Section 4: Lab 1 Hints, Modular Arithmetic and 2DES

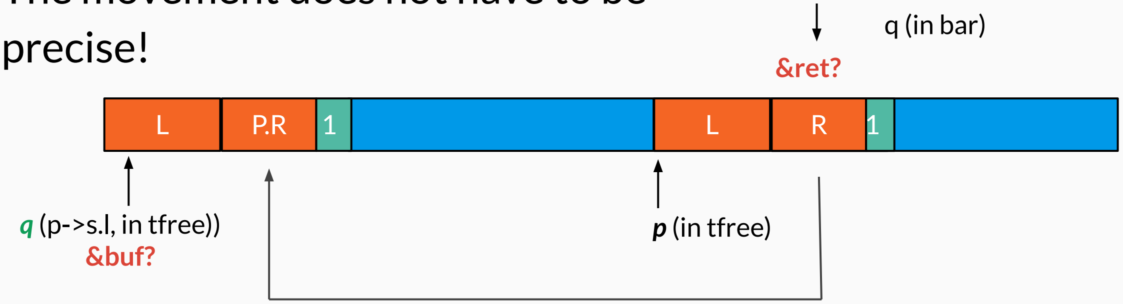
Administrivia

- Final deadline for lab1 is Friday, April 30 @ 11:59pm
  - Run the md5sum command on your last 4 exploits
  - Put the outputs in <netid>_<netid>_<netid>.txt
  - Submit on Canvas
- Homework 2 to be released early next week
  - Hands-on work with cryptography
  - Individual assignment

Lab 1 Notes/Hints

```
108    q = p->s.l;
109    if (q != NULL && GET_FREEBIT(q))
110      {
111        CLR_FREEBIT(q);
112        q->s.r        = p->s.r;
113        p->s.r->s.l = q;
114        SET_FREEBIT(q);
115        p = q;
```

- Sploit 5: See tfree from last section.
    - Make sure the free bit of the left chunk is set
    - The 2nd four bytes of ***q*** will be overwritten by line 112
    - How can you move past this?
        i. Point to an assembly instruction?
        ii. Hardcode an instruction code?
        iii. The movement does not have to be precise!

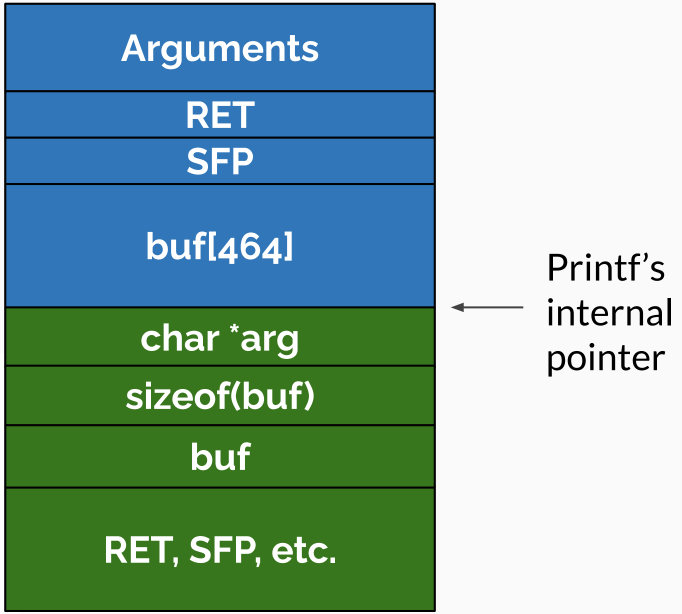


Lab 1 Notes/Hints

- Sploit 6: snprintf to a location.
    - Overwrite ret with %n (will need > 1)
    - Pad %u, %d, %x to get the value to write
    - %u, %d, %x, %n all expect an argument
    - Internal pointer begins after (char *) arg

```c
int foo(char *arg)
{
  char buf[312];
  snprintf(buf, sizeof buf, arg);
  return 0;
}
```

Blue: foo's stack frame
Green: snprintf's stack frame



Additional arguments to `snprintf` would (normally) be after `arg`.

```c
int snprintf ( char * s, size_t n, const char * format, ... );
```

Lab 1 Notes/Hints

- Sploit 7: Similar to sploit 2.
  - However, you can’t overwrite RET since `foo` calls `_exit` before returning.
  - Where can you take over execution?
    - Hint: Think about `*p = a`
  - Try disassembling `_exit`

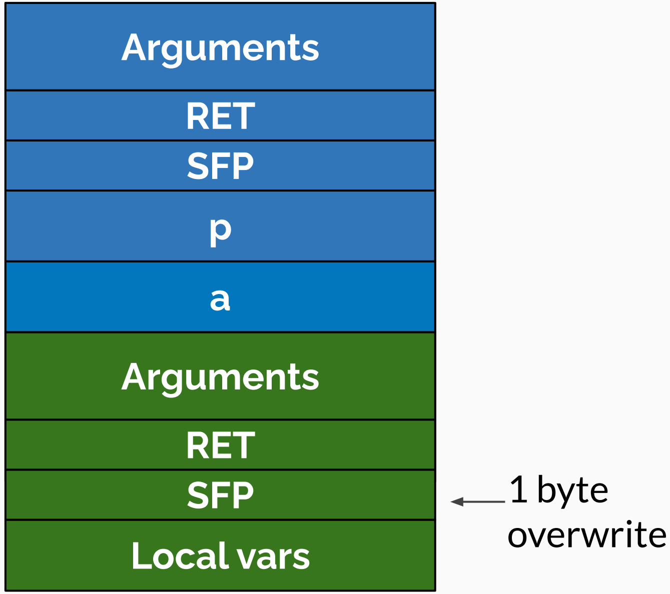
```c
void foo(char *argv[])
{
    int *p;
    int a = 0;
    p = &a;

    bar(argv[1]);
```

```c
    *p = a;

    _exit(0);
    /* not reached */
}
```

Blue: Foo’s stack frame
Green: bar’s stack frame



Program expects the stack to look like the layout of `foo` when returning from `bar`.

Homework 2 Pointers

- RSA functionality (more next section)
- Block modes: CTR, ECB
- Diffie-Hellman (lecture, soon)
- Certificate Authorities (lecture, soon)
- Meet-in-the-middle vs 2DES (lecture 10)
  - Python quickstart guide: https://learnxinyminutes.com/docs/python/
  - Python DES package: https://pypi.org/project/des/

Modular Arithmetic

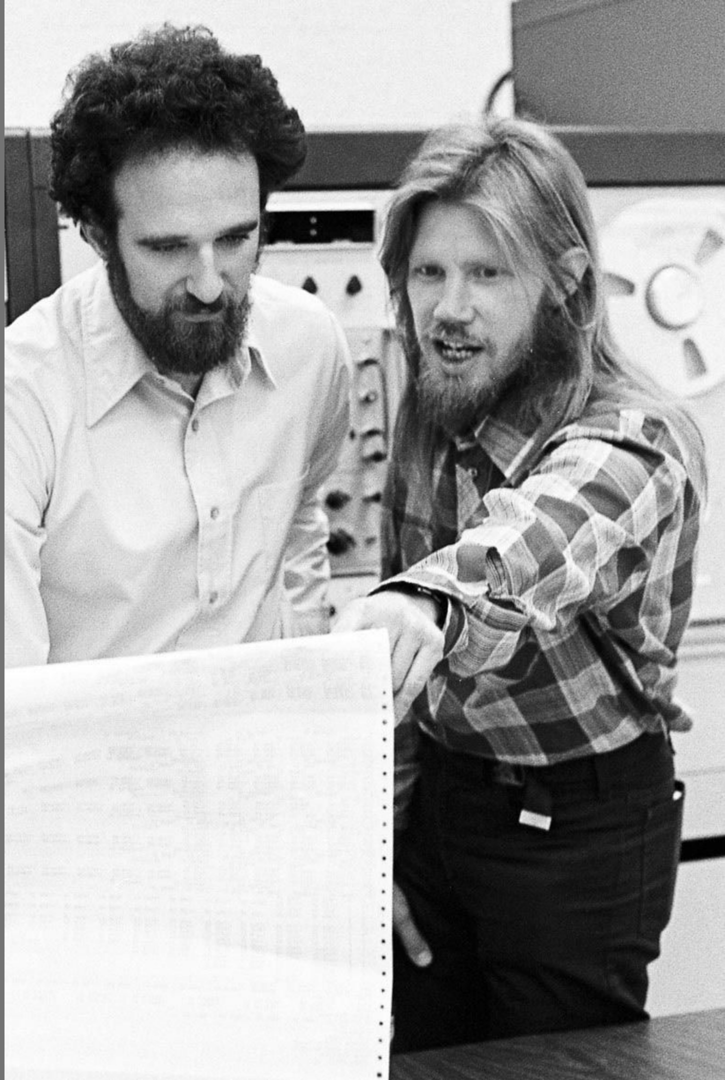
- The modulo:

  a mod b

  =

  the remainder of *a*÷*b*

- Many parts of cryptography depend on properties of modular arithmetic
- We'll talk more about it in lecture soon™ - public key cryptography, Diffie-Hellman Protocol (1976)



Modular Exponentiation

How would we compute something like this?

Let p = 11. Let g = 7.
Compute g^{400} mod p

$7^{400} \approx 1.09 \times 10^{338}$...

$$(a*b) \bmod p = (a \bmod p * b \bmod p) \bmod p$$

Q1

Let $p = 11$. Let $g = 10$.
Compute $g^1 \bmod p$, $g^2 \bmod p$, $g^3 \bmod p$, ..., $g^{100} \bmod p$.

$$(a*b) \bmod \mathbf{p}$$
$$=$$
$$(a \bmod \mathbf{p} * b \bmod \mathbf{p}) \bmod \mathbf{p}$$

Q1 Solution

Let p = 11. Let g = 10.
Compute g^1 mod p, g^2 mod p, g^3 mod p, …, g^{100} mod p.

10^1 mod 11 = 10   10^2 mod 11 = 1
10^3 mod 11 = (10^1 mod 11 * 10^2 mod 11) mod 11 = (10 * 1) mod 11 = 10
10^4 mod 11 = (10^2 mod 11 * 10^2 mod 11) mod 11= (1 * 1) mod 11 = 1
10^5 mod 11 = (10^1 mod 11 * 10^4 mod 11) mod 11 = (10 * 1) mod 11 = 10

…. Etc.

Creates cyclic group {10, 1}.

(a*b) mod **p**
=
(a mod **p** * b mod **p**) mod **p**

Q2

Let $p = 11$. Let $g = 7$.
Compute $g^1 \bmod p$, $g^2 \bmod p$, $g^3 \bmod p$, …, $g^{100} \bmod p$.

$$(a*b) \bmod p$$
$$=$$
$$(a \bmod p * b \bmod p) \bmod p$$

Q2 Solution

Let p = 11. Let g = 7.
Compute g^1 mod p, g^2 mod p, g^3 mod p, ..., g^{100} mod p.

7^1 mod 11 = 7	7^2 mod 11 = 5	7^3 mod 11 = 2	7^4 mod 11 = 3
7^5 mod 11 = 10	7^6 mod 11 = 4	7^7 mod 11 = 6	7^8 mod 11 = 9
7^9 mod 11 = 8	7^10 mod 11 = 1		
7 ^11 mod 11 = 7	7^12 mod 11 = 5	…. Etc.	

Creates cyclic group {7,5,2,3,10,4,6,9,8,1}.
This is generating all positive integers < p.

(a*b) mod **p**
=
(a mod **p** * b mod **p**) mod **p**

Q3

Let p = 11. Let g = 7.
Compute g^{400} mod p, without using a calculator.

(a*b) mod **p**

=

(a mod **p** * b mod **p**) mod **p**

Q3 Solution

Note that 400 = 256 + 128 + 16.

7^2 mod 11 = 5

7^4 mod 11 = (7^2 mod 11 * 7^2 mod 11) mod 11 = 5 * 5 mod 11 = 3

7^8 mod 11 = (7^4 mod 11 * 7^4 mod 11) mod 11 = 3 * 3 mod 11 = 9

7^16 mod 11 = (7^8 mod 11 * 7^8 mod 11) mod 11 = 9 * 9 mod 11 = 4

… … …

7^128 mod 11 = (7^64 mod 11 * 7^64 mod 11) mod 11 = 3 * 3 mod 11 = 9

7^256 mod 11 = (7^128 mod 11 * 7^128 mod 11) mod 11 = 9 * 9 mod 11 = 4

Thus, 7^400 mod 11 = (7^256 mod 11 * 7^128 mod 11 * 7^16 mod 11) mod 11

= (4 * 9 * 4) mod 11

= 1 mod 11

= 1

Modular Exponentiation

$$a = g^x \bmod p$$

Given a, g, and p, what is x?

Calculate using a ***discrete logarithm*** - computationally very hard

- Why is this hard? There's not much we can learn from cyclical groups - very little is understood about the sequence of values
- You can base cryptographic schemes around the hardness of calculating the discrete logarithm, especially if you pick large values

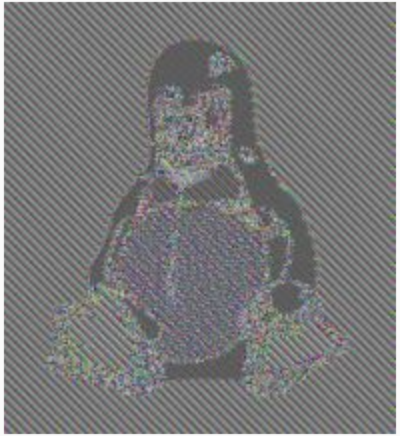
Thinking about encryption

Which symmetric encryption mode would you use for the following situations? Why?

- You are going to send a small one-time command to fire to your nukes.
- You are living in the 1970s and want to send a long letter to your lover on ARPANET.
- Everything else (given the tools we've learned)

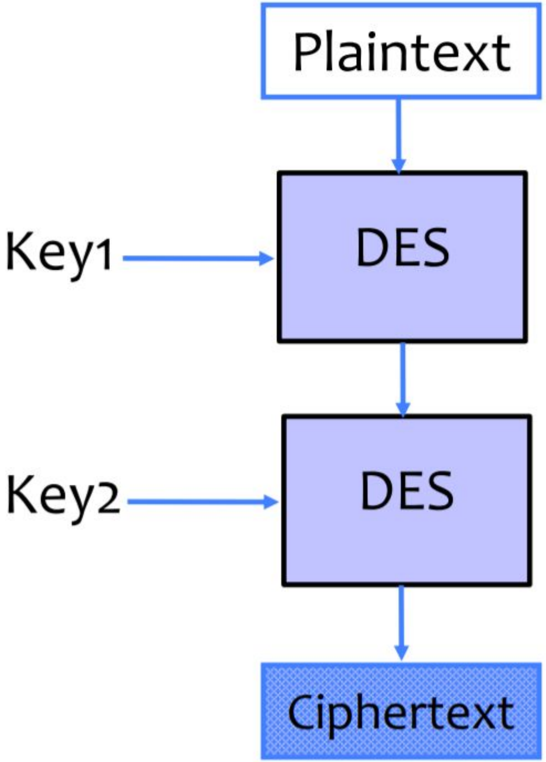
Thinking about encryption

What is a flaw with ECB encryption?

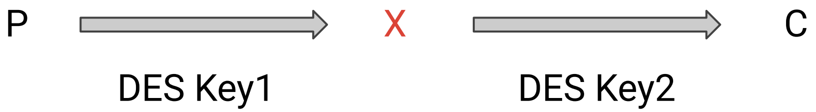


2DES

- Key1 and key2 are 56-bit keys
- Adversary knows the plaintext and the ciphertext
- Strategy 1: brute force attack - 2^{112} possibilities
- Strategy 2: meet-in-the-middle attack - precompute 2 tables for Encrypt (P, Key1) and Decrypt (C, Key2) and find the matching output, $2^{56} * 2 = 2^{57}$ possibilities



Meet-in-the-middle attack



K1	Encrypt(P, K1)
1	Y_1
2	Y_2
...	...
2^{56}	$Y_{2^{56}}$

Decrypt(C, K2)	K2
Z_1	1
Z_2	2
...	...
$Z_{2^{56}}$	2^{56}

If Y□ = Z□, We have found X. K1 = K□ and K2 = K□

Tips on HW2 Q9

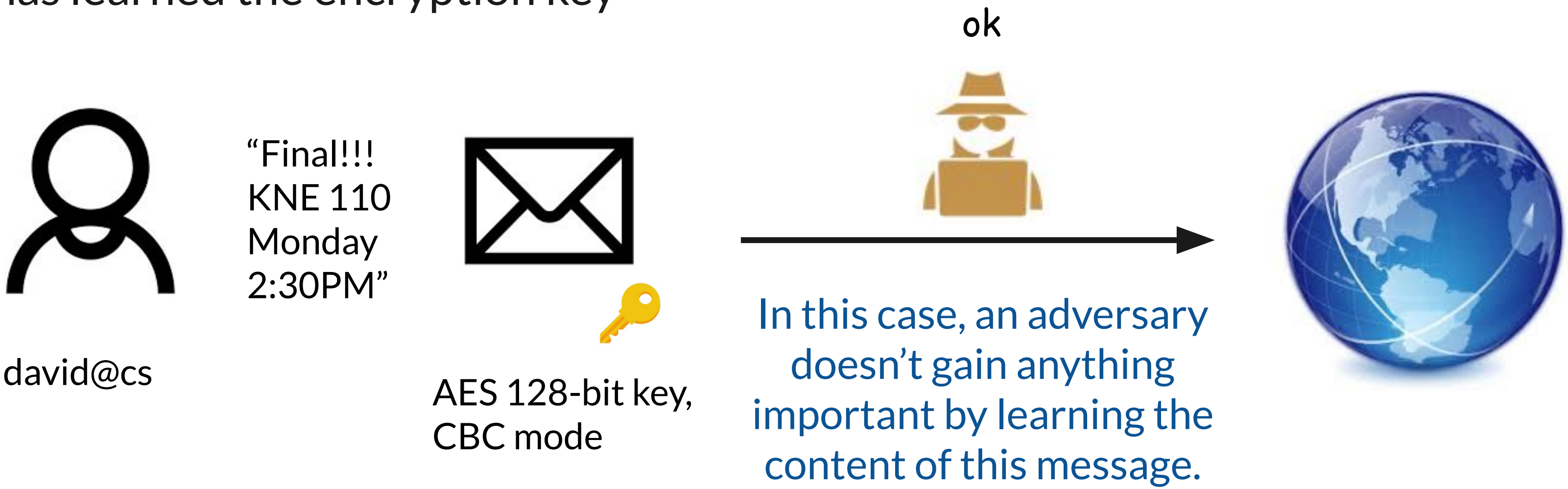
- Shorter key length 2^{14}
- You are given a plaintext/ciphertext pair for finding the key, and another ciphertext to decrypt and obtain the message
- Use des package with the function provided to you

```
from des import DesKey
def expandkey(val):
    if(val >= (2**14)):
        print("Key too large! Must fit in 14 bits")
        exit()
    k = val | (val << 14) | (val << 28) | (val << 42)
    return DesKey(bytearray.fromhex("{v:016X}".format(v=k)))
```

- Other functions that might be helpful from des: encrypt(plaintext), decrypt(ciphertext), bytearray.fromhex()

Is encryption (confidentiality) enough?

Scenario: Yoshi wants to send out an email about exam times - and a hacker has learned the encryption key

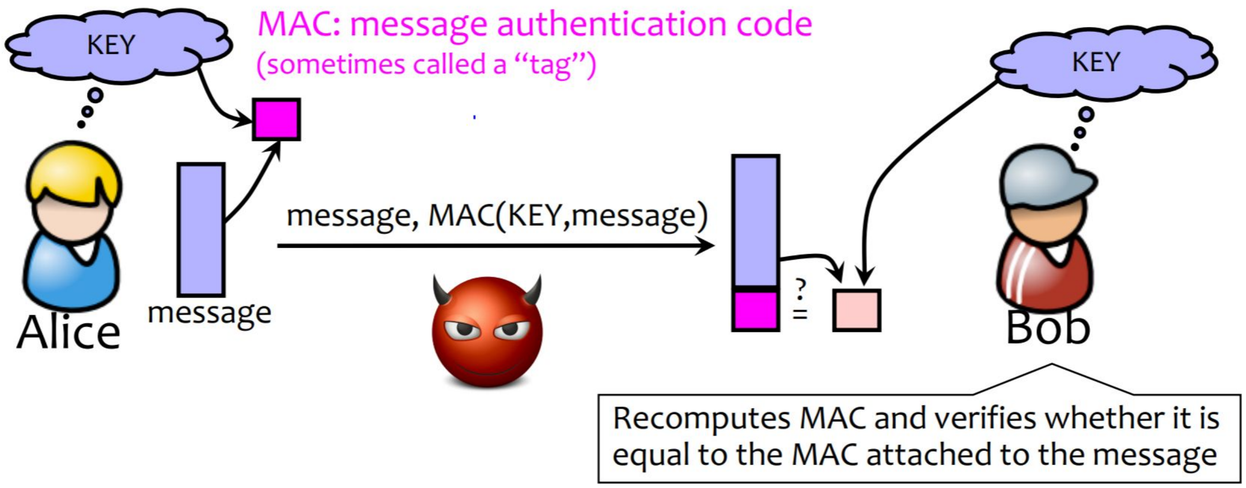


Is encryption (confidentiality) enough?

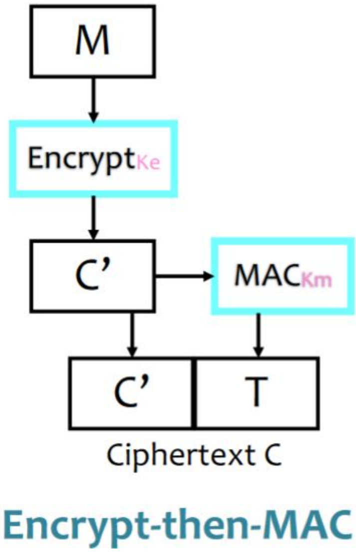
But, the attacker could tamper with the message during transmission, and the recipient would not know - so we need to ensure **integrity**

MAC (Message Authentication Code)

Provides integrity and authentication: only someone who knows the KEY can compute correct MAC for a given message.



When do we MAC?



The good:

Encrypt-then-MAC

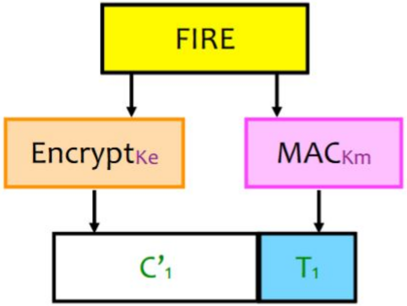
MAC-then-encrypt

Not as good as Encrypt-then-MAC

The bad (& ugly):

Encrypt-and-MAC

MAC is deterministic! Same plaintext → same MAC

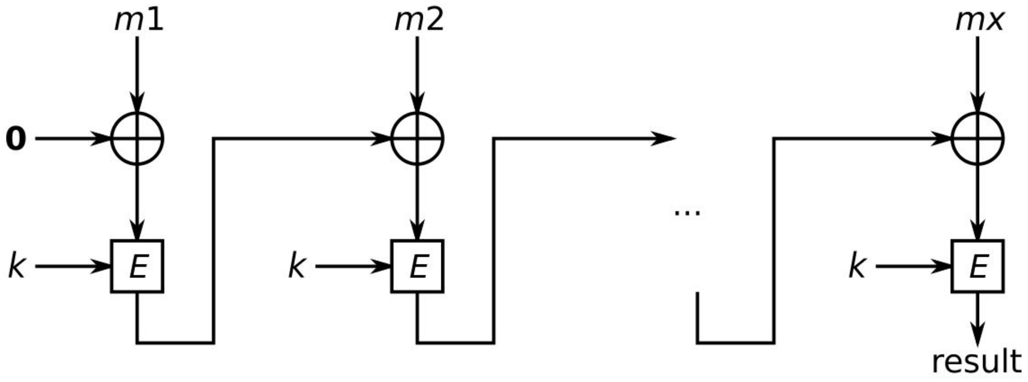


How do we create a MAC?

CBC-MAC: Encrypt the message in CBC mode, use the last block as the MAC

Initialization vector is 0

k = secret key



Last block of ciphertext used as MAC

*CBC-MAC is not the only MAC algorithm - today most use HMAC; we'll show why next

Is CBC-MAC vulnerable?

- How could we find out?
    - Cryptanalysis: using mathematical analysis to rigorously reason about a cryptographic system

- Let's use cryptanalysis to find a collision
    - two different inputs leading to the same MAC tag
    - (violating collision resistance)

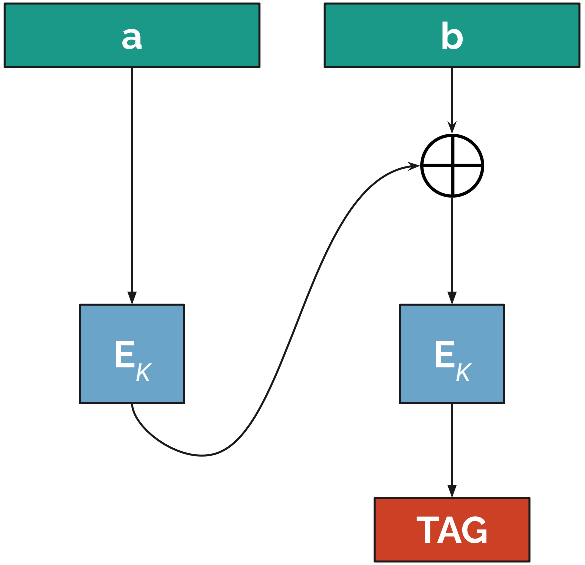
Exercise: CBC-MAC collision vulnerability

Suppose a and b are both one block long, and suppose the sender MACs a, b, and $a \,||\, b$ with CBC-MAC.

An attacker who intercepts the MAC tags for these messages can now forge the MAC for the message

$$b \,||\, (M_K(b) \oplus M_K(a) \oplus b)$$

which the sender never sent. The forged tag for this message is equal to $M_K(a \,||\, b)$, the tag for $a \,||\, b$. Justify mathematically why this is true.



a || b: a and b concatenated
M_K(a): MAC for message a
E_K(a): ciphertext for message a

(Ferguson, Schneier, & Kohno. *Cryptography Engineering: Design Principles and Practical Applications*. Wiley Publishing 2010. Exercise 6.3 p. 97)

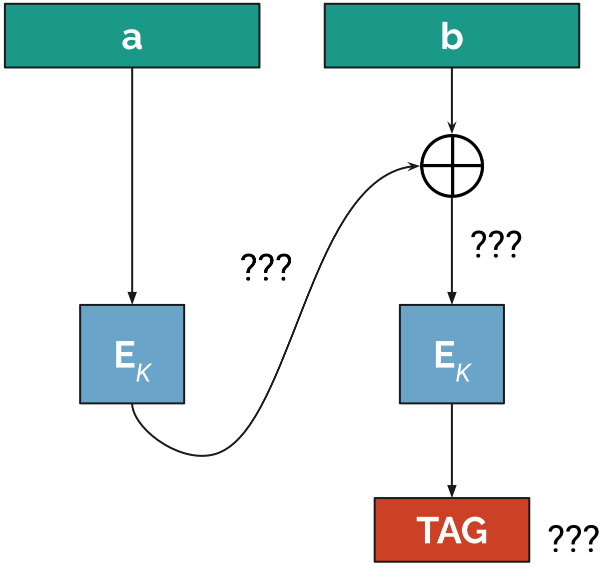
Exercise: CBC-MAC collision vulnerability

Prove:

$$\mathbf{M_K}(b \,||\, (M_K(b) \oplus M_K(a) \oplus b)) = \mathbf{M_K}(a \,||\, b)$$

Step 1: Figure out what M_K(a), M_K(b), and M_K(a || b) in terms of the encryption key.

Annotate sketch with the sender's messages and MACs.



(Ferguson, Schneier, & Kohno. *Cryptography Engineering: Design Principles and Practical Applications*. Wiley Publishing 2010. Exercise 6.3 p. 97)

Exercise: CBC-MAC collision vulnerability

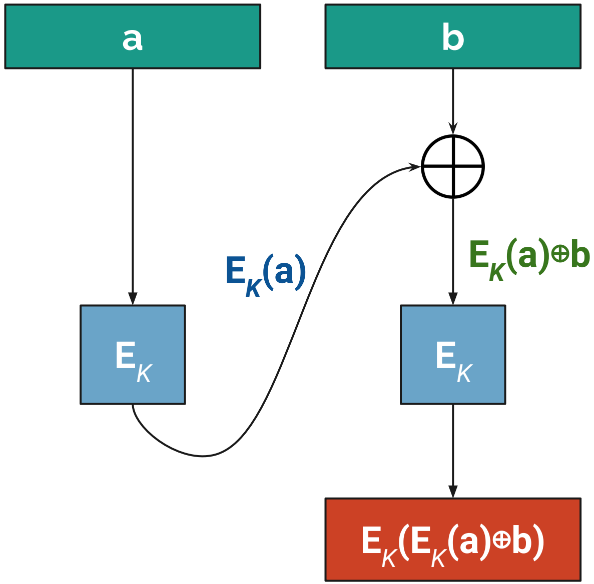
Prove:

$$\boldsymbol{M_K}(b \,||\, (M_K(b) \oplus M_K(a) \oplus b)) = \boldsymbol{M_K}(a \,||\, b)$$

$M_K(a) = E_K(a)$

$M_K(b) = E_K(b)$ (not shown)

$M_K(a \,||\, b) = E_K(E_K(a) \oplus b)$



(Ferguson, Schneier, & Kohno. *Cryptography Engineering: Design Principles and Practical Applications*. Wiley Publishing 2010. Exercise 6.3 p. 97)

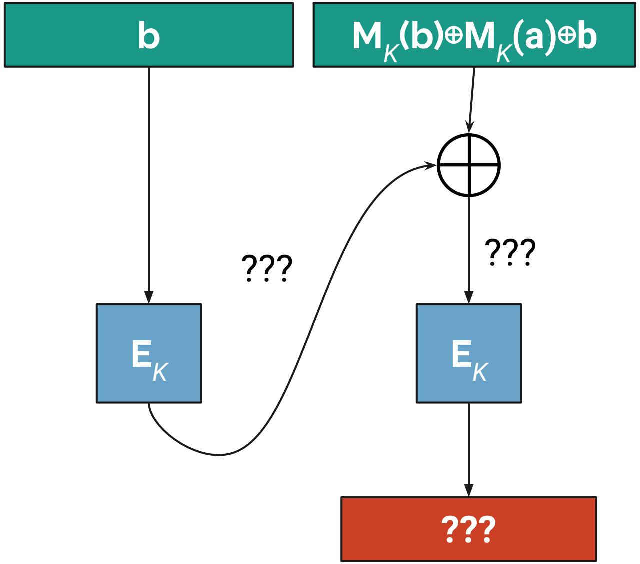
Exercise: CBC-MAC collision vulnerability

Prove:

$$\mathbf{M_K}(b \,||\, (M_K(b) \oplus M_K(a) \oplus b)) = \mathbf{M_K}(a \,||\, b)$$

Step 2: Figure out $\mathbf{M_K}(b \,||\, (M_K(b) \oplus M_K(a) \oplus b))$.

For the MAC of the attacker's message $b \,||\, (M_K(b) \oplus M_K(a) \oplus b)$, what are the values of the ???'s?



(Ferguson, Schneier, & Kohno. *Cryptography Engineering: Design Principles and Practical Applications*. Wiley Publishing 2010. Exercise 6.3 p. 97)

Exercise: CBC-MAC collision vulnerability

Prove:

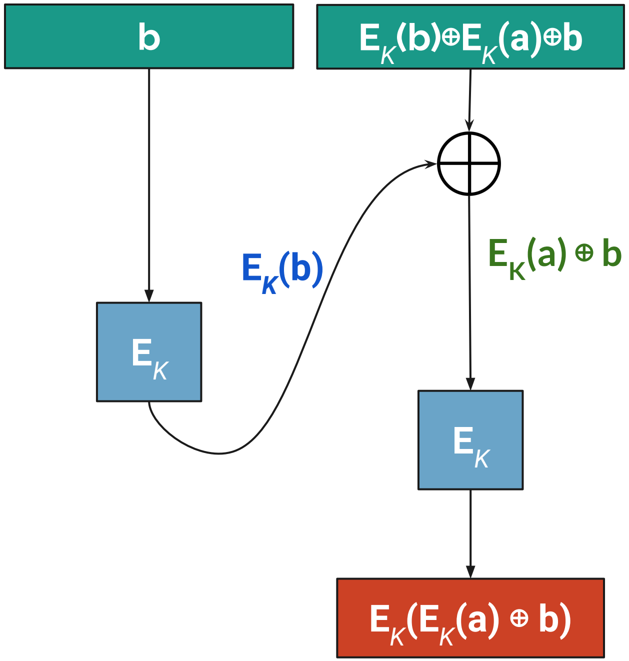
$$M_K(b \,||\, (M_K(b) \oplus M_K(a) \oplus b)) = M_K(a \,||\, b)$$

$$M_K(b \,||\, (M_K(b) \oplus M_K(a) \oplus b))$$
$$= M_K(b \,||\, (E_K(b) \oplus E_K(a) \oplus b))$$
$$= E_K(E_K(b) \oplus E_K(b) \oplus E_K(a) \oplus b)$$

These terms cancel out

$$= E_K(E_K(a) \oplus b)$$

This is the same as $M_K(a \,||\, b)$!



So what?

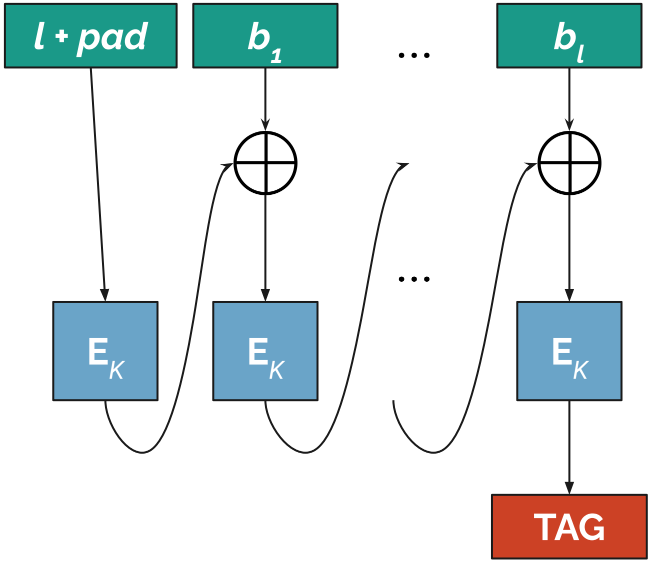
- We can prove, just using the specification of CBC-MAC, that the messages $b \,||\, (M(b) \oplus M(a) \oplus b)$ and $a \,||\, b$ share the same tag. This approach is a common method used in cryptanalysis.
- We broke the *theoretical* guarantee that no two different messages will never share a tag.
- If you were to use CBC-MAC in a protocol, it provides information about specific weaknesses and how not to use it.

Safer CBC-MAC for variable length messages

For a message *m* of length *l*:

1. Construct *s* by prepending the length of *m* to the message: *s = concat(l, m)*
2. Pad *s* until the length is a multiple of the block size
3. Apply CBC-MAC to the padded string *s*.
4. Output the last ciphertext block, or a part of it. Don't output intermediates.

- *Warning*: Appending to end is just as broken as what we showed!
- Or encrypt output with another block cipher under a different key (CMAC). Or use HMAC, UMAC, GMAC.
- Follow latest guidance very carefully!



Good luck with the rest of lab 1!