipedia]

Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:
  given g^x *mod p*, it's hard to extract x
  - There is no known <u>efficient</u> algorithm for doing this
  - This is <u>not</u> enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:
  given g^x and g^y, it's hard to compute g^{xy} *mod p*
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:
  given g^x and g^y, it's hard to tell the difference between
  g^{xy} *mod p* and g^r *mod p* where r is random

Diffie-Hellman Caveats (1)

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose p=2q+1, where q is also a large prime
    - Choose g that generates a subgroup of order q in Z_p*
    - DDH is hard in this group
  - Eavesdropper can't tell the difference between the established key and a random value
  - In practice, often hash g^{xy} *mod p*, and use the hash as the key
  - Can use the new key for symmetric cryptography

Diffie-Hellman Caveats (2)

- Diffie-Hellman protocol (by itself) does not provide authentication (against <u>active</u> attackers)
  - Person in the middle attack (aka "man in the middle attack")

Diffie-Hellman Key Exchange Today

- Important Note:
  - We have discussed discrete logs modulo integers
  - Significant advantages in using **elliptic curve groups**
    - Groups with some similar mathematical properties (i.e., are “groups”) but have better security and performance (size) properties
    - Today’s de-facto standard

Stepping Back: Asymmetric Crypto

- We've just seen session key establishment
  - Can then use shared key for symmetric crypto
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For authenticity

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function $\varphi(n)$ $(n \geq 1)$ is the number of integers in the $[1,n]$ interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: $\varphi(p) = p-1$
  - Note that $\varphi(ab) = \varphi(a)\,\varphi(b)$ if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 2048 bits each (need primality testing, too)
  - Compute **n**=pq and φ(**n**)=(p-1)(q-1)
  - Choose small **e**, relatively prime to $\varphi(n)$
    - Typically, **e=3** or **e=2^{16}+1=65537**
  - Compute unique **d** such that ed $\equiv$ 1 mod $\varphi(n)$
    - Modular inverse: d $\equiv$ e^{-1} mod $\varphi(n)$
  - Public key = (e,n); private key = (d,n)
- Encryption of m: c = m^e mod n
- Decryption of c: c^d mod n = $(m^e)^d$ mod n = m

How to compute?

- Extended Euclidian algorithm
- Wolfram Alpha ☺
- Brute force for small values