CSE 484: Computer Security and Privacy

Cryptography
[Asymmetric Cryptography]

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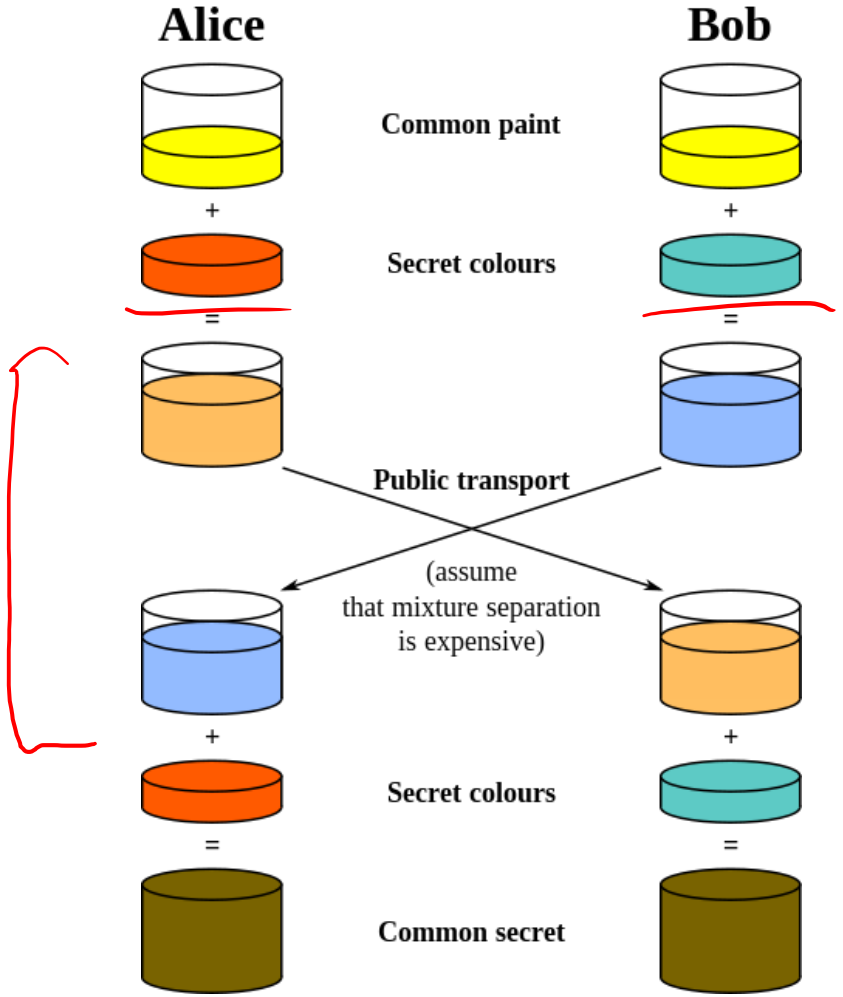
Admin

- **No class next Monday** (8^{th})
- **Lab 1 due tonight**
- In-class course feedback/evaluation on Friday

Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:

$g^x \bmod p$

$g^y \bmod p$

Common secret: $g^{xy} \bmod p$

[from Wikipedia]

Diffie-Hellman Caveats

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose p=2q+1, where q is also a large prime
    - Choose g that generates a subgroup of order q in Z_p*
  - Eavesdropper can't tell the difference between the established key and a random value
  - In practice, often hash g^{xy} *mod p*, and use the hash as the key
  - Can use the new key for symmetric cryptography
- Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
  - Person in the middle attack (also called "man in the middle attack")

Remember our troubles with randomness?

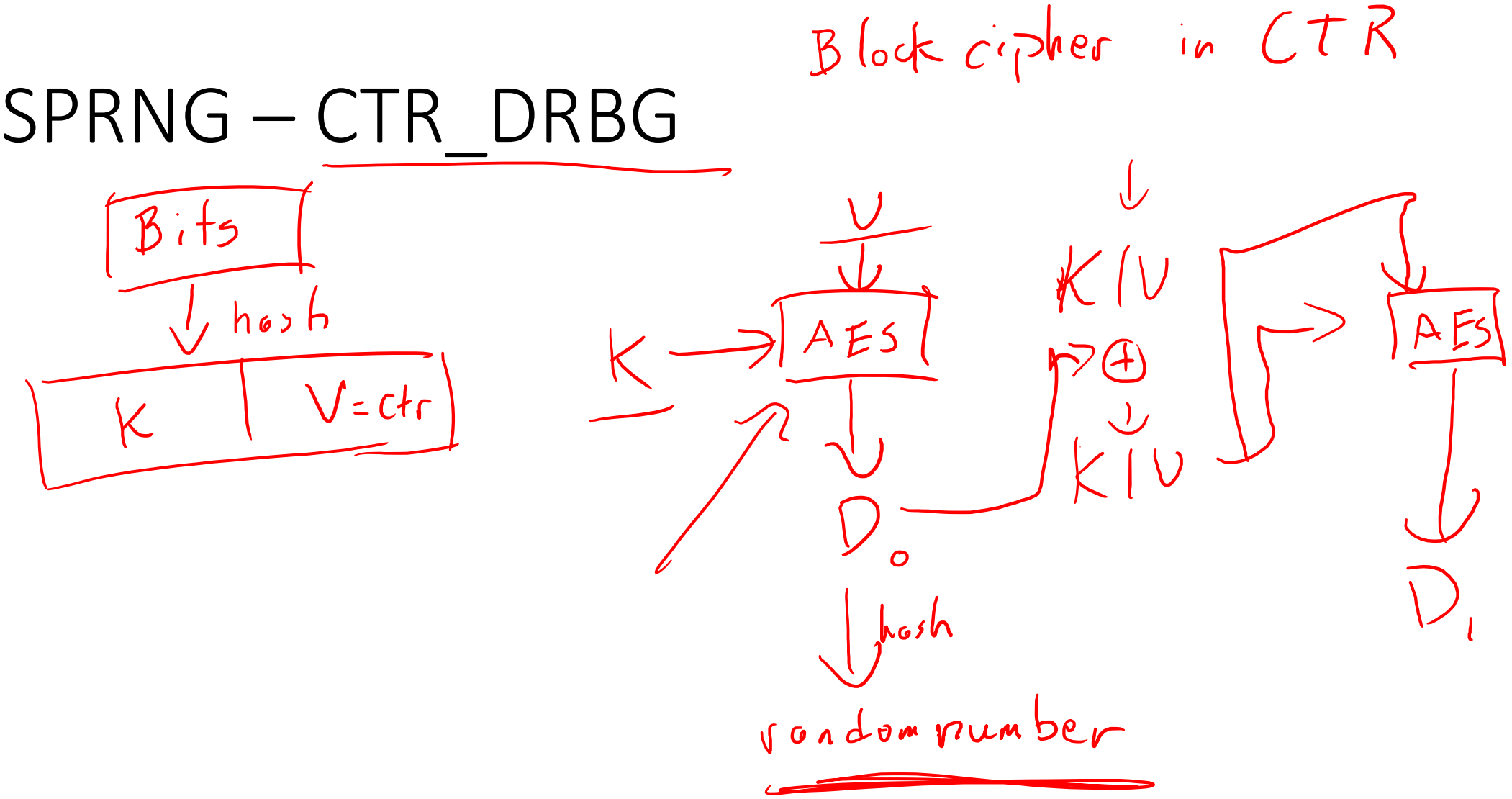
CSPRNGs in practice (or DRBG)

- Gather some good entropy (256 bits?)
- Use a block cipher/HMAC/Hash to 'stretch' this entropy
- Regularly mix in more entropy!

Check out NIST.SP.800-90

3 CSPRNGs

CSPRNG – CTR_DRBG



Why does this work for CSPRNGs?

- To 'break' the CSPRNG (that is, predict the next output)
  - Must know state of CSPRNG (key, inputs)
  - Requires breaking the security of the primitive!

- Your CSPRNG is just as secure as the scheme you use the output for!
  - Never 'loses entropy', same guarantee as block cipher

- Why mix in new entropy?
  - Can't hurt, prevents a single bug from breaking the future

CSPRNGs gone bad

4th rec. CSPRNG

2004 - 2013?

- DUAL_EC_DRBG – Dual Elliptic Curve Deterministic Random Bit Generator
  - CSPRNG based on elliptic curve math
  - NSA designed

juniper VPNs

- DUAL_EC_DRBG has a *backdoor*
  - Special mathematical construction that allows recovery of state!

- Remember DES's s-boxes?
  - This is the opposite

Stepping Back: Asymmetric Crypto

- We've just seen session key establishment
  - Can then use shared key for symmetric crypto → cause fast
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For authenticity

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Encrypt

Some Number Theory Facts

- Euler totient function φ(n) (n≥1) is the number of integers in the [1,n] interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: φ(p) = p-1
  - Note that φ(ab) = φ(a) φ(b) if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 1024 bits each (need primality testing, too)
  - Compute **n**=pq and φ**(n)**=(p-1)(q-1)
  - Choose small **e**, relatively prime to φ(n)
    - Typically, **e=3** or **e=2^{16}+1=65537**
  - Compute unique **d** such that ed ≡ 1 mod φ(n)
    - Modular inverse: $d \equiv e^{-1} \bmod \varphi(n)$
  - Public key = (e,n); private key = (d,n)
- Encryption of m: $c = m^e \bmod n$
- Decryption of c: $c^d \bmod n = (m^e)^d \bmod n = m$

How to compute?

Why is RSA Secure?

n = p q

eff key strength
1024 bits = 80 bits of sec.

- RSA problem: given c, n=pq, and e such that gcd(e, φ(n))=1, find m such that $m^e = c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n = p_1^{e1} p_2^{e2} \ldots p_k^{ek}$

- If factoring is easy, then RSA problem is easy (knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n -- but if it is, we don't know how

RSA Encryption Caveats

$M^e \mod N$

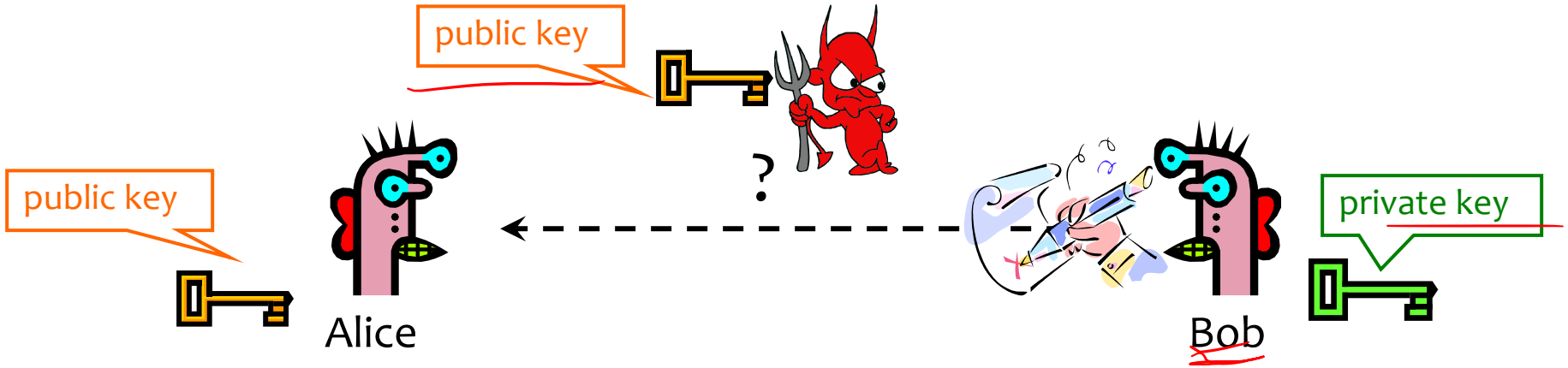
- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow
- Plain RSA also does not provide integrity
  - Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt

$M \oplus G(r) \,||\, r \oplus H(M \oplus G(r))$

- r is random and fresh, G and H are hash functions

Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

e d

$M^{e^d} = M \bmod N$

$M^{d^e} \bmod N = M$

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: $s = m^d \bmod n$
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m:

  verify that $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

- Digital Signature Standard (DSS)
  - U.S. government standard (1991, most recent rev. 2013)
- Public key: $(p, q, g, y=g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from $g^x \bmod p$ (public key)
- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Cryptography Summary

- Goal: Privacy
  - Symmetric keys:
    - One-time pad, Stream ciphers
    - Block ciphers (e.g., DES, AES) → modes: EBC, CBC, CTR
  - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
  - MACs, often using hash functions (e.g, SHA-256)
- Goal: Privacy and Integrity
  - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
  - Digital signatures (e.g., RSA, DSS)

Want More Crypto?

- Some suggestions:
  - CSE 490C (Rachel Lin): https://courses.cs.washington.edu/courses/cse490c/20au/
  - Stanford Coursera (Dan Boneh): https://www.coursera.org/learn/crypto