CSE 484 / CSE M 584: More Asymmetric Cryptography

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Announcements

- Things due
  - Lab 1: tomorrow
  - Homework 2: Next Friday
    - Individual assignment (no groups)
  - CSE 584M: Don't forget about weekly research readings
- In-class activities
  - 5 "freebies"
  - In-section activities not graded

Stepping Back: Asymmetric Crypto

- Last time we saw session key establishment (Diffie-Hellman)
  - Can then use shared key for symmetric crypto
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For authenticity

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function $\varphi(n)$ $(n \geq 1)$ is the number of integers in the $[1,n]$ interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: $\varphi(p) = p-1$
  - Note that $\varphi(ab) = \varphi(a)\,\varphi(b)$ if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 2048 bits each (need primality testing, too)
  - Compute **n**=pq and φ(**n**)=(p-1)(q-1)
  - Choose small **e**, relatively prime to $\varphi(n)$
    - Typically, **e=3** or **e=2^{16}+1=65537**
  - Compute unique **d** such that ed $\equiv$ 1 mod $\varphi(n)$
    - Modular inverse: d $\equiv$ e^{-1} mod $\varphi(n)$
  - Public key = (e,n); private key = (d,n)
- Encryption of m: c = m^e mod n
- Decryption of c: c^d mod n = $(m^e)^d$ mod n = m

How to compute?

- Extended Euclidian algorithm
- Wolfram Alpha ☺
- Brute force for small values

Why is RSA Secure?

- RSA problem: given c, n=pq, and e such that gcd(e, φ(n))=1, find m such that $m^e = c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this *without* knowing p and q
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n = p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$

- If factoring is easy, then RSA problem is easy
  (knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n – but if it is, we don't know how

RSA Encryption Caveats

- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow.
- Plain RSA also does not provide integrity
  - Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt

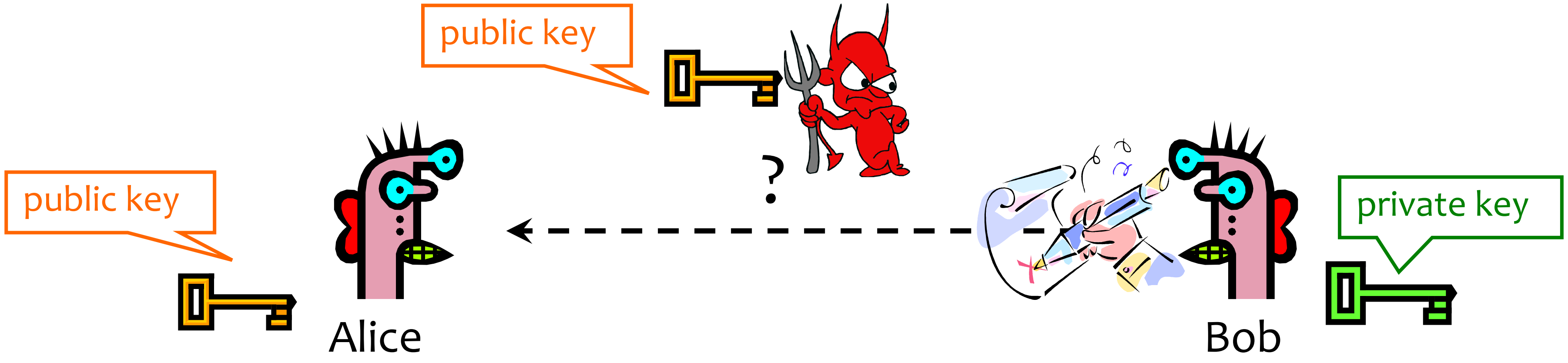
$M \oplus G(r) \| r \oplus H(M \oplus G(r))$

- r is random and fresh, G and H are hash functions

Stepping Back: Asymmetric Crypto

- Last time we saw session key establishment (Diffie-Hellman)
  – Can then use shared key for symmetric crypto
- We just saw: public key encryption
  – For confidentiality
- Finally, now: digital signatures
  – For authenticity

Digital Signatures: Basic Idea



<u>Given</u>: Everybody knows Bob's public key
Only Bob knows the corresponding private key

<u>Goal</u>: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: s = $m^d \bmod n$
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m: verify that $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Without padding and hashing: Consider multiplying two signatures together
  - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

- Digital Signature Standard (DSS)
  - U.S. government standard (1991, most recent rev. 2013)
- Public key: $(p, q, g, y=g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from $g^x \bmod p$ (public key)
- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Post-Quantum Cryptography

- If quantum computers become a reality
  - It becomes much more efficient to break conventional asymmetric encryption schemes (e.g., factoring becomes “easy”)
  - For block ciphers (symmetric encryption), use 128-bit keys for 256-bits of security
- There exists efforts to make quantum-resilient asymmetric encryption schemes

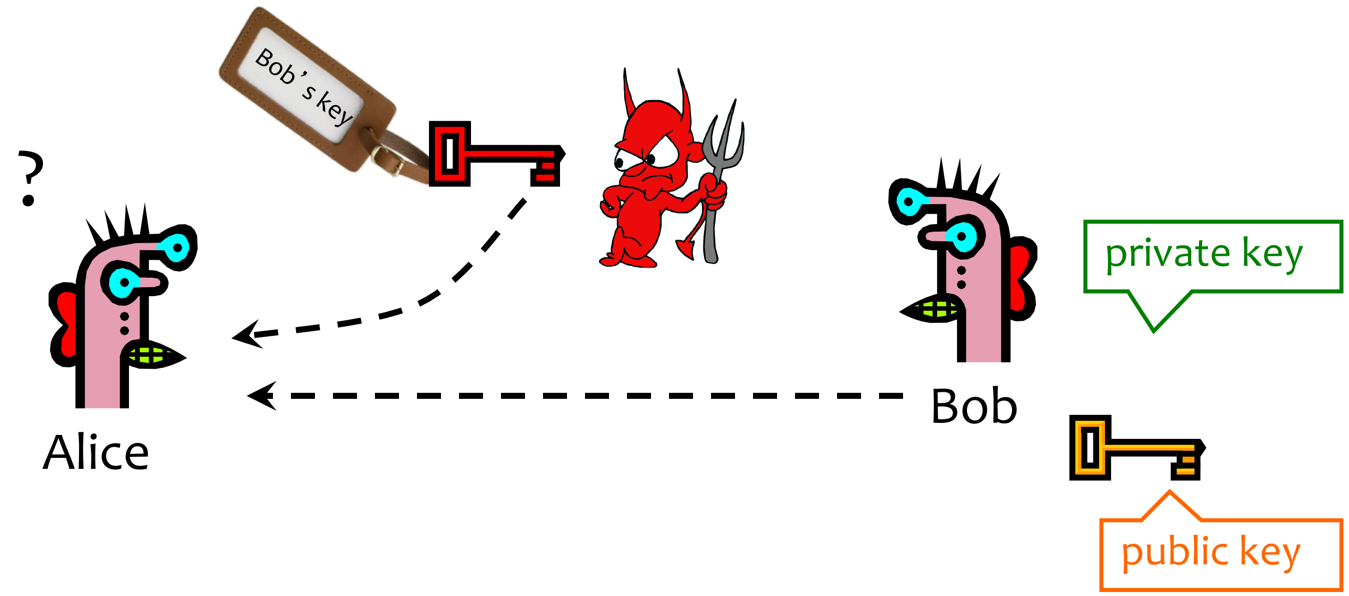
Cryptography Summary

- Goal: Privacy
  - Symmetric keys:
    - One-time pad, Stream ciphers
    - Block ciphers (e.g., DES, AES) → modes: EBC, CBC, CTR
  - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
  - MACs, often using hash functions (e.g, SHA-256)
- Goal: Privacy and Integrity ("authenticated encryption")
  - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
  - Digital signatures (e.g., RSA, DSS)

Want More Crypto?

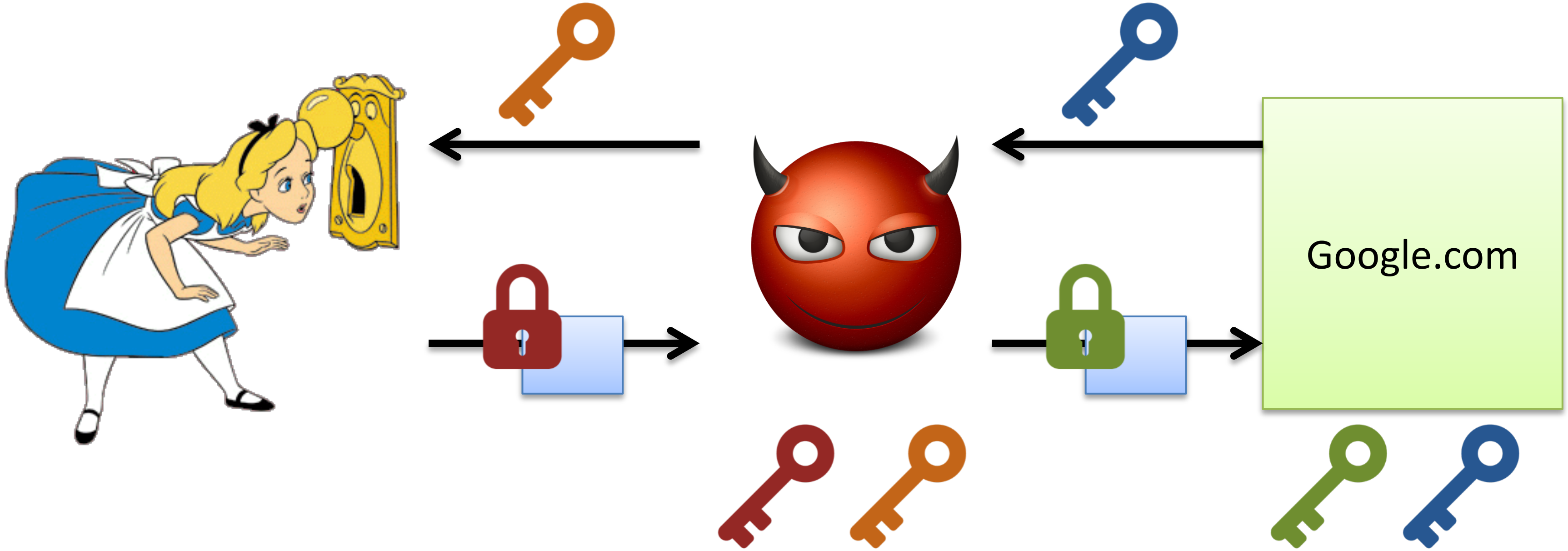
- Some suggestions:
  - CSE 490C, likely becoming CSE 426: Cryptography
  - Stanford Coursera (Dan Boneh): https://www.coursera.org/learn/crypto

Authenticity of Public Keys



Problem: How does Alice know that the public key she received is really Bob's public key?

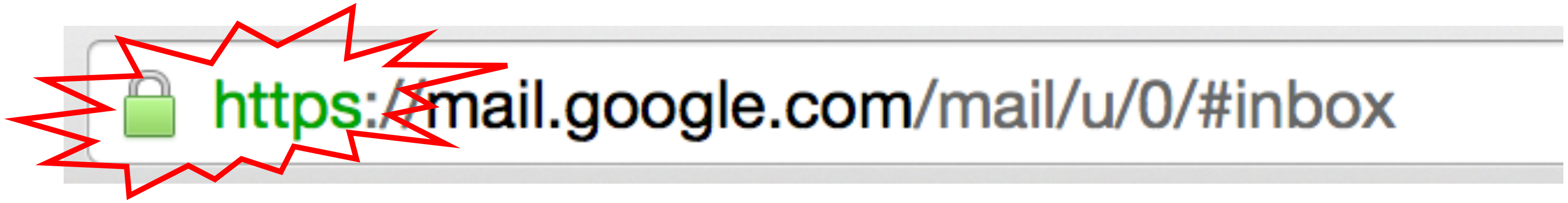
Threat: Person-in-the Middle



Distribution of Public Keys

- Public announcement or public directory
  - Risks: forgery and tampering
- Public-key certificate
  - Signed statement specifying the key and identity
    - sig_{CA}(“Bob”, PK_B)
- Common approach: certificate authority (CA)
  - Single agency responsible for certifying public keys
  - After generating a private/public key pair, user proves his identity and knowledge of the private key to obtain CA’s certificate for the public key (offline)
  - Every computer is pre-configured with CA’s public key

You encounter this every day…



SSL/TLS: Encryption & authentication for connections