CSE 484 : Computer Security and Privacy

Cryptography

[Finish Hash Functions; Start Asymmetric Cryptography]

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David Kohlbrenner

dkohlbre@cs.washington.edu

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Admin

• Lab 1 due on Wednesday!
  • Check your group settings on Canvas!
• Remember to do your ‘in-class’ activities, even if you watch the recordings, they are nearly free points
• Homework 2 (crypto) out now (due Feb 10)
Recall: Achieving Integrity

**Message authentication schemes:** A tool for protecting integrity.

**Integrity and authentication:** only someone who knows KEY can compute correct MAC for a given message.
HMAC (older hashes)

- Construct MAC from a cryptographic hash function
  - Invented by Bellare, Canetti, and Krawczyk (1996)
  - Used in SSL/TLS, mandatory for IPsec
- Construction:
  - $\text{HMAC}(k,m) = \text{Hash}((k \oplus \text{ipad}) \ || \ \text{Hash}(k \oplus \text{opad} \ || \ m))$
- Why not block ciphers (at the time it was designed)?
  - Hashing is faster than block ciphers in software
  - Can easily replace one hash function with another
  - There used to be US export restrictions on encryption
MAC with SHA3

• SHA3(Key || Message)

• SHA3 has some nice features that prevent the class of attacks HMAC prevents
Authenticated Encryption

• What if we want both privacy and integrity?
• Natural approach: combine encryption scheme and a MAC.
Authenticated Encryption

• What if we want both privacy and integrity?
• Natural approach: combine encryption scheme and a MAC.
• But be careful!
  • Obvious approach: Encrypt-and-MAC
  • Problem: MAC is deterministic! same plaintext $\rightarrow$ same MAC

\[\begin{align*}
&\text{Encrypt}_{K_e} \quad \text{MAC}_{K_m} \\
&\quad \downarrow \quad \downarrow \\
&C_1^' \quad T_1 \\
&\text{Encrypt}_{K_e} \quad \text{MAC}_{K_m} \\
&\quad \downarrow \quad \downarrow \\
&C_2^' \quad T_2 \\
&\text{Encrypt}_{K_e} \quad \text{MAC}_{K_m} \\
&\quad \downarrow \quad \downarrow \\
&C_3^' \quad T_3
\end{align*}\]
Authenticated Encryption

• Instead: Encrypt \textit{then} MAC.

• (Not as good: MAC-then-Encrypt)
Back to cryptography land
Stepping Back: Flavors of Cryptography

• Symmetric cryptography
  • Both communicating parties have access to a shared random string $K$, called the key.

• Asymmetric cryptography
  • Each party creates a public key $pk$ and a secret key $sk$. 
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Asymmetric Setting

Each party creates a public key $pk$ and a secret key $sk$. 

Alice
$pk_A, sk_A$

Bob
$pk_B, sk_B$

Adversary
$pk_B, sk_A$

$pk_A, sk_B$
Given: Everybody knows Bob’s **public key**
Only Bob knows the corresponding **private key**

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate himself
Applications of Public Key Crypto

• Encryption for confidentiality
  • Anyone can encrypt a message
    • With symmetric crypto, must know secret key to encrypt
  • Only someone who knows private key can decrypt
  • Key management is simpler (or at least different)
    • Secret is stored only at one site: good for open environments

• Digital signatures for authentication
  • Can “sign” a message with your private key

• Session key establishment
  • Exchange messages to create a secret session key
  • Then switch to symmetric cryptography (why?)
Session Key Establishment
Modular Arithmetic

• Given $g$ and prime $p$, compute: $g^1 \mod p$, $g^2 \mod p$, ... $g^{100} \mod p$
  • For $p=11$, $g=10$
    • $10^1 \mod 11 = 10$, $10^2 \mod 11 = 1$, $10^3 \mod 11 = 10$, ...
    • Produces cyclic group $\{10, 1\}$ (order=2)
  • For $p=11$, $g=7$
    • $7^1 \mod 11 = 7$, $7^2 \mod 11 = 5$, $7^3 \mod 11 = 2$, ...
    • Produces cyclic group $\{7,5,2,3,10,4,6,9,8,1\}$ (order = 10)
    • $g=7$ is a “generator” of $\mathbb{Z}_{11}^*$
Diffie-Hellman Protocol (1976)
Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- **Public info:** $p$ and $g$
  - $p$ is a large prime, $g$ is a **generator** of $\mathbb{Z}_p^*$
    - $\mathbb{Z}_p^* = \{1, 2, \ldots, p-1\}$; an $i$ such that $a = g^i \mod p$
    - **Modular arithmetic:** numbers “wrap around” after they reach $p$

**Diagram:**

- Alice picks secret, random $X$
- Bob picks secret, random $Y$
- Alice computes $k = (g^y)^x = g^{xy} \mod p$
- Bob computes $k = (g^x)^y = g^{xy} \mod p$
Example Diffie Hellman Computation
Why is Diffie-Hellman Secure?

• Discrete Logarithm (DL) problem:
  - given $g^x \mod p$, it’s hard to extract $x$
    • There is no known efficient algorithm for doing this
    • This is not enough for Diffie-Hellman to be secure!

• Computational Diffie-Hellman (CDH) problem:
  - given $g^x$ and $g^y$, it’s hard to compute $g^{xy} \mod p$
    • ... unless you know $x$ or $y$, in which case it’s easy

• Decisional Diffie-Hellman (DDH) problem:
  - given $g^x$ and $g^y$, it’s hard to tell the difference between $g^{xy} \mod p$ and $g^r \mod p$ where $r$ is random
Diffie-Hellman: Conceptually

**Common paint:** $p$ and $g$

**Secret colors:** $x$ and $y$

**Send over public transport:**
- $g^x \mod p$
- $g^y \mod p$

**Common secret:** $g^{xy} \mod p$
Properties of Diffie-Hellman

• Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  • Common recommendation:
    • Choose $p=2q+1$, where $q$ is also a large prime
    • Choose $g$ that generates a subgroup of order $q$ in $\mathbb{Z}_p^*$
  • Eavesdropper can’t tell the difference between the established key and a random value
    • In practice, often hash $g^{xy} \mod p$, and use the hash as the key
    • Can use the new key for symmetric cryptography
• Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
  • Person in the middle attack (also called “man in the middle attack”)
Person In The Middle Attack
More on Diffie-Hellman Key Exchange

• **Important Note:**
  • We have discussed discrete logs modulo integers
  • Significant advantages in using *elliptic curve groups*
    • Groups with some similar mathematical properties (i.e., are “groups”) but have better security and performance (size) properties