CSE 484 / CSE M 584: Computer Security and Privacy

Cryptography
[Finish Hash Functions; Start Asymmetric Cryptography]

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Admin

- Lab 1 due Monday
- Coming up
  - Monday: Adversarial ML (Ivan Evtimov)
  - Today/Wednesday: Finish crypto
  - Friday: start web security!

Common Hash Functions

- MD5 – Don't Use!
  - 128-bit output
  - Designed by Ron Rivest, used very widely
  - Collision-resistance broken (summer of 2004)
- RIPEMD-160
  - 160-bit variant of MD5
- SHA-1 (Secure Hash Algorithm)
  - 160-bit output
  - US government (NIST) standard as of 1993-95
  - Theoretically broken 2005; practical attack 2017!
- SHA-256, SHA-512, SHA-224, SHA-384
- SHA-3: standard released by NIST in August 2015

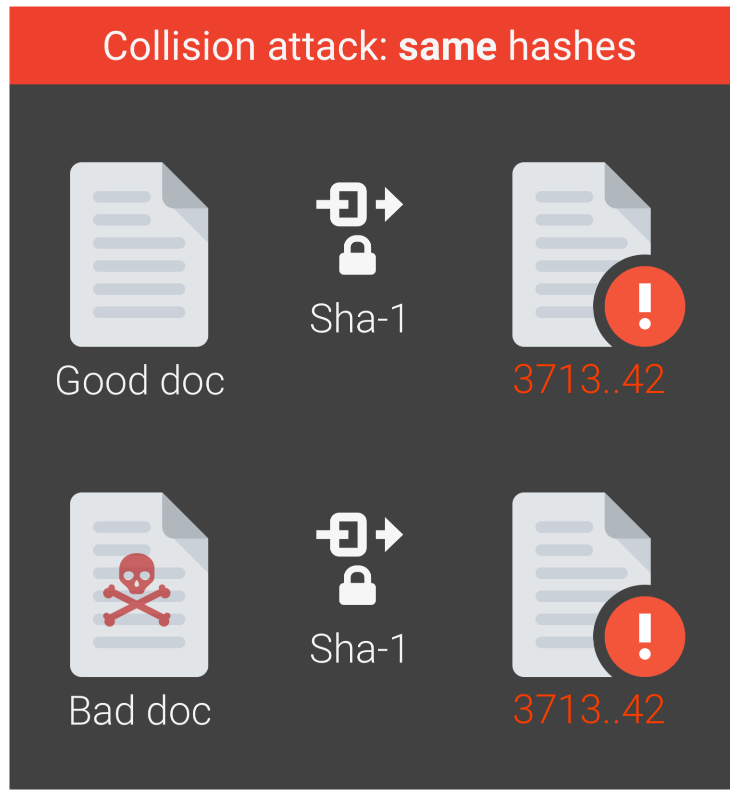
SHA-1 Broken in Practice (2017)

Google just cracked one of the building blocks of web encryption (but don't worry)

It's all over for SHA-1

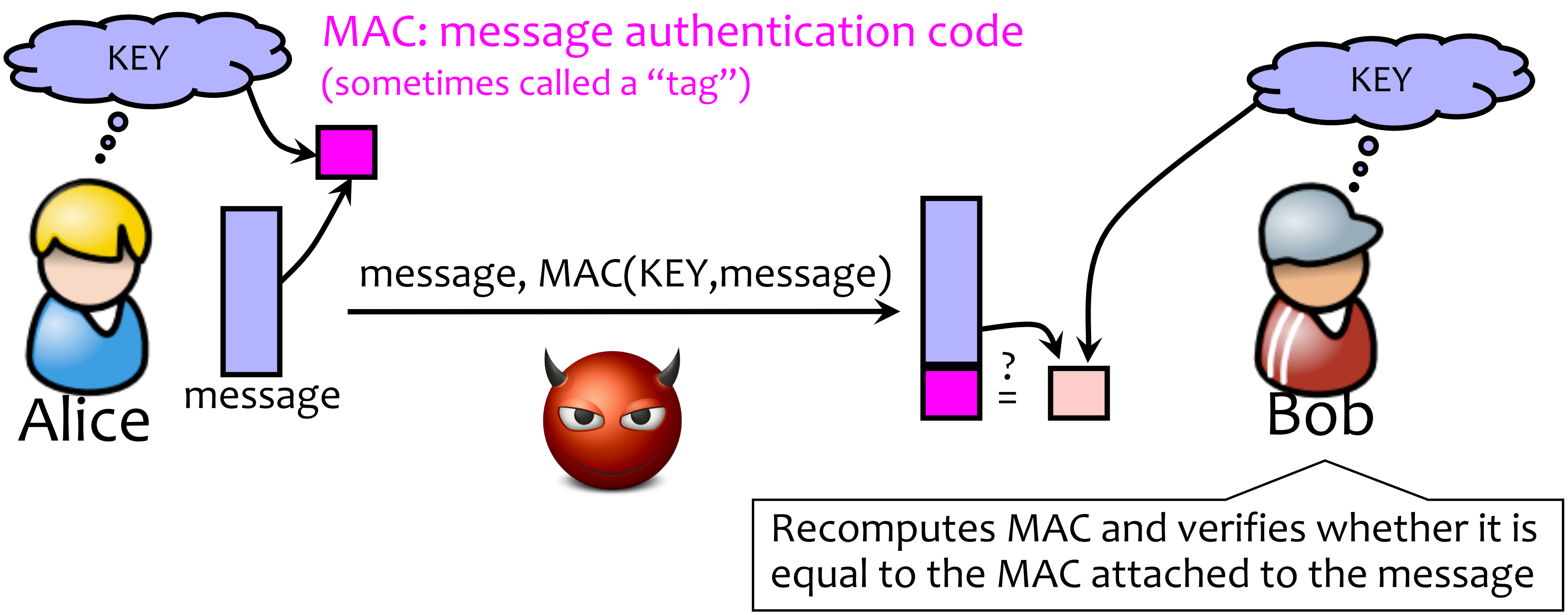
by Russell Brandom | @russellbrandom | Feb 23, 2017, 11:49am EST

https://shattered.io



Recall: Achieving Integrity

Message authentication schemes: A tool for protecting integrity.



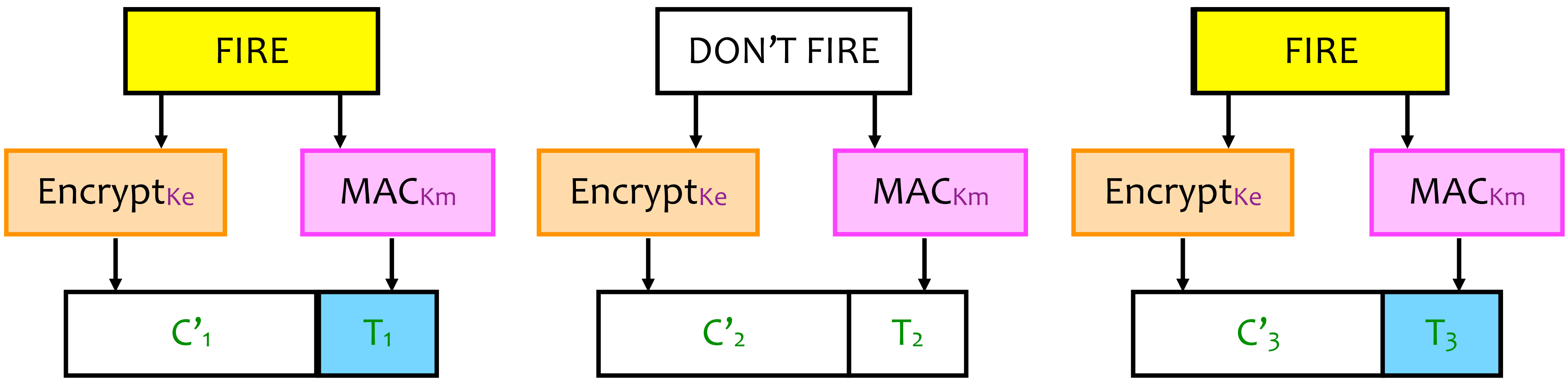
Integrity and authentication: only someone who knows KEY can compute correct MAC for a given message.

HMAC

- Construct MAC from a cryptographic hash function
  - Invented by Bellare, Canetti, and Krawczyk (1996)
  - Used in SSL/TLS, mandatory for IPsec
- Why not encryption?
  - Hashing is faster than block ciphers in software
  - Can easily replace one hash function with another
  - There used to be US export restrictions on encryption

Authenticated Encryption

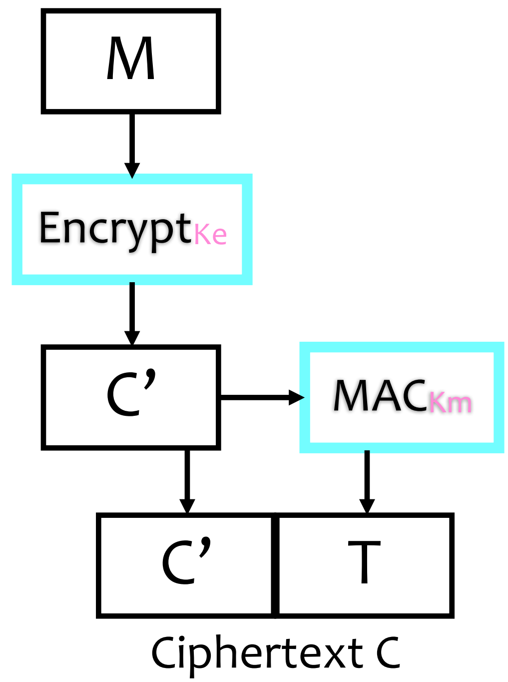
- What if we want both privacy and integrity?
- Natural approach: combine encryption scheme and a MAC.
- But be careful!
  - Obvious approach: Encrypt-and-MAC
  - Problem: MAC is deterministic! same plaintext → same MAC



Authenticated Encryption

- Instead:
  Encrypt *then* MAC.

- (Not as good:
  MAC-then-Encrypt)



Encrypt-then-MAC

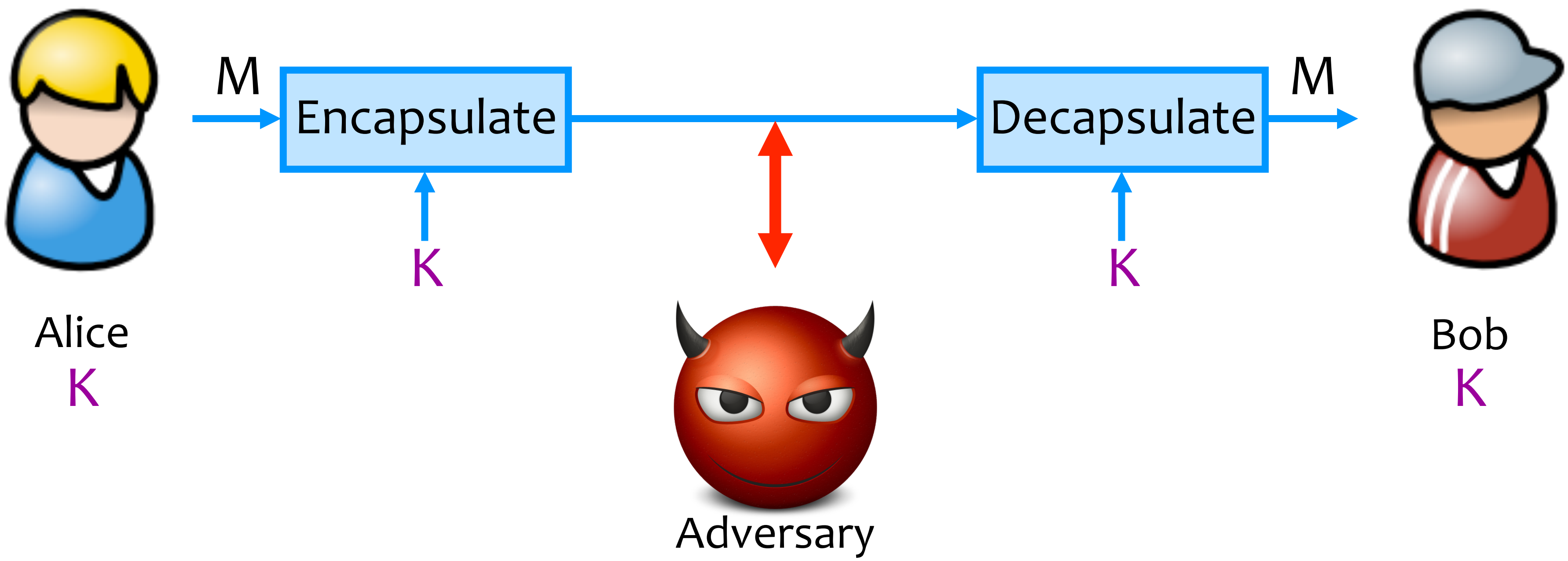
Stepping Back: Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string K, called the key.

- Asymmetric cryptography
  - Each party creates a public key pk and a secret key sk.

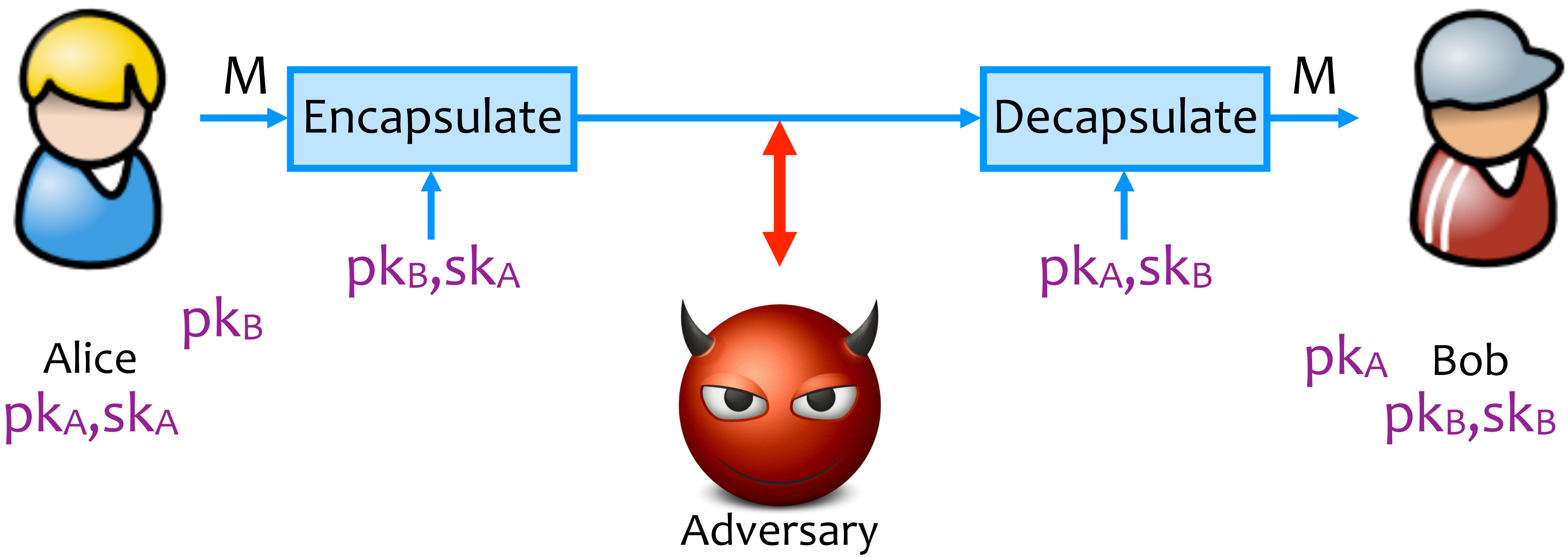
Symmetric Setting

Both communicating parties have access to a shared random string K, called the key.

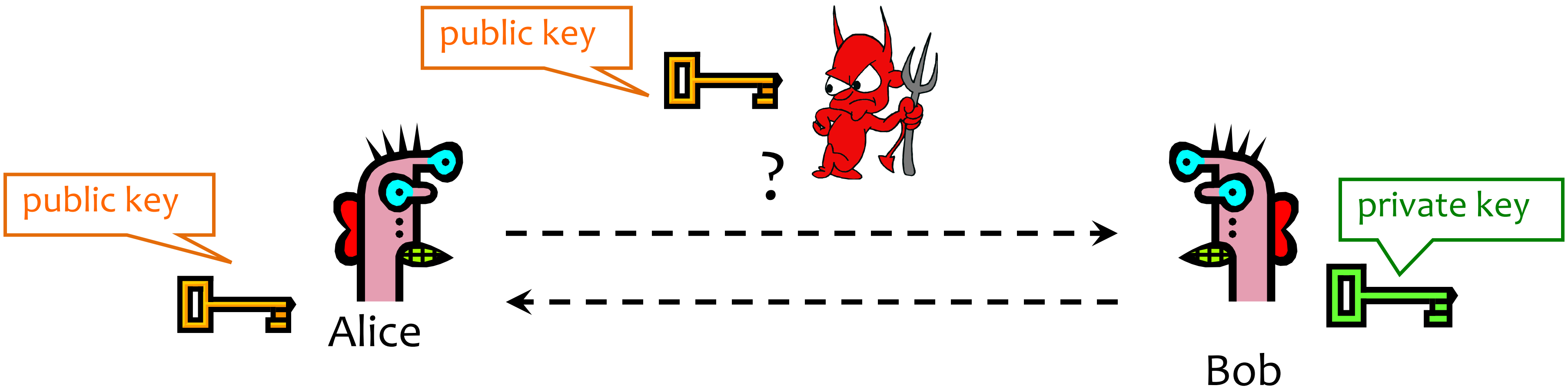


Asymmetric Setting

Each party creates a public key pk and a secret key sk.



Public Key Crypto: Basic Problem



Ignore for now: How do we know it's REALLY Bob's??

Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate himself

Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can "sign" a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

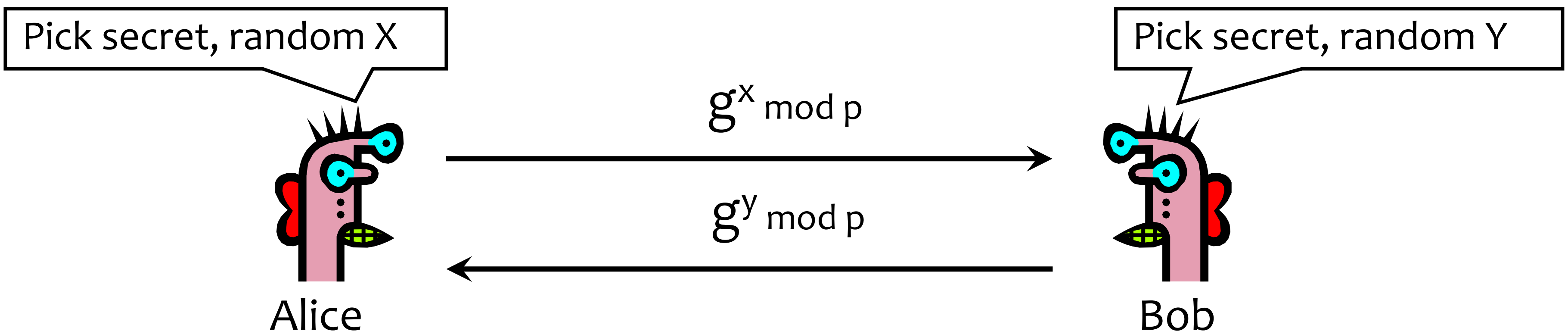
Session Key Establishment

Modular Arithmetic

- Refresher in section yesterday
- Given g and prime p, compute:
  $g^1 \bmod p$, $g^{100} \bmod p$, … $g^{100} \bmod p$
  - For p=11, g=10
    - $10^1 \bmod 11 = 10$, $10^2 \bmod 11 = 1$, $10^3 \bmod 11 = 10$, …
    - Produces cyclic group {10, 1} (order=2)
  - For p=11, g=7
    - $7^1 \bmod 11 = 7$, $7^2 \bmod 11 = 5$, $7^3 \bmod 11 = 2$, …
    - Produces cyclic group {7,5,2,3,10,4,6,9,8,1} (order = 10)
    - g=7 is a "generator" of Z_{11}^*

Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info: p and g
  - p is a large prime, g is a **generator** of Z_p^*
    - $Z_p^*=\{1, 2 \ldots p-1\}$; $\forall a \in Z_p^* \;\; \exists i$ such that $a=g^i \bmod p$
    - Modular arithmetic: numbers "wrap around" after they reach p



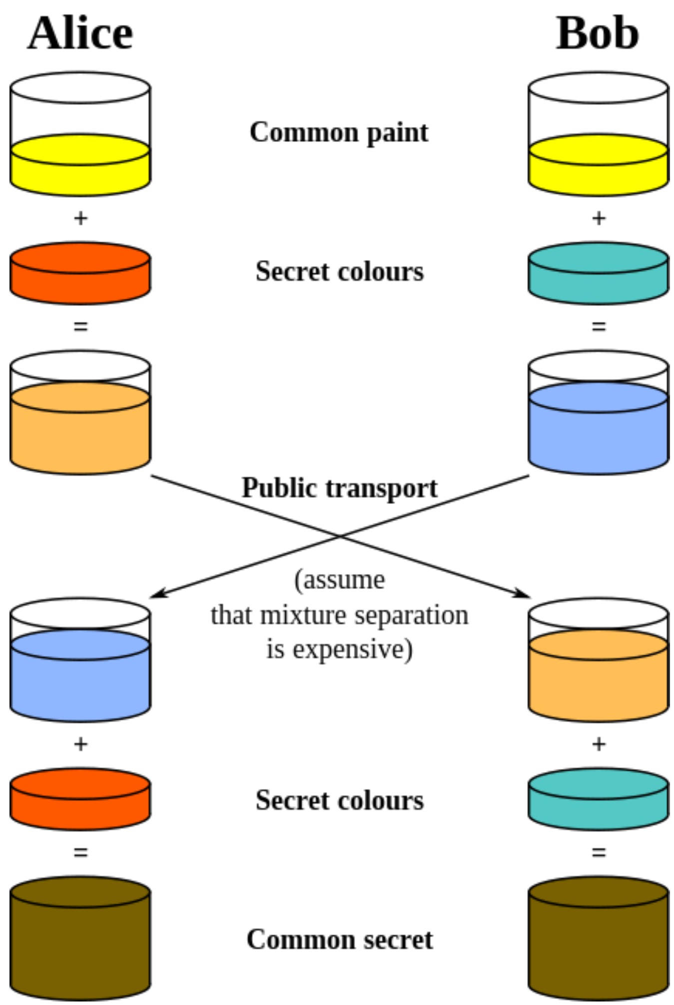
Compute $k=(g^y)^x=g^{xy} \bmod p$

Compute $k=(g^x)^y=g^{xy} \bmod p$

Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:
  given g^x *mod p*, it's hard to extract x
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:
  given g^x and g^y, it's hard to compute g^{xy} *mod p*
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:
  given g^x and g^y, it's hard to tell the difference between g^{xy} *mod p* and g^r *mod p* where r is random

Diffie-Hellman: Conceptually



Common paint: p and g

Secret colors: x and y

Send over public transport:
g^x mod p
g^y mod p

Common secret: g^{xy} mod p

[from Wikipedia]

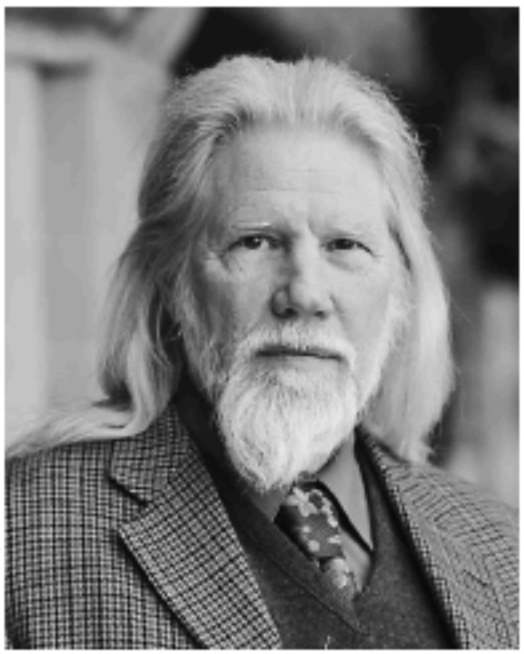
Properties of Diffie-Hellman

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose p=2q+1, where q is also a large prime
    - Choose g that generates a subgroup of order q in Z_p*
  - Eavesdropper can't tell the difference between the established key and a random value
  - Often hash $g^{xy} \bmod p$, and use the hash as the key
  - Can use the new key for symmetric cryptography
- Diffie-Hellman protocol (by itself) does not provide authentication
  - Party in the middle attack (often called "man in the middle attack")

More on Diffie-Hellman Key Exchange

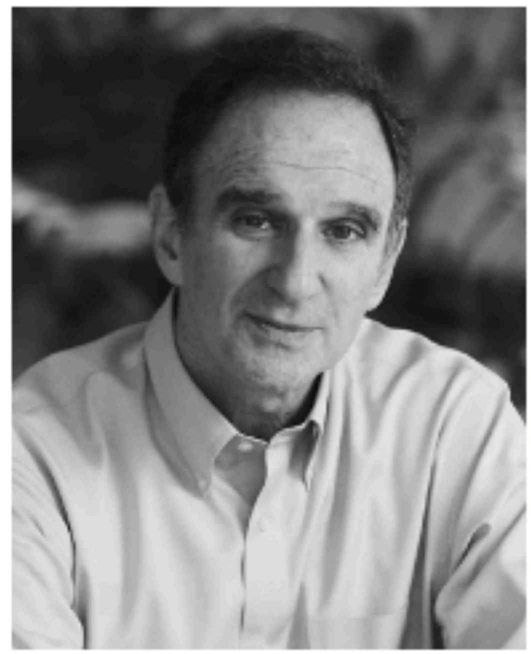
- Important Note:
  - We have discussed discrete logs modulo integers
  - Significant advantages in using elliptic curve groups
    - Groups with some similar mathematical properties (i.e., are "groups") but have better security and performance (size) properties

Diffie and Hellman Receive 2015 Turing Award



Rod Searcey/Stanford University.

Whitfield Diffie



Linda A. Cicero/Stanford News Service.

Martin E. Hellman

Public Key Encryption

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function $\varphi(n)$ ($n \geq 1$) is the number of integers in the $[1,n]$ interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
  - Easy to compute for primes: $\varphi(p) = p-1$
  - Note that $\varphi(ab) = \varphi(a)\,\varphi(b)$

RSA Cryptosystem [Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 1024 bits each (need primality testing, too)
  - Compute **n**=pq and **φ(n)**=(p-1)(q-1)
  - Choose small e, relatively prime to φ(n)
    - Typically, e=3 or $e=2^{16}+1=65537$
  - Compute unique d such that ed ≡ 1 mod φ(n)
    - Modular inverse: $d \equiv e^{-1} \bmod \varphi(n)$

How to compute?

  - Public key = (e,n); private key = (d,n)
- Encryption of m: $c = m^e \bmod n$
- Decryption of c: $c^d \bmod n = (m^e)^d \bmod n = m$

Why RSA Decryption Works (FYI)

$e \cdot d = 1 \bmod \varphi(n)$, thus $e \cdot d = 1 + k \cdot \varphi(n)$ for some k

Let m be any integer in Z_n^* (not all of Z_n)

$c^d \bmod n = (m^e)^d \bmod n = m^{1+k \cdot \varphi(n)} \bmod n$

$= (m \bmod n) * (m^{k \cdot \varphi(n)} \bmod n)$

Recall: Euler's theorem: if $a \in Z_n^*$, then $a^{\varphi(n)} = 1 \bmod n$

$c^d \bmod n = (m \bmod n) * (1 \bmod n)$

$= m \bmod n$

Proof omitted: True for all m in Z_n, not just m in Z_n^*

Why RSA Decryption Works (FYI)

- Decryption of c: $c^d \bmod n = (m^e \bmod n)^d \bmod n = (m^e)^d \bmod n = m$
- Recall **n**=pq and φ**(n)**=(p-1)(q-1) and ed ≡ 1 mod φ(n)

- Chinese Remainder Theorem: To show $m^{ed} \bmod n \equiv m \bmod n$, sufficient to show:
  - $m^{ed} \bmod p \equiv m \bmod p$
  - $m^{ed} \bmod q \equiv m \bmod q$

- If $m \equiv 0 \bmod p \rightarrow m^{ed} \equiv 0 \bmod p$

- Else $m^{ed} = m^{ed-1}m = m^{k(q-1)(p-1)}m = m^{h(p-1)}m$ for some k, and h=k(q-1). Why? Recall how d was chosen and the definition of mod.
- Fermat Little Theorem: $m^{(p-1)h}m \equiv 1^h m \bmod p \equiv m \bmod p$

Why is RSA Secure?

- RSA problem: given c, n=pq, and e such that $\gcd(e, \varphi(n))=1$, find m such that $m^e=c \bmod n$
  - In other words, recover m from ciphertext c and public key (n,e) by taking e^{th} root of c modulo n
  - There is no known efficient algorithm for doing this
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n=p_1^{e_1}p_2^{e_2}\ldots p_k^{e_k}$

- If factoring is easy, then RSA problem is easy (knowing factors means you can compute d = inverse of e mod (p-1)(q-1))
  - It may be possible to break RSA without factoring n -- but if it is, we don't know how

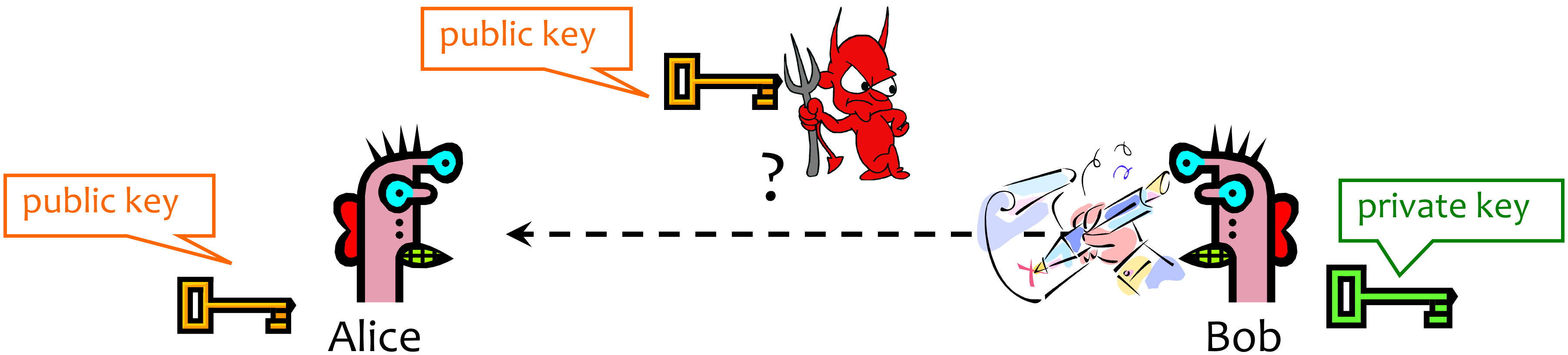
RSA Encryption Caveats

- Encrypted message needs to be interpreted as an integer less than n
- Don't use RSA **directly** for privacy – output is deterministic! Need to pre-process input somehow
- Plain RSA also does <u>not</u> provide integrity
  - Can tamper with encrypted messages

In practice, OAEP is used: instead of encrypting M, encrypt $M \oplus G(r) ; r \oplus H(M \oplus G(r))$

- r is random and fresh, G and H are hash functions

Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, only the public key is needed

RSA Signatures

- Public key is **(n,e)**, private key is **(n,d)**
- To sign message m: $s = m^d \bmod n$
  - Signing & decryption are same **underlying** operation in RSA
  - It's infeasible to compute **s** on **m** if you don't know **d**
- To verify signature s on message m:

  verify that $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption (for RSA primitive)
  - Anyone who knows **n** and **e** (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Standard padding/hashing schemes exist for RSA signatures

DSS Signatures

- Digital Signature Standard (DSS)
  - U.S. government standard (1991, most recent rev. 2013)
- Public key: $(p, q, g, y = g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from $g^x \bmod p$ (public key)

- Again: We've discussed discrete logs modulo integers; significant advantages to using elliptic curve groups instead.

Cryptography Summary

- Goal: Privacy
  - Symmetric keys:
    - One-time pad, Stream ciphers
    - Block ciphers (e.g., DES, AES) → modes: EBC, CBC, CTR
  - Public key crypto (e.g., Diffie-Hellman, RSA)
- Goal: Integrity
  - MACs, often using hash functions (e.g, MD5, SHA-256)
- Goal: Privacy and Integrity
  - Encrypt-then-MAC
- Goal: Authenticity (and Integrity)
  - Digital signatures (e.g., RSA, DSS)