CSE 484: Computer Security and Privacy

Cryptography
[Finish Hash Functions; Start Asymmetric Cryptography]

Winter 2021

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Admin

- Lab 1 due on **Wednesday**!
  - Check your group settings on Canvas!
- Remember to do your ‘in-class’ activities, even if you watch the recordings, they are nearly free points
- Homework 2 (crypto) out now (due Feb 10)

*Feb 8th* no live lecture
Recall: Achieving Integrity

Message authentication schemes: A tool for protecting integrity.

Integrity and authentication: only someone who knows KEY can compute correct MAC for a given message.
HMAC (older hashes)

• Construct MAC from a cryptographic hash function
  • Invented by Bellare, Canetti, and Krawczyk (1996)
  • Used in SSL/TLS, mandatory for IPsec

• Construction:
  • $\text{HMAC}(k, m) = \text{Hash}((k \oplus \text{ipad}) || \text{Hash}(k \oplus \text{opad} || m))$

• Why not block ciphers (at the time it was designed)?
  • Hashing is faster than block ciphers in software
  • Can easily replace one hash function with another
  • There used to be US export restrictions on encryption
MAC with SHA3

• \( \text{SHA3} (\text{Key} \ || \ \text{Message}) = \text{MAC} \)

• SHA3 has some nice features that prevent the class of attacks HMAC prevents
Authenticated Encryption

• What if we want both privacy and integrity?
• Natural approach: combine encryption scheme and a MAC.

Encrypt_{K_e} M_1 \rightarrow MAC_{K_m} \rightarrow C'_1 T_1
Encrypt_{K_e} M_2 \rightarrow MAC_{K_m} \rightarrow C'_2 T_2
Encrypt_{K_e} M_3 \rightarrow MAC_{K_m} \rightarrow C'_3 T_3
Authenticated Encryption

• What if we want both privacy and integrity?
• Natural approach: combine encryption scheme and a MAC.
• But be careful!
  • Obvious approach: Encrypt-and-MAC
  • Problem: MAC is deterministic! same plaintext → same MAC
Authenticated Encryption

• Instead: Encrypt \textit{then} MAC.

• (Not as good: MAC-then-Encrypt)
Back to cryptography land
Stepping Back:
Flavors of Cryptography

• Symmetric cryptography
  • Both communicating parties have access to a shared random string K, called the key.

• Asymmetric cryptography
  • Each party creates a public key pk and a secret key sk.
Both communicating parties have access to a shared random string $K$, called the key.
Asymmetric Setting

Each party creates a public key $pk$ and a secret key $sk$. 

- Alice: $pk_A, sk_A$
- Bob: $pk_B, sk_B$
Public Key Crypto: Basic Problem

Given: Everybody knows Bob’s public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate himself

Ignore for now: How do we know it’s REALLY Bob’s??
Applications of Public Key Crypto

• Encryption for confidentiality
  • Anyone can encrypt a message
    • With symmetric crypto, must know secret key to encrypt
  • Only someone who knows private key can decrypt
  • Key management is simpler (or at least different)
    • Secret is stored only at one site: good for open environments

• Digital signatures for authentication
  • Can “sign” a message with your private key

• Session key establishment
  • Exchange messages to create a secret session key
  • Then switch to symmetric cryptography (why?)
Session Key Establishment

A \rightarrow Public info \rightarrow B

clear text

adversary

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Modular Arithmetic

• Given $g$ and prime $p$, compute: $g^1 \mod p$, $g^2 \mod p$, … $g^{100} \mod p$
  • For $p=11$, $g=10$
    • $10^1 \mod 11 = 10$, $10^2 \mod 11 = 1$, $10^3 \mod 11 = 10$ …
    • Produces cyclic group $\{10, 1\}$ (order=2)
  • For $p=11$, $g=7$
    • $7^1 \mod 11 = 7$, $7^2 \mod 11 = 5$, $7^3 \mod 11 = 2$ …
    • Produces cyclic group $\{7, 5, 2, 3, 10, 4, 6, 9, 8, 1\}$ (order = 10)
  • $g=7$ is a “generator” of $\mathbb{Z}_{11}^*$
Diffie-Hellman Protocol (1976)

Diffie and Hellman Receive 2015 Turing Award

Whitfield Diffie

Martin E. Hellman
Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info: $p$ and $g$
  - $p$ is a large prime, $g$ is a generator of $\mathbb{Z}_p^*$
    - $\mathbb{Z}_p^* = \{1, 2 \ldots p-1\}$; an element $a$ such that $a = g^i \mod p$
    - Modular arithmetic: numbers “wrap around” after they reach $p$

Alice:
- Pick secret, random $X$
- Compute $k = (g^y)^x = g^{xy} \mod p$

Bob:
- Pick secret, random $Y$
- Compute $k = (g^x)^y = g^{xy} \mod p$
Example Diffie Hellman Computation

\[ p = 11 \quad g = 2 \]

Alice
\[ x = 9 \]
\[ 5^9 \mod 11 = 9 \]
\[ \text{KDF}(9) = \text{Key} \]

Bob
\[ y = 4 \]
\[ g^4 \mod 11 = 5 \]
\[ 6^4 \mod 11 = 9 \]
\[ \text{KDF}(9) = \text{Key} \]
Why is Diffie-Hellman Secure?

- **Discrete Logarithm (DL) problem:**
  - Given $g^x \mod p$, it’s hard to extract $x$
  - There is no known efficient algorithm for doing this
  - This is **not** enough for Diffie-Hellman to be secure!

- **Computational Diffie-Hellman (CDH) problem:**
  - Given $g^x$ and $g^y$, it’s hard to compute $g^{xy} \mod p$
  - ... unless you know $x$ or $y$, in which case it’s easy

- **Decisional Diffie-Hellman (DDH) problem:**
  - Given $g^x$ and $g^y$, it’s hard to tell the difference between $g^{xy} \mod p$ and $g^r \mod p$ where $r$ is random
Diffie-Hellman: Conceptually

Common paint: $p$ and $g$

Secret colors: $x$ and $y$

Send over public transport:
$g^x \mod p$
$g^y \mod p$

Common secret: $g^{xy} \mod p$
Properties of Diffie-Hellman

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose \( p=2q+1 \), where \( q \) is also a large prime
    - Choose \( g \) that generates a subgroup of order \( q \) in \( \mathbb{Z}_p^* \)
  - Eavesdropper can’t tell the difference between the established key and a random value
  - In practice, often hash \( g^{xy} \mod p \), and use the hash as the key
  - Can use the new key for symmetric cryptography
- Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
  - Person in the middle attack (also called “man in the middle attack”)

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Person In The Middle Attack

\[ A \rightarrow g^x \mod p \left| g^{xz} \right| g^{yz} \left| g^z \mod p \right| B \]

\[ g^z \mod p \left| g^{yz} \mod p \right| \rightarrow \text{read mod} \]
More on Diffie-Hellman Key Exchange

• **Important Note:**
  • We have discussed discrete logs modulo integers
  • Significant advantages in using elliptic curve groups
    • Groups with some similar mathematical properties (i.e., are “groups”) but have better security and performance (size) properties