

CSE 484 / CSE M 584: Computer Security and Privacy

Cryptography

[Asymmetric Cryptography]

Autumn 2020

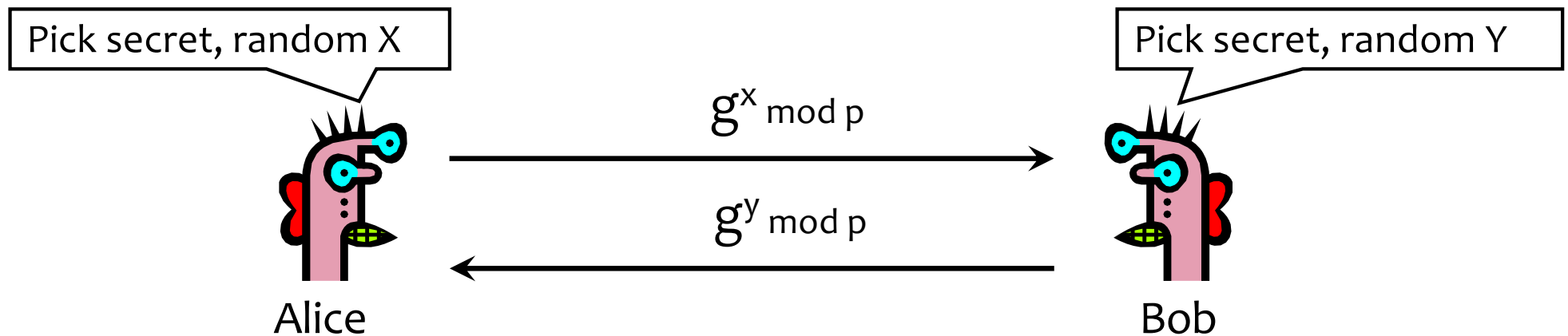
Franziska (Franzi) Roesner

franzi@cs.washington.edu

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Diffie-Hellman Key Exchange

- Alice and Bob never met and share no secrets
- Public info: p and g
 - p is a large prime, g is a **generator** of Z_p^*
 - $Z_p^* = \{1, 2 \dots p-1\}$; $\forall a \in Z_p^* \exists i$ such that $a = g^i \pmod p$
 - Modular arithmetic: numbers “wrap around” after they reach p



Compute $k = (g^y)^x = g^{xy} \pmod p$

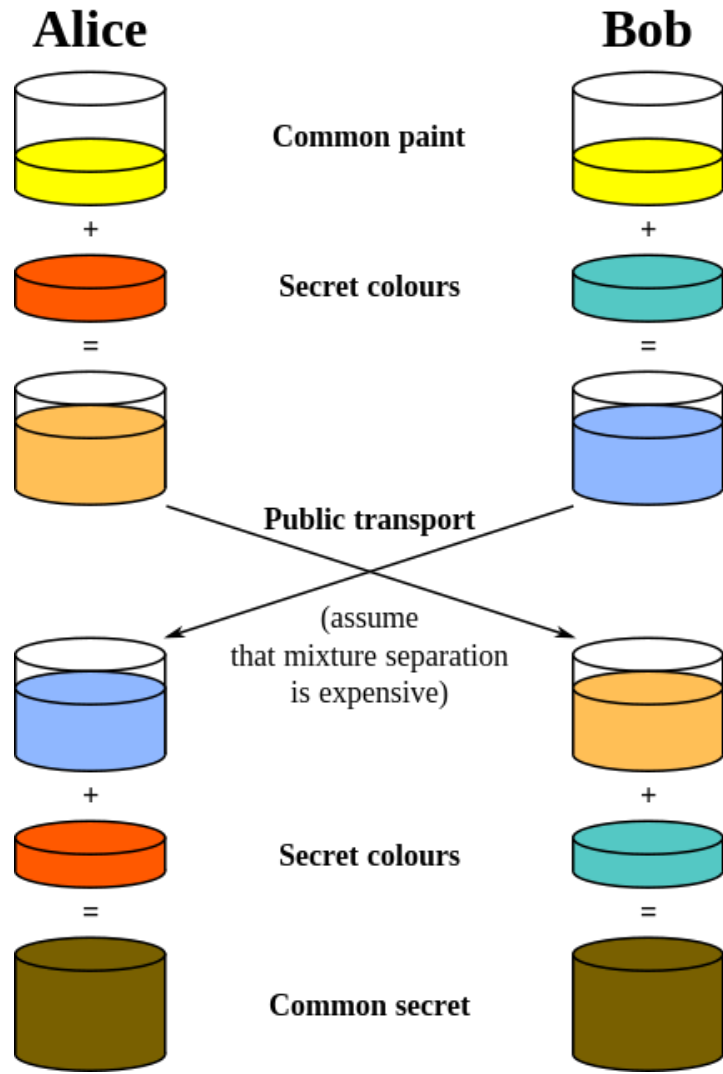
Compute $k = (g^x)^y = g^{xy} \pmod p$

Example Diffie Hellman Computation

Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:
given $g^x \bmod p$, it's hard to extract x
 - There is no known efficient algorithm for doing this
 - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:
given g^x and g^y , it's hard to compute $g^{xy} \bmod p$
 - ... unless you know x or y , in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:
given g^x and g^y , it's hard to tell the difference between $g^{xy} \bmod p$ and $g^r \bmod p$ where r is random

Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:

$g^x \bmod p$

$g^y \bmod p$

Common secret: $g^{xy} \bmod p$

[from Wikipedia]

Diffie-Hellman Caveats

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
 - Common recommendation:
 - Choose $p=2q+1$, where q is also a large prime
 - Choose g that generates a subgroup of order q in Z_p^*
 - Eavesdropper can't tell the difference between the established key and a random value
 - In practice, often hash $g^{xy} \bmod p$, and use the hash as the key
 - Can use the new key for symmetric cryptography
- Diffie-Hellman protocol (by itself) does not provide authentication (against active attackers)
 - Person in the middle attack (also called “man in the middle attack”)

Person In The Middle Attack

More on Diffie-Hellman Key Exchange

- Important Note:
 - We have discussed discrete logs modulo integers
 - Significant advantages in using elliptic curve groups
 - Groups with some similar mathematical properties (i.e., are “groups”) but have better security and performance (size) properties

Stepping Back: Asymmetric Crypto

- We've just seen **session key establishment**
 - Can then use shared key for symmetric crypto
- Next: **public key encryption**
 - For confidentiality
- Then: **digital signatures**
 - For authenticity

Requirements for Public Key Encryption

- **Key generation:** computationally easy to generate a pair (public key PK , private key SK)
- **Encryption:** given plaintext M and public key PK , easy to compute ciphertext $C = E_{PK}(M)$
- **Decryption:** given ciphertext $C = E_{PK}(M)$ and private key SK , easy to compute plaintext M
 - Infeasible to learn anything about M from C without SK
 - Trapdoor function: $Decrypt(SK, Encrypt(PK, M)) = M$

Some Number Theory Facts

- Euler totient function $\varphi(n)$ ($n \geq 1$) is the number of integers in the $[1, n]$ interval that are relatively prime to n
 - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
 - Easy to compute for primes: $\varphi(p) = p-1$
 - Note that $\varphi(ab) = \varphi(a) \varphi(b)$ if a & b are relatively prime

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
 - Generate large primes p, q
 - Say, 1024 bits each (need primality testing, too)
 - Compute $n=pq$ and $\phi(n)=(p-1)(q-1)$
 - Choose small e , relatively prime to $\phi(n)$
 - Typically, $e=3$ or $e=2^{16}+1=65537$
 - Compute unique d such that $ed \equiv 1 \pmod{\phi(n)}$
 - Modular inverse: $d \equiv e^{-1} \pmod{\phi(n)}$ ← How to compute?
 - Public key = (e,n) ; private key = (d,n)
- Encryption of m : $c = m^e \pmod n$
- Decryption of c : $c^d \pmod n = (m^e)^d \pmod n = m$

Why is RSA Secure?

- **RSA problem:** given c , $n=pq$, and e such that $\gcd(e, \varphi(n))=1$, find m such that $m^e=c \pmod n$
 - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
 - There is no known efficient algorithm for doing this
- **Factoring problem:** given positive integer n , find primes p_1, \dots, p_k such that $n=p_1^{e_1}p_2^{e_2}\dots p_k^{e_k}$
- If factoring is easy, then RSA problem is easy (knowing factors means you can compute $d = \text{inverse of } e \pmod{(p-1)(q-1)}$)
 - It may be possible to break RSA without factoring n -- but if it is, we don't know how

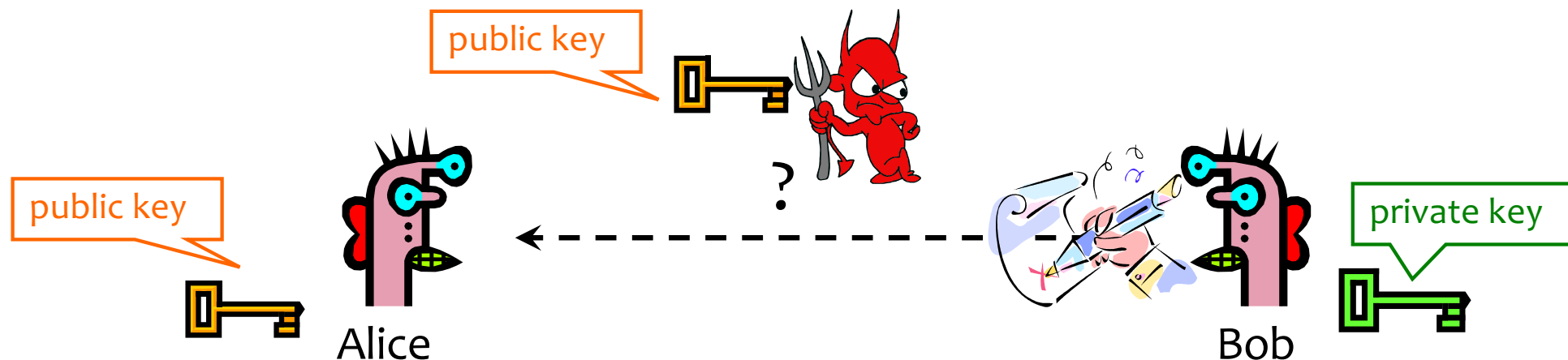
RSA Encryption Caveats

- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – **output is deterministic!** Need to pre-process input somehow
- Plain RSA also does not provide integrity
 - Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M , encrypt $M \oplus G(r); r \oplus H(M \oplus G(r))$

- r is random and fresh, G and H are hash functions

Digital Signatures: Basic Idea



Given: Everybody knows Bob's **public key**
Only Bob knows the corresponding **private key**

Goal: Bob sends a “digitally signed” message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is (n,e) , private key is (n,d)
- To **sign** message m : $s = m^d \bmod n$
 - Signing & decryption are same **underlying** operation in RSA
 - It's infeasible to compute s on m if you don't know d
- To **verify** signature s on message m :
verify that $s^e \bmod n = (m^d)^e \bmod n = m$
 - Just like encryption (for RSA primitive)
 - Anyone who knows n and e (public key) can verify signatures produced with d (private key)
- **In practice, also need padding & hashing**
 - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

- Digital Signature Standard (DSS)
 - U.S. government standard (1991, most recent rev. 2013)
- Public key: $(p, q, g, y=g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
 - If could solve discrete logarithm problem, would extract x (private key) from $g^x \bmod p$ (public key)
- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Cryptography Summary

- Goal: Privacy
 - Symmetric keys:
 - One-time pad, Stream ciphers
 - Block ciphers (e.g., DES, AES) → modes: EBC, CBC, CTR
 - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
 - MACs, often using hash functions (e.g, SHA-256)
- Goal: Privacy and Integrity
 - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
 - Digital signatures (e.g., RSA, DSS)

Want More Crypto?

- Some suggestions:
 - CSE 490C (Rachel Lin):
<https://courses.cs.washington.edu/courses/cse490c/20au/>
 - Stanford Coursera (Dan Boneh):
<https://www.coursera.org/learn/crypto>