

CSE 484 : Computer Security and Privacy

# Cryptography

[Finish Hash Functions; *— and MACs*  
Start Asymmetric Cryptography]

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# Admin

Monday Feb 1<sup>st</sup>

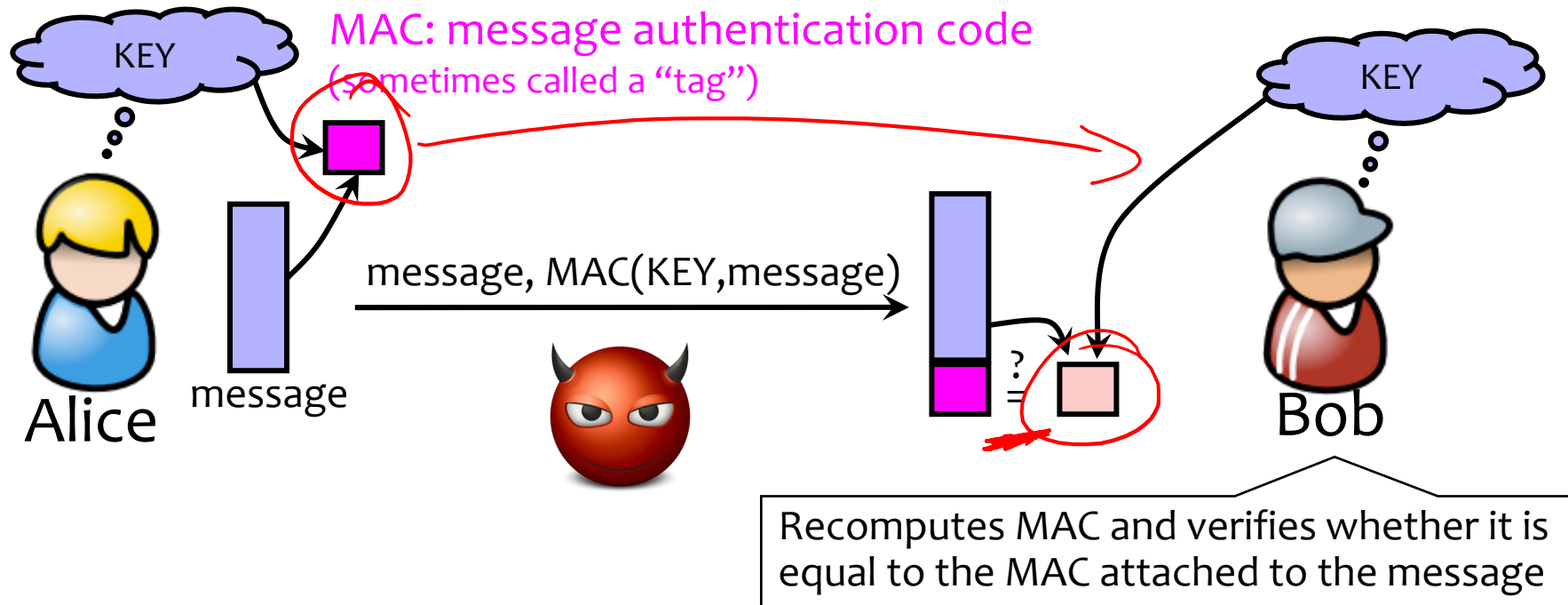
- Lab 1 due on ~~Wednesday~~!
  - Check your group settings on Canvas!
- Remember to do your 'in-class' activities, even if you watch the recordings, they are nearly free points
- Homework 2 (crypto) out now (due Feb 10)

wednesday

Feb 8<sup>th</sup> no live lecture

# Recall: Achieving Integrity

Message authentication schemes: A tool for protecting integrity.



Integrity and authentication: only someone who knows KEY can compute correct MAC for a given message.

$|| \rightarrow \text{concatenate}$   
 $+ \rightarrow ?$

# HMAC (older hashes)

- Construct MAC from a cryptographic hash function

- Invented by Bellare, Canetti, and Krawczyk (1996)
- Used in SSL/TLS, mandatory for IPsec

- Construction:

$\downarrow 0x36 \quad \downarrow 0x5c$

$$\text{HMAC}(k, m) = \text{Hash}((k \oplus \text{ipad}) || \text{Hash}(k \oplus \text{opad} || m))$$

- Why not block ciphers (at the time it was designed)?

- Hashing is faster than block ciphers in software
- Can easily replace one hash function with another

→ There used to be US export restrictions on encryption

Hash(Key || msg)  
SHA-1/2  
MD5  
↑

# MAC with SHA3

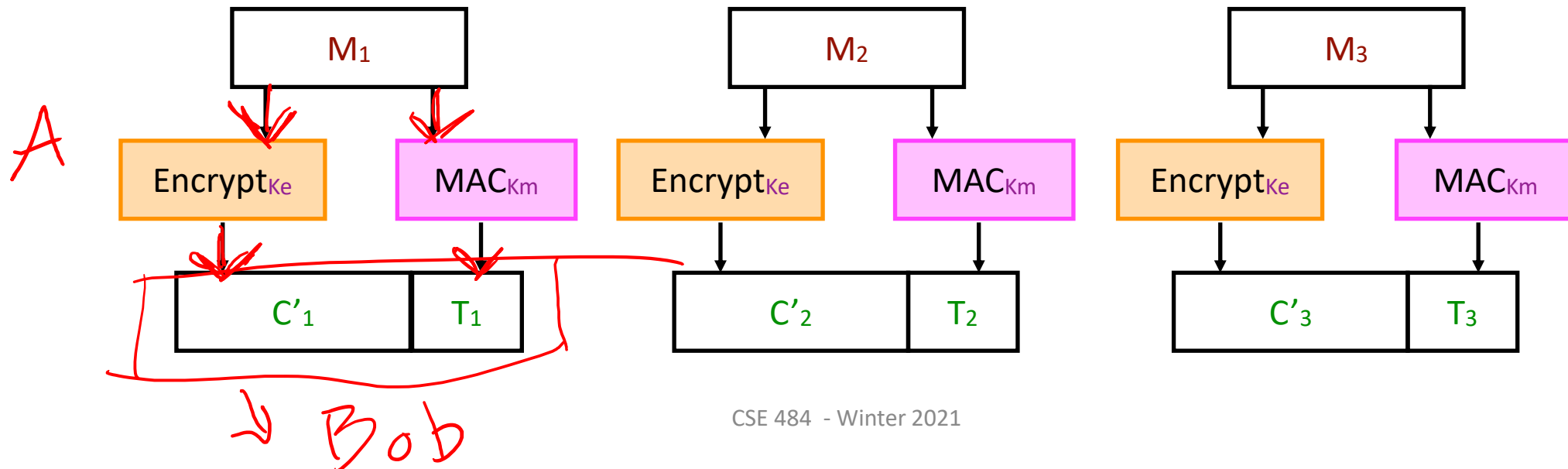
- $\text{SHA3}(\text{Key} || \text{Message}) = \text{MAC}$
- SHA3 has some nice features that prevent the class of attacks HMAC prevents

# Authenticated Encryption

CIA  
↑

- What if we want both privacy and integrity?
- Natural approach: combine **encryption scheme** and a **MAC**.

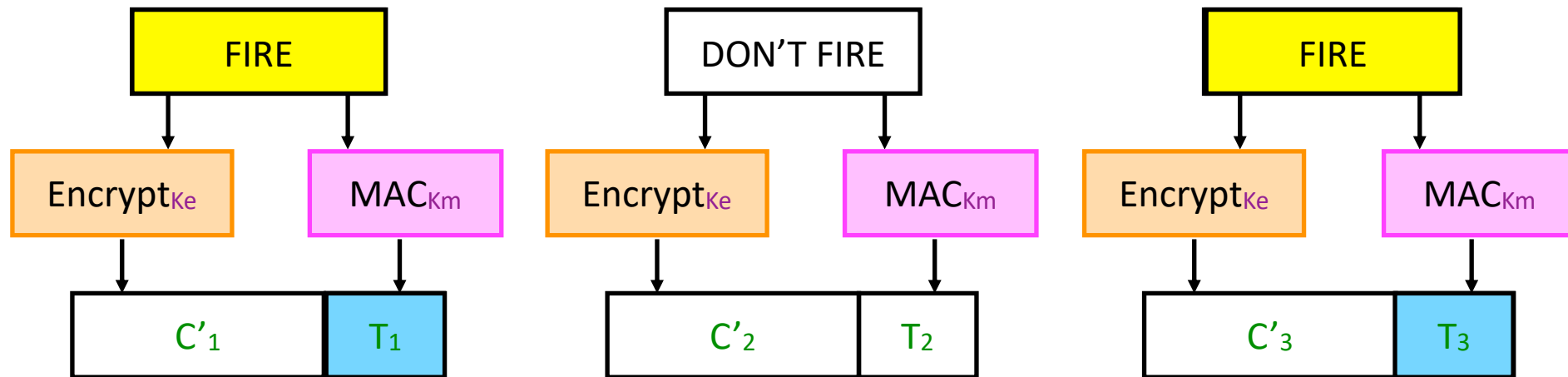
Encrypt and MAC



# Authenticated Encryption

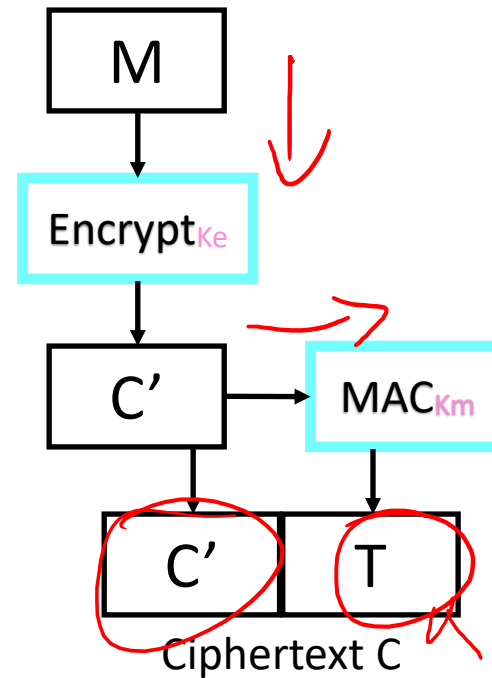
$$C'_1 \neq C_3$$
$$T_1 = T_3$$

- What if we want both privacy and integrity?
- Natural approach: combine **encryption scheme** and a **MAC**.
- But be careful!
  - Obvious approach: Encrypt-and-MAC
  - Problem: MAC is deterministic! same plaintext  $\rightarrow$  same MAC

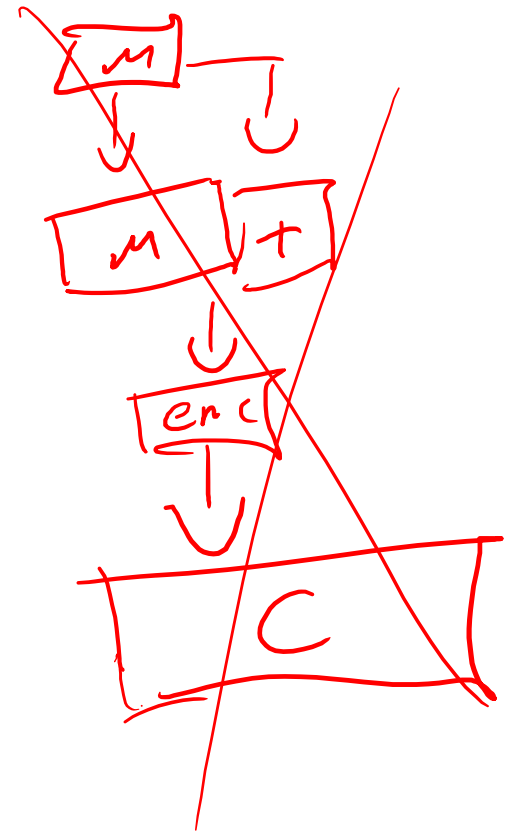


# Authenticated Encryption

- Instead:  
*Encrypt then MAC.*
- (Not as good:  
MAC-then-Encrypt)



**Encrypt-then-MAC**





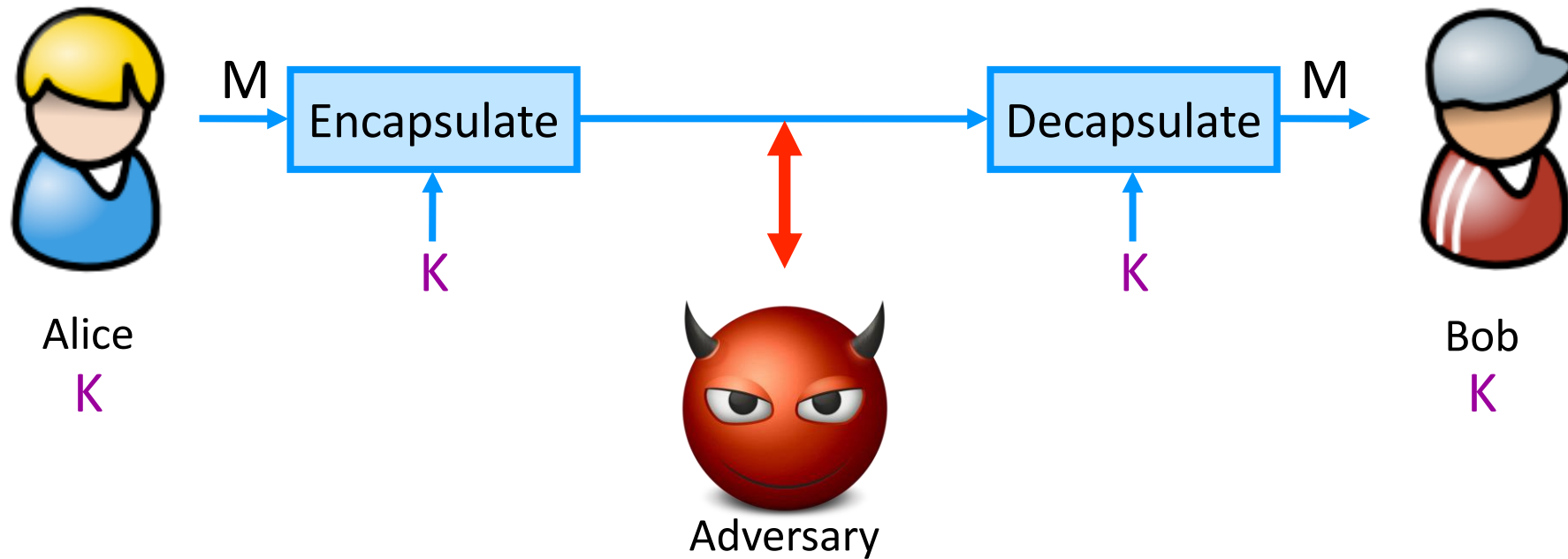
# Back to cryptography land

# Stepping Back: Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a **shared random string  $K$** , called the **key**.
- Asymmetric cryptography
  - Each party creates a public key  **$pk$**  and a secret key  **$sk$** .

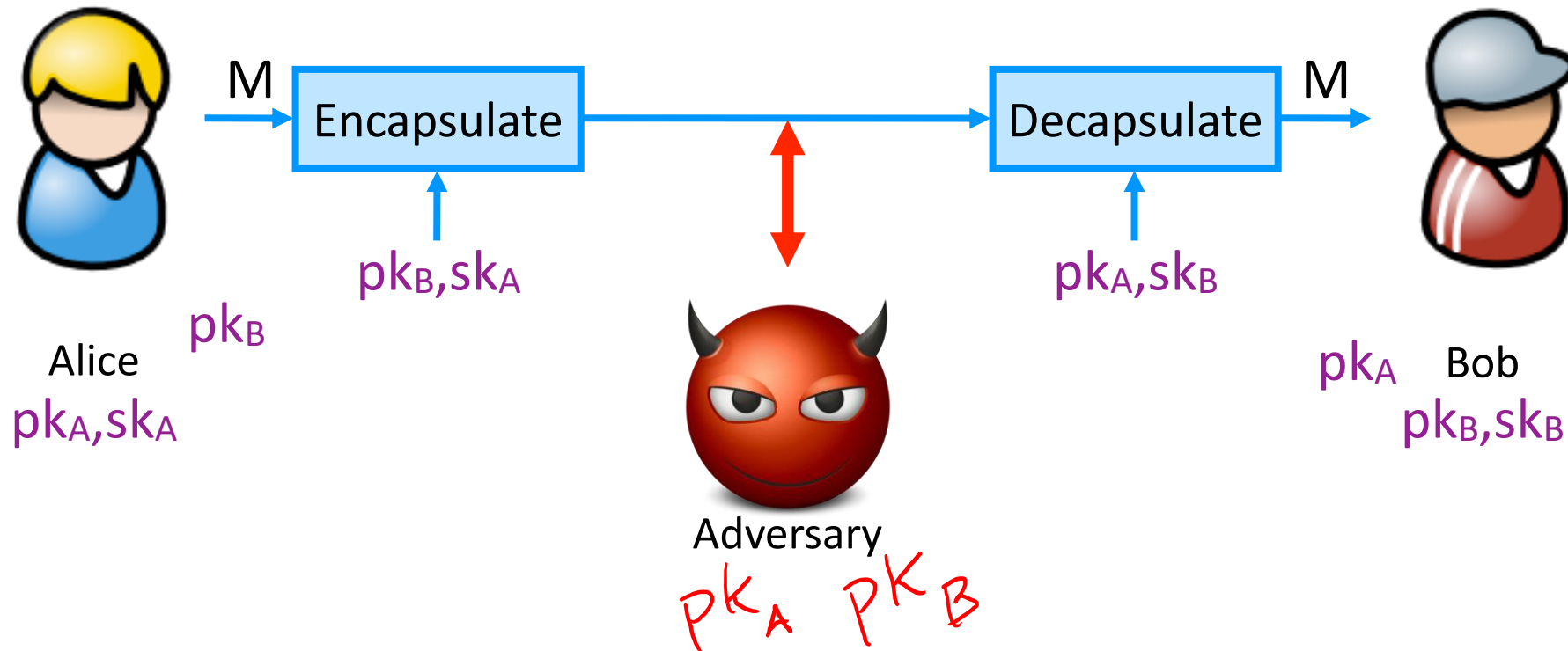
# Symmetric Setting

Both communicating parties have access to a **shared random string  $K$** , called the **key**.

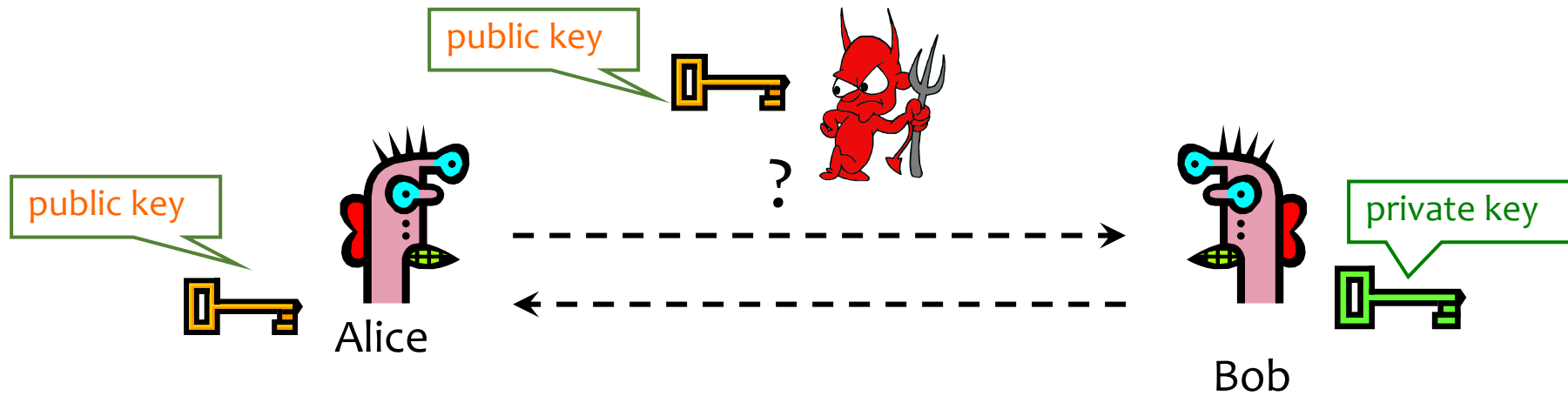


# Asymmetric Setting

Each party creates a public key  $pk$  and a secret key  $sk$ .



# Public Key Crypto: Basic Problem



Given: Everybody knows Bob's **public key**  
Only Bob knows the corresponding **private key**

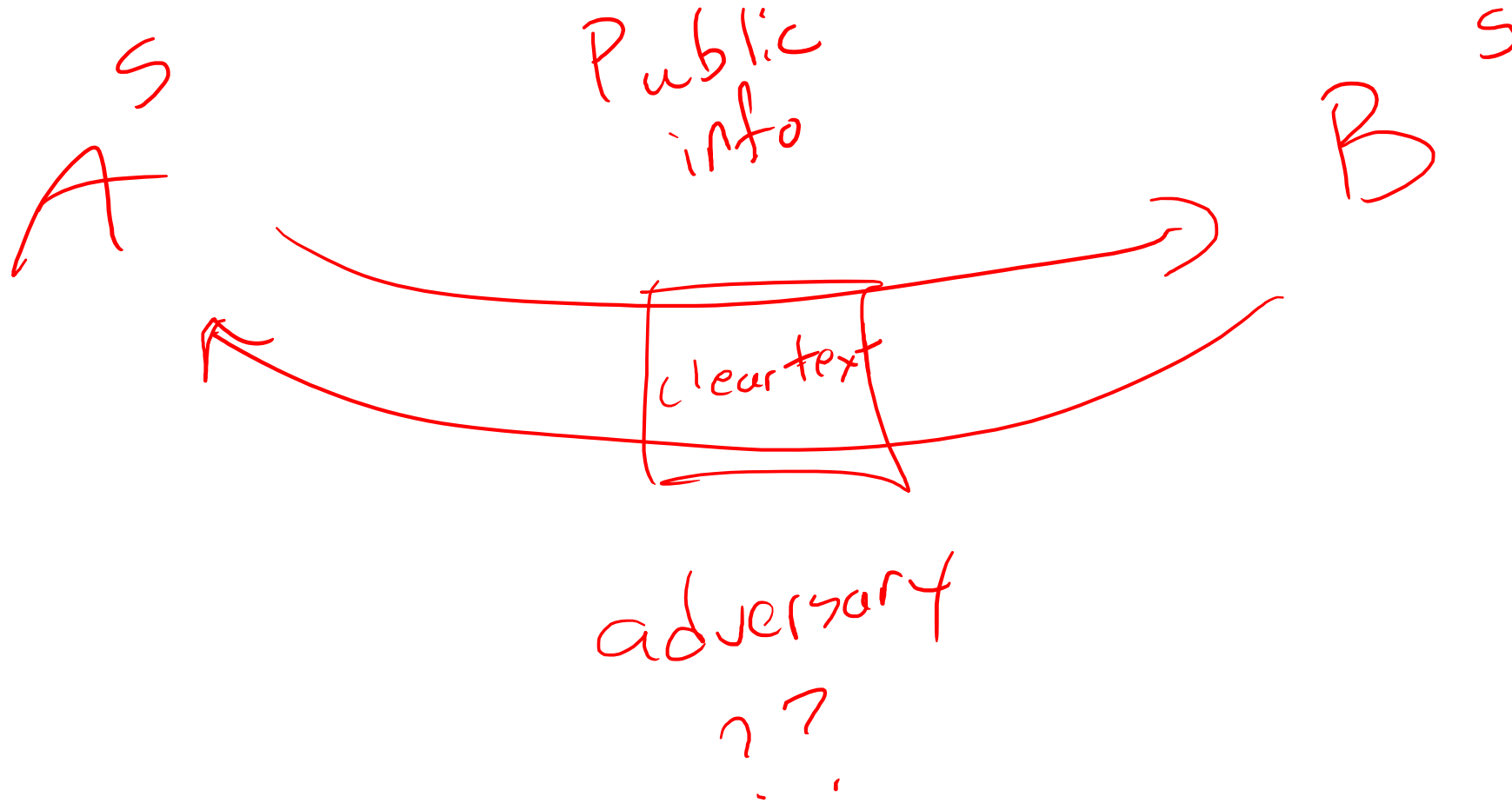
Ignore for now: How do we know it's REALLY Bob's??

Goals: 1. Alice wants to send a secret message to Bob  
2. Bob wants to authenticate himself

# Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can “sign” a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key  $\rightarrow$  random secret value
  - Then switch to symmetric cryptography (why?)

# Session Key Establishment



# Modular Arithmetic

Algebra

- Given  $g$  and prime  $p$ , compute:  $g^1 \bmod p$ ,  $g^2 \bmod p$ , ...  $g^{100} \bmod p$

- For  $p=11$ ,  $g=10$

- $10^1 \bmod 11 = 10$ ,  $10^2 \bmod 11 = 1$ ,  $10^3 \bmod 11 = 10$  ...

- Produces cyclic group  $\{10, 1\}$  (order=2)

- For  $p=11$ ,  $g=7$

- $7^1 \bmod 11 = 7$ ,  $7^2 \bmod 11 = 5$ ,  $7^3 \bmod 11 = 2$ , ...

- Produces cyclic group  $\{7, 5, 2, 3, 10, 4, 6, 9, 8, 1\}$  (order = 10)

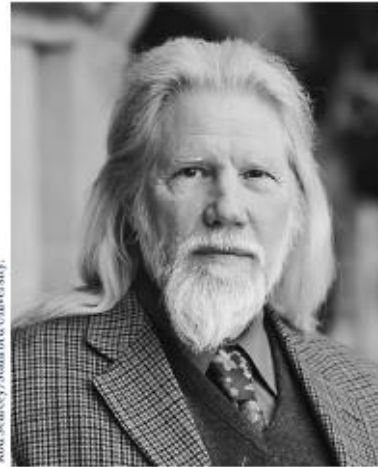
- $g=7$  is a "generator" of  $\mathbb{Z}_{11}^*$

$g^n \bmod p$



# Diffie-Hellman Protocol (1976)

## Diffie and Hellman Receive 2015 Turing Award



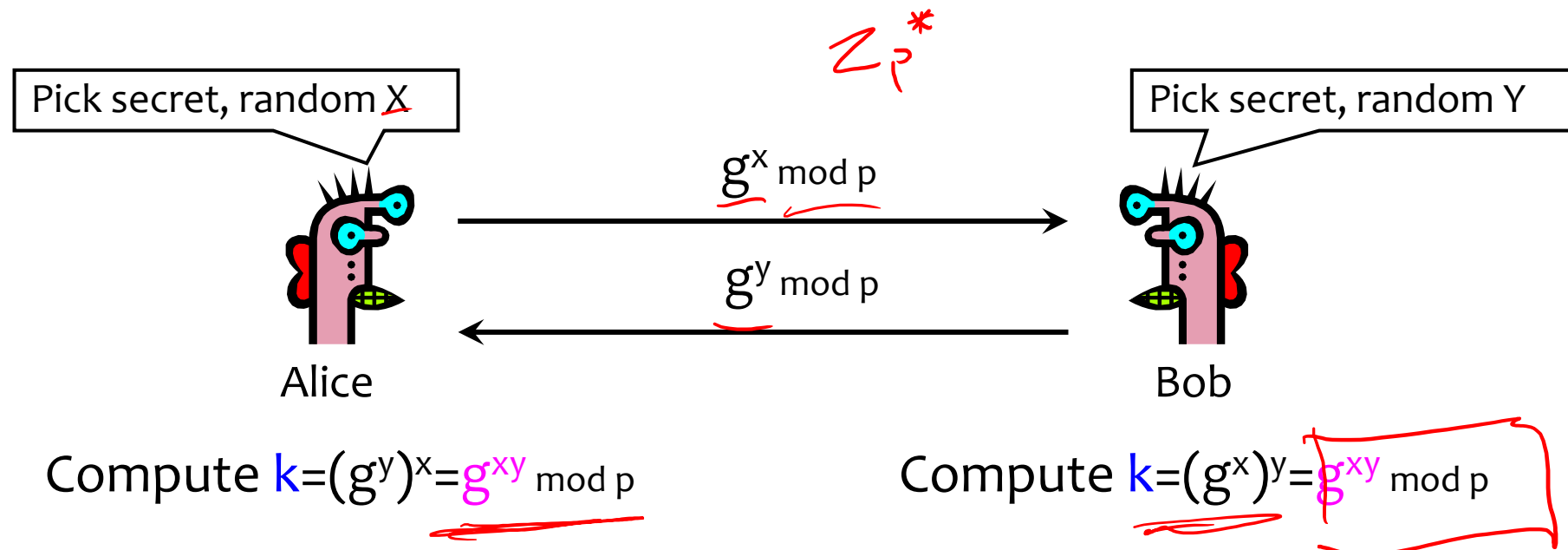
Whitfield Diffie



Martin E. Hellman

# Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info:  $p$  and  $g$ 
  - $p$  is a large prime,  $g$  is a **generator** of  $Z_p^*$ 
    - $Z_p^* = \{1, 2 \dots p-1\}$ ; a  $Z_p^*$   $i$  such that  $a = g^i \bmod p$
    - Modular arithmetic: numbers “wrap around” after they reach  $p$



# Example Diffie Hellman Computation

$$p = 11 \quad g = 2$$

Alice

$$x = 9$$

$$5^9 \bmod 11 = \textcircled{9}$$

$$\text{KDF}(9) = \underline{\text{key}}$$

$$\xrightarrow{g^a \bmod 11 = 6}$$

$$\xleftarrow{g^4 \bmod 11 = 5}$$

6, 5
??
..

Bob  
 $y = 4$

$$6^4 \bmod 11 = \textcircled{9}$$

$$\text{KDF}(9) = \underline{\text{key}}$$

# Why is Diffie-Hellman Secure?

$$\underline{g^x \bmod p = 6}$$

- Discrete Logarithm (DL) problem:

given  $g^x \bmod p$ , it's hard to extract  $x$

- There is no known efficient algorithm for doing this
- This is not enough for Diffie-Hellman to be secure!

- Computational Diffie-Hellman (CDH) problem:

→ given  $g^x$  and  $g^y$ , it's hard to compute  $g^{xy} \bmod p$

- ... unless you know  $x$  or  $y$ , in which case it's easy

- Decisional Diffie-Hellman (DDH) problem:

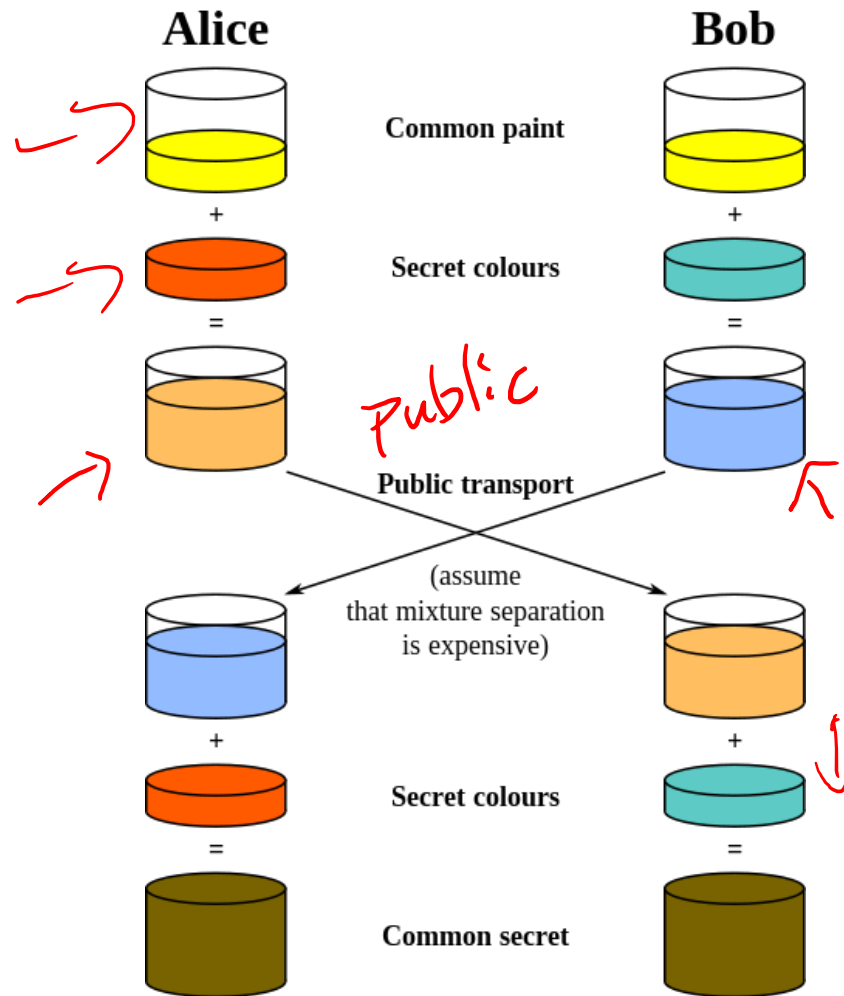
Stronger

given  $g^x$  and  $g^y$ , it's hard to tell the difference between  
where  $r$  is random

$$(g^x)^y \bmod p = (g^y)^x \bmod p$$

$$\underline{g^{xy} \bmod p} \text{ and } \underline{g^r \bmod p}$$

# Diffie-Hellman: Conceptually



**Common paint:**  $p$  and  $g$

**Secret colors:**  $x$  and  $y$

**Send over public transport:**

$g^x \bmod p$

$g^y \bmod p$

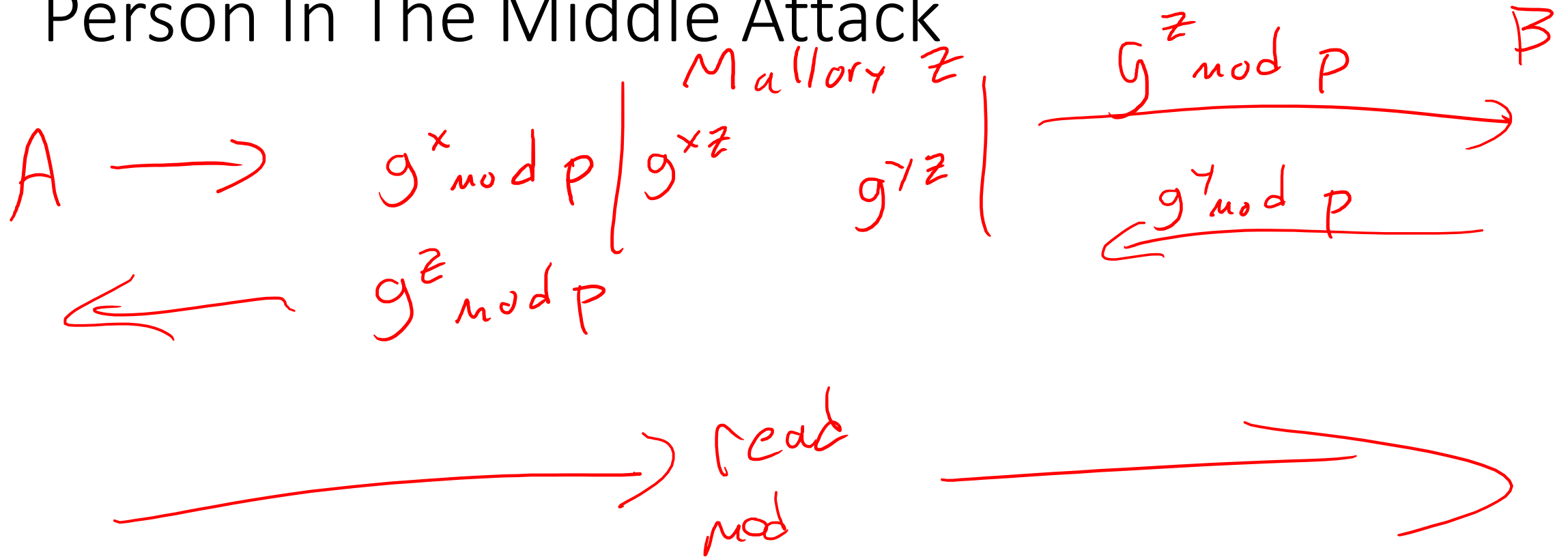
**Common secret:**  $g^{xy} \bmod p$

[from Wikipedia]

# Properties of Diffie-Hellman

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose  $p=2q+1$ , where  $q$  is also a large prime
    - Choose  $g$  that generates a subgroup of order  $q$  in  $\mathbb{Z}_p^*$  *f y i s*
  - Eavesdropper can't tell the difference between the established key and a random value
  - In practice, often hash  $g^{xy} \bmod p$ , and use the hash as the key
  - Can use the new key for symmetric cryptography *KDF*
- Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
  - Person in the middle attack (also called “man in the middle attack”)

# Person In The Middle Attack



# More on Diffie-Hellman Key Exchange

$$g^x \quad g^y \quad (x, y)$$
$$g^{xy}$$

- Important Note:

- We have discussed discrete logs modulo integers
- Significant advantages in using elliptic curve groups
  - Groups with some similar mathematical properties (i.e., are “groups”) but have better security and performance (size) properties

ECDH  
the standard