CSE 484 / CSE M 584: Computer Security and Privacy

Cryptography
[Finish Asymmetric Cryptography]

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Franziska (Franzi) Roesner
franzi@cs.washington.edu

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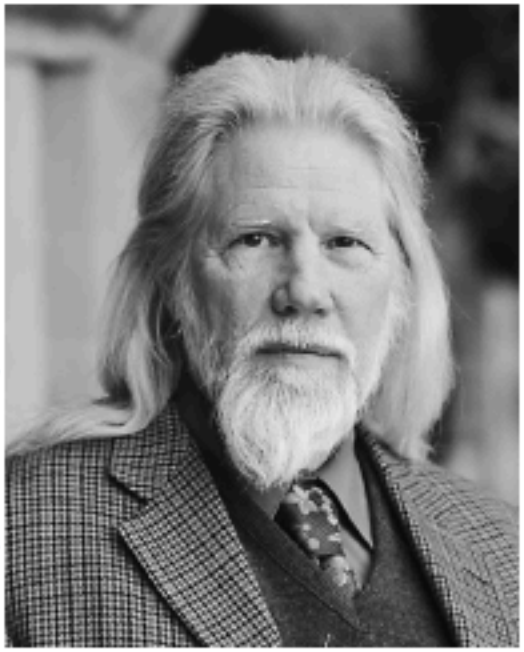
Admin

- Lab 1 done ☺
- Homework 2 (crypto) out
  - Get started now; shouldn't need the whole time
- Lab 2 (web security) out next week
  - Stay tuned for group signup instructions

More on Diffie-Hellman Key Exchange

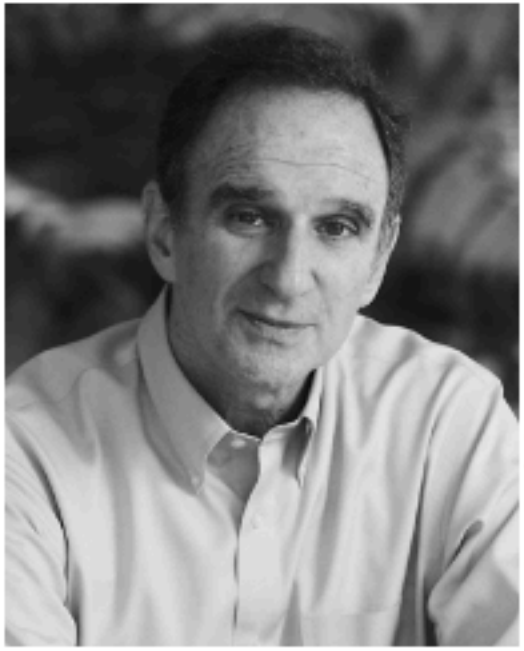
- Important Note:
  - We have discussed discrete logs modulo integers
  - Significant advantages in using elliptic curve groups
    - Groups with some similar mathematical properties (i.e., are "groups") but have better security and performance (size) properties

Diffie and Hellman Receive 2015 Turing Award



Rod Searcey/Stanford University.

Whitfield Diffie



Linda A. Cicero/Stanford News Service.

Martin E. Hellman

Public Key Encryption

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function $\varphi(n)$ ($n \geq 1$) is the number of integers in the [1,n] interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: $\varphi(p) = p-1$
  - Note that $\varphi(ab) = \varphi(a)\,\varphi(b)$

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 1024 bits each (need primality testing, too)
  - Compute **n**=pq and **φ(n)**=(p-1)(q-1)
  - Choose small e, relatively prime to φ(n)
    - Typically, **e=3** or **$e=2^{16}+1=65537$**
  - Compute unique d such that ed ≡ 1 mod φ(n)
    - Modular inverse: $d \equiv e^{-1} \bmod \varphi(n)$
  - Public key = (e,n); private key = (d,n)

How to compute?

- Encryption of m: $c = m^e \bmod n$
- Decryption of c: $c^d \bmod n = (m^e)^d \bmod n = m$

Why RSA Decryption Works (FYI)

$e \cdot d = 1 \bmod \varphi(n)$, thus $e \cdot d = 1 + k \cdot \varphi(n)$ for some k

Let m be any integer in Z_n^* (not all of Z_n)

$$c^d \bmod n = (m^e)^d \bmod n = m^{1+k\cdot\varphi(n)} \bmod n$$
$$= (m \bmod n) * (m^{k\cdot\varphi(n)} \bmod n)$$

Recall: Euler's theorem: if $a \in Z_n^*$, then $a^{\varphi(n)} = 1 \bmod n$

$$c^d \bmod n = (m \bmod n) * (1 \bmod n)$$
$$= m \bmod n$$

Proof omitted: True for all m in Z_n, not just m in Z_n^*

Why RSA Decryption Works (FYI)

- Decryption of c: $c^d \bmod n = (m^e \bmod n)^d \bmod n = (m^e)^d \bmod n = m$
- Recall **n**=pq and $\varphi(\mathbf{n})=(p-1)(q-1)$ and $ed \equiv 1 \bmod \varphi(n)$

- Chinese Remainder Theorem: To show $m^{ed} \bmod n \equiv m \bmod n$, sufficient to show:
  - $m^{ed} \bmod p \equiv m \bmod p$
  - $m^{ed} \bmod q \equiv m \bmod q$

- If $m \equiv 0 \bmod p \rightarrow m^{ed} \equiv 0 \bmod p$

- Else $m^{ed} = m^{ed-1}m = m^{k(q-1)(p-1)}m = m^{h(p-1)}m$ for some k, and h=k(q-1). Why? Recall how d was chosen and the definition of mod.
- Fermat Little Theorem: $m^{(p-1)h}m \equiv 1^h m \bmod p \equiv m \bmod p$

Why is RSA Secure?

- RSA problem: given c, n=pq, and e such that gcd(e, φ(n))=1, find m such that $m^e = c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n = p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$
- If factoring is easy, then RSA problem is easy (knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n -- but if it is, we don't know how

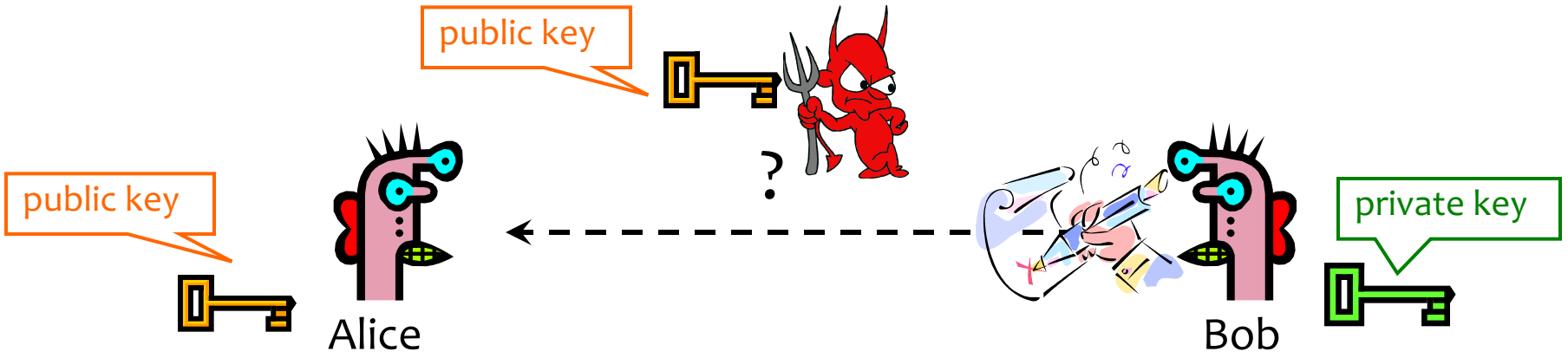
RSA Encryption Caveats

- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow
- Plain RSA also does not provide integrity
  - Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt $M \oplus G(r)$; $r \oplus H(M \oplus G(r))$

- r is random and fresh, G and H are hash functions

Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: $s = m^d \bmod n$
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m:

  verify that $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

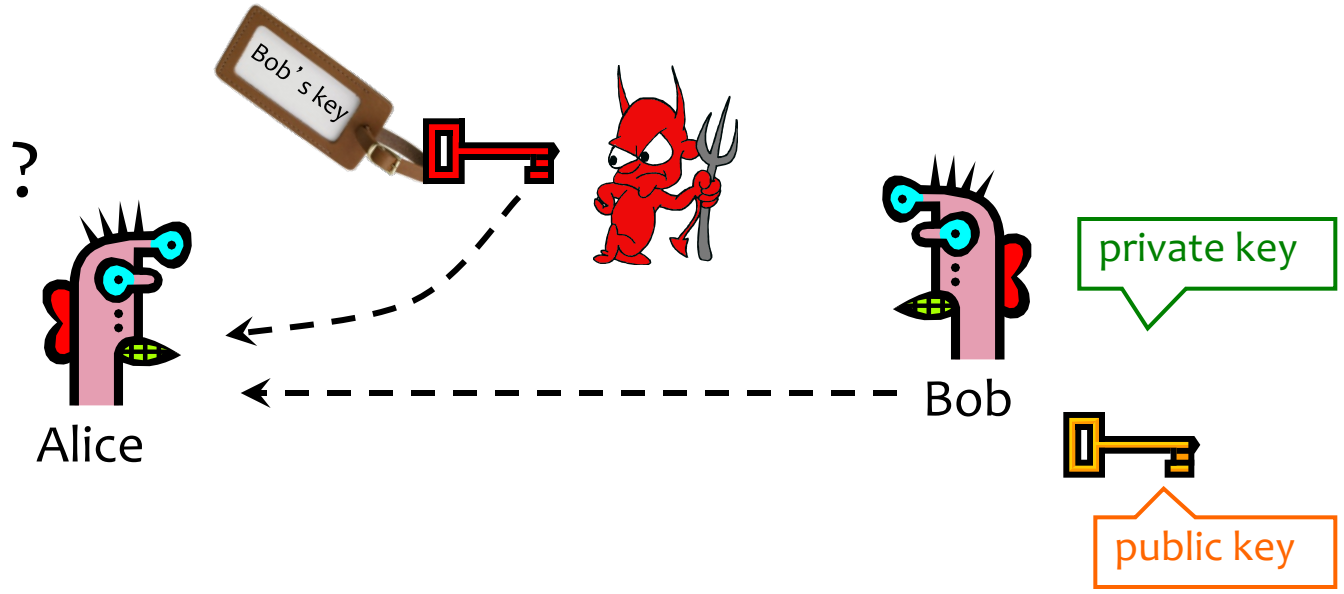
- Digital Signature Standard (DSS)
  - U.S. government standard (1991, most recent rev. 2013)
- Public key: $(p, q, g, y=g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from $g^x \bmod p$ (public key)

- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Cryptography Summary

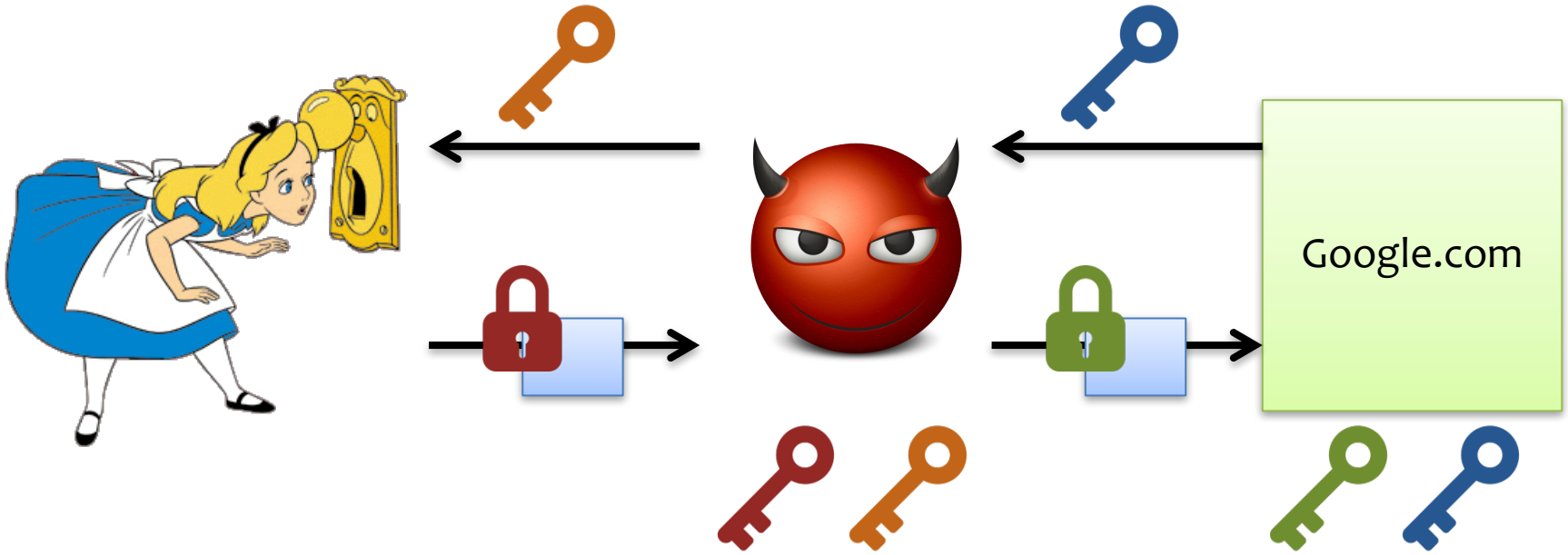
- Goal: Privacy
  - Symmetric keys:
    - One-time pad, Stream ciphers
    - Block ciphers (e.g., DES, AES) → modes: EBC, CBC, CTR
  - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
  - MACs, often using hash functions (e.g, SHA-256)
- Goal: Privacy and Integrity
  - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
  - Digital signatures (e.g., RSA, DSS)

Authenticity of Public Keys



Problem: How does Alice know that the public key she received is really Bob's public key?

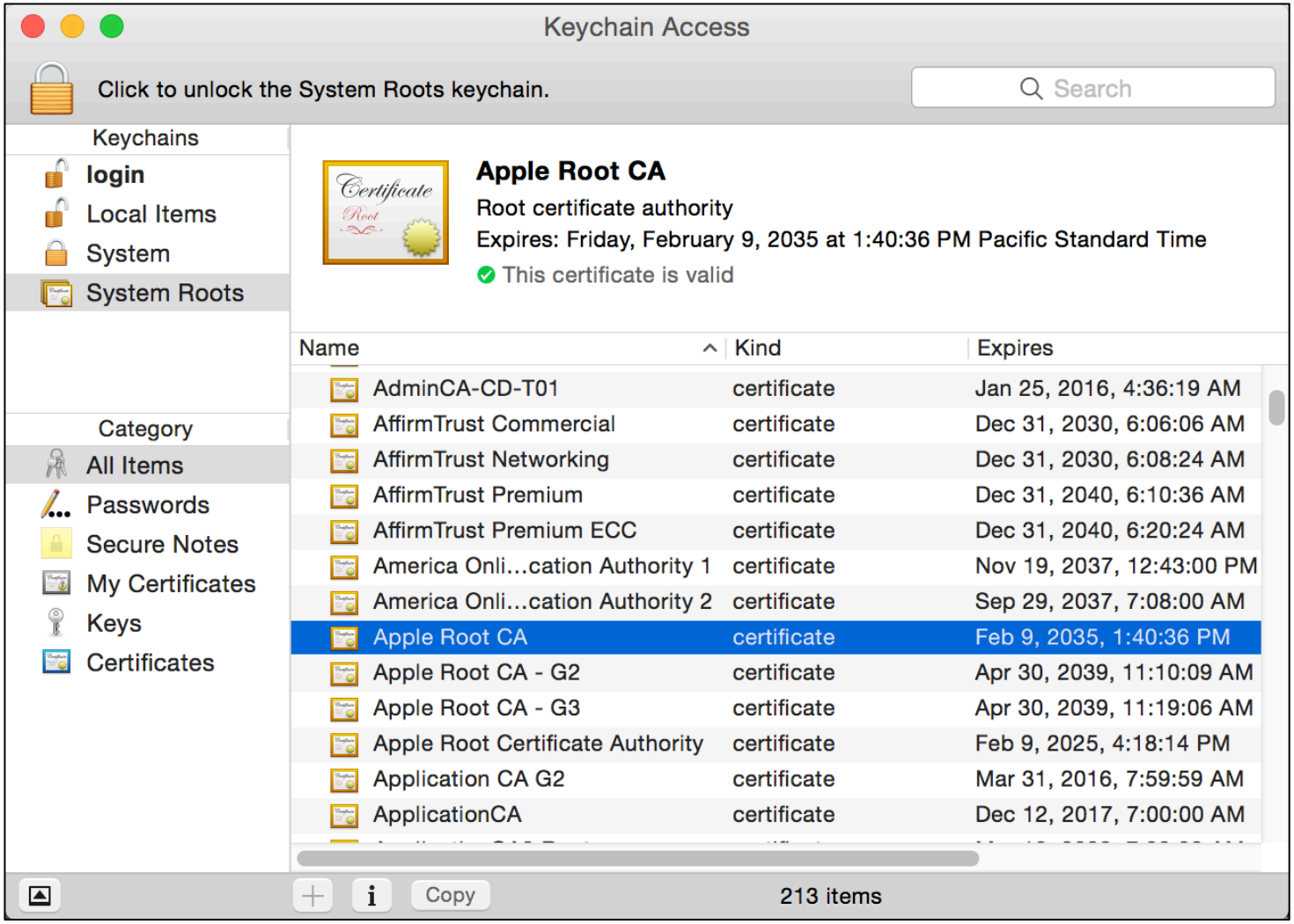
Threat: Man-In-The-Middle (MITM)



Distribution of Public Keys

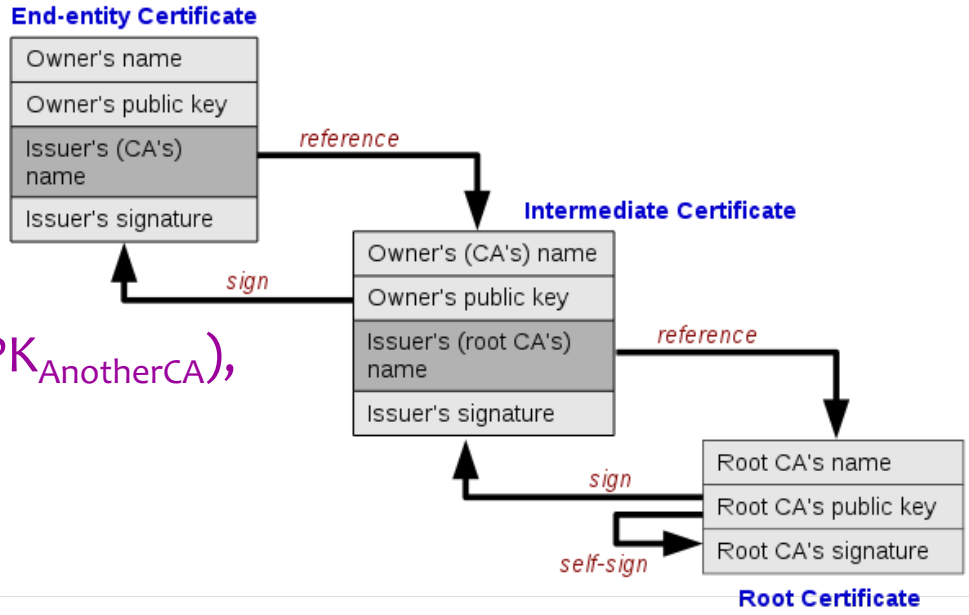
- Public announcement or public directory
  - Risks: forgery and tampering
- Public-key certificate
  - Signed statement specifying the key and identity
    - sig_{CA}("Bob", PK_B)
- Common approach: certificate authority (CA)
  - Single agency responsible for certifying public keys
  - After generating a private/public key pair, user proves his identity and knowledge of the private key to obtain CA's certificate for the public key (offline)
  - Every computer is pre-configured with CA's public key

Trusted(?) Certificate Authorities



Hierarchical Approach

- Single CA certifying every public key is impractical
- Instead, use a trusted root authority (e.g., Verisign)
  - Everybody must know the root's public key
  - Instead of single cert, use a certificate chain
    - $sig_{Verisign}$("AnotherCA", $PK_{AnotherCA}$), $sig_{AnotherCA}$("Alice", PK_A)



  - What happens if root authority is ever compromised?