CSE 484 : Computer Security and Privacy

Asymmetric Cryptography

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Administrivia

- Lab 1 due on Wednesday
- HW2 next week

Remember our troubles with randomness?

CSPRNGs in practice

- Gather some good entropy (256 bits?)
- Use a block cipher/HMAC/Hash to ‘stretch’ this entropy
- Regularly mix in more entropy!

Check out NIST.SP.800-90

CSPRNG – CTR_DRBG

Why does this work for CSPRNGs?

- To ‘break’ the CSPRNG (that is, predict the next output)
  - Must know state of CSPRNG (key, inputs)
  - Requires breaking the security of the primitive!

- Your CSPRNG is just as secure as the scheme you use the output for!
  - Never ‘loses entropy’, same guarantee as block cipher

- Why mix in new entropy?
  - Can’t hurt, prevents a single bug from breaking the future

CSPRNGs gone bad

- DUAL_EC_DRBG – Dual Elliptic Curve Deterministic Random Bit Generator
    - CSPRNG based on elliptic curve math
    - NSA designed
- DUAL_EC_DRBG has a *backdoor*
    - Special mathematical construction that allows recovery of state!
- Remember DES's s-boxes?
    - This is the opposite

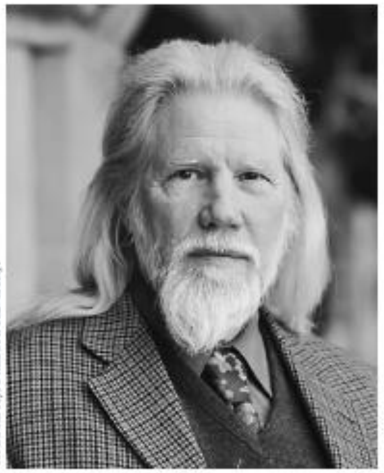
Session Key Establishment

Modular Arithmetic

- Given g and prime p, compute: g^1 mod p, g^2 mod p, … g^{100} mod p
  - For p=11, g=10
    - 10^1 mod 11 = 10, 10^2 mod 11 = 1, 10^3 mod 11 = 10, …
    - Produces cyclic group {10, 1} (order=2)
  - For p=11, g=7
    - 7^1 mod 11 = 7, 7^2 mod 11 = 5, 7^3 mod 11 = 2, …
    - Produces cyclic group {7,5,2,3,10,4,6,9,8,1} (order = 10)
    - g=7 is a "generator" of Z_{11}*

Diffie-Hellman Protocol (1976)

Diffie and Hellman Receive 2015 Turing Award

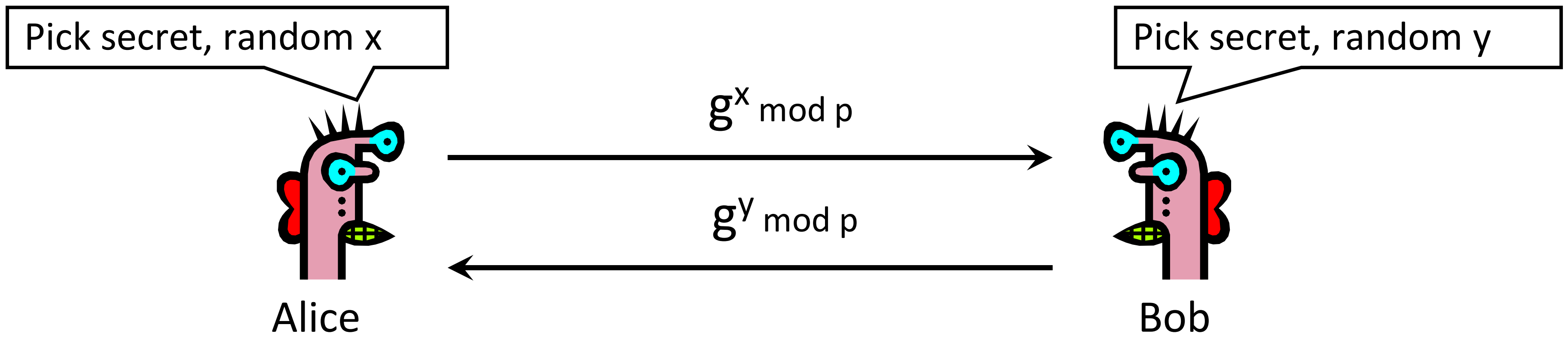


Whitfield Diffie

Martin E. Hellman

Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info: p and g
  - p is a large prime, g is a **generator** of Z_p^*
    - $Z_p^*=\{1, 2 \ldots p-1\}$; a Z_p^* i such that $a=g^i \bmod p$
    - Modular arithmetic: numbers "wrap around" after they reach p



Compute $k=(g^y)^x=g^{xy} \bmod p$

Compute $k=(g^x)^y=g^{xy} \bmod p$

Example Diffie Hellman Computation

Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:

  given $g^x \; mod \; p$, it's hard to extract x
  - There is no known <u>efficient</u> algorithm for doing this
  - This is <u>not</u> enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:

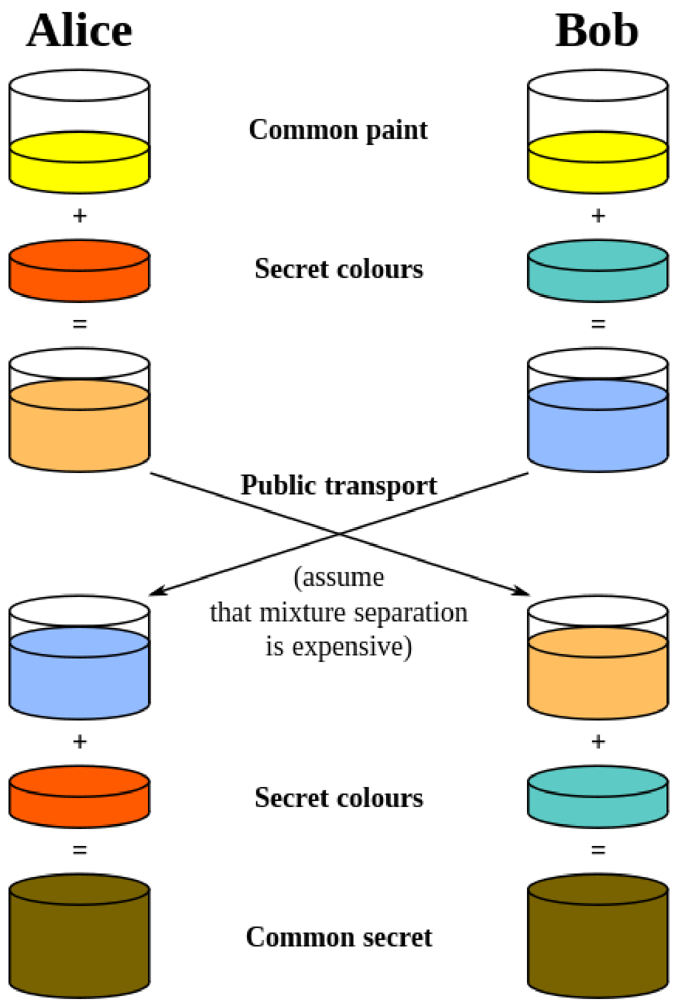
  given g^x and g^y, it's hard to compute $g^{xy} \; mod \; p$
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:

  given g^x and g^y, it's hard to tell the difference between $g^{xy} \; mod \; p$ and $g^r \; mod \; p$ where r is random

More on Diffie-Hellman Key Exchange

- Important Note:
  - We have discussed discrete logs modulo integers
  - Significant advantages in using elliptic curve groups
    - Groups with some similar mathematical properties (i.e., are "groups") but have better security and performance (size) properties

Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:
$g^x \bmod p$
$g^y \bmod p$

Common secret: $g^{xy} \bmod p$

[from Wikipedia]

Diffie-Hellman Caveats

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose p=2q+1, where q is also a large prime
    - Choose g that generates a subgroup of order q in Z_p*
    - DDH is hard in this group
  - Eavesdropper can't tell the difference between the established key and a random value
  - In practice, often hash g^{xy} *mod p*, and use the hash as the key
  - Can use the new key for symmetric cryptography
- Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
  - Person in the middle attack (also called "man in the middle attack")

Example from Earlier

- Given g and prime p, compute: $g^1 \bmod p$, $g^2 \bmod p$, … $g^{100} \bmod p$
  - For p=11, g=10
    - $10^1 \bmod 11 = 10$, $10^2 \bmod 11 = 1$, $10^3 \bmod 11 = 10$, …
    - Produces cyclic group {10, 1} (order=2)
  - For p=11, g=7
    - $7^1 \bmod 11 = 7$, $7^2 \bmod 11 = 5$, $7^3 \bmod 11 = 2$, …
    - Produces cyclic group {7,5,2,3,10,4,6,9,8,1} (order = 10)
    - g=7 is a "generator" of Z_{11}^*
  - For p=11, g=3
    - $3^1 \bmod 11 = 3$, $3^2 \bmod 11 = 9$, $3^3 \bmod 11 = 5$, …
    - Produces cyclic group {3,9,5,4,1} (order = 5) (5 is a prime)
    - g=3 generates a group of prime order

Stepping Back: Asymmetric Crypto

- We’ve just seen session key establishment
  - Can then use shared key for symmetric crypto
- Next: public key encryption
  - For confidentiality
- Then: digital signatures
  - For authenticity

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function $\varphi(n)$ $(n \geq 1)$ is the number of integers in the [1,n] interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: $\varphi(p) = p-1$
  - Note that $\varphi(ab) = \varphi(a)\, \varphi(b)$ if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 2048 bits each (need primality testing, too)
  - Compute **n**=pq and **φ(n)**=(p-1)(q-1)
  - Choose small **e**, relatively prime to φ(n)
    - Typically, **e=3** or **$e=2^{16}+1=65537$**
  - Compute unique **d** such that ed ≡ 1 mod φ(n)
    - Modular inverse: $d \equiv e^{-1}$ mod φ(n)
  - Public key = (e,n); private key = (d,n)
- Encryption of m: c = m^e mod n
- Decryption of c: c^d mod n = $(m^e)^d$ mod n = m

$\leftarrow$ How to compute?

Why is RSA Secure?

- RSA problem: given c, n=pq, and e such that gcd(e, φ(n))=1, find m such that $m^e=c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking eth root of c modulo n
  - There is no known efficient algorithm for doing this *without* knowing p and q
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n=p_1^{e_1}p_2^{e_2}\ldots p_k^{e_k}$

- If factoring is easy, then RSA problem is easy (knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n -- but if it is, we don't know how

RSA Encryption Caveats

- Encrypted message needs to be interpreted as an integer less than n
- Don’t use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow
- Plain RSA also does not provide integrity
  - Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt

$M \oplus G(r) \,||\, r \oplus H(M \oplus G(r))$

- r is random and fresh, G and H are hash functions

RSA OAEP

$M \oplus G(r) \,||\, r \oplus H(M \oplus G(r))$

Review: RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 2048 bits each (need primality testing, too)
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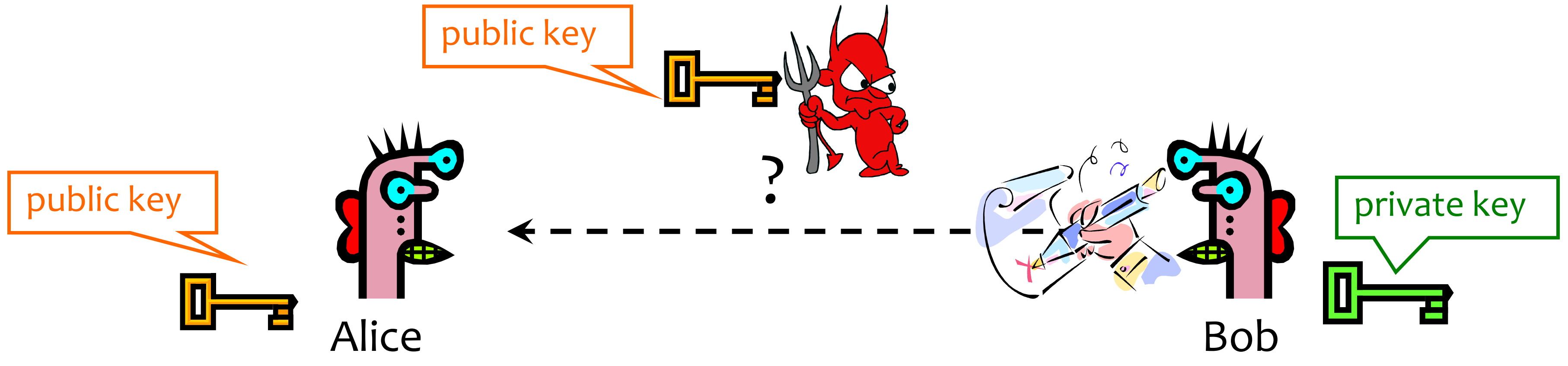
  - Compute unique **d** such that ed ≡ 1 mod φ(n)
    - Modular inverse: $d \equiv e^{-1} \bmod \varphi(n)$ $\longleftarrow$ How to compute?
  - Public key = (e,n); private key = (d,n)
- Encryption of m: $c = m^e \bmod n$
- Decryption of c: $c^d \bmod n = (m^e)^d \bmod n = m$

Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: s = m^d mod n
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m:

  verify that s^e mod n = $(m^d)^e$ mod n = m
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Without padding and hashing: Consider multiplying two signatures together
  - Standard padding/hashing schemes exist for RSA signatures