CSE 484 / CSE M 584: Computer Security and Privacy

Cryptography

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Admin

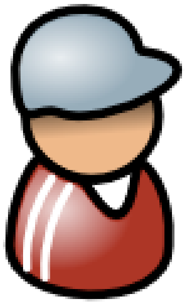
- Lab 1:
  - Due Oct 24, 4:30pm (Today!)

- TA Office Hours (especially for Lab 1): M 2:30, W 1:30, F 12
- My office hours (especially for crypto, research readings, administrivia, worksheet pick up): M 11:30

Challenge Question from Last Time

- Alice and Bob are both cryptographers, and they are talking on the phone. They want to randomly flip a coin. If they were together, in person, they would flip a real coin and see if it was Heads or Tails. But they are not together, in person, and they don't trust each other enough to have one of them flip a coin and tell the other person the answer.

- Using the techniques we've discussed so far in class, how can Alice and Bob effectively flip a random coin together, over the phone, such that they both trust the answer even though they don't trust each other?



Pick bit bA at random

C1 = Locked box with bA

Pick bit bB at random

C2 = Locked box with bB

Info to open first box C1

Now knows bA

Info to open second box C2

Now knows bB

Both compute random bit at bA xor bB

|| denotes concatenation

Pick bit bA at random

Pick RA as long random string

$C_1 = H(bA \,||\, RA)$ →

Pick bit bB at random

Pick RB as long random string

← $C_2 = H(bB \,||\, RB)$

Send bA || RA →

Verify that has of message equals C_1

← Send bB || RB

Verify that has of message equals C_2

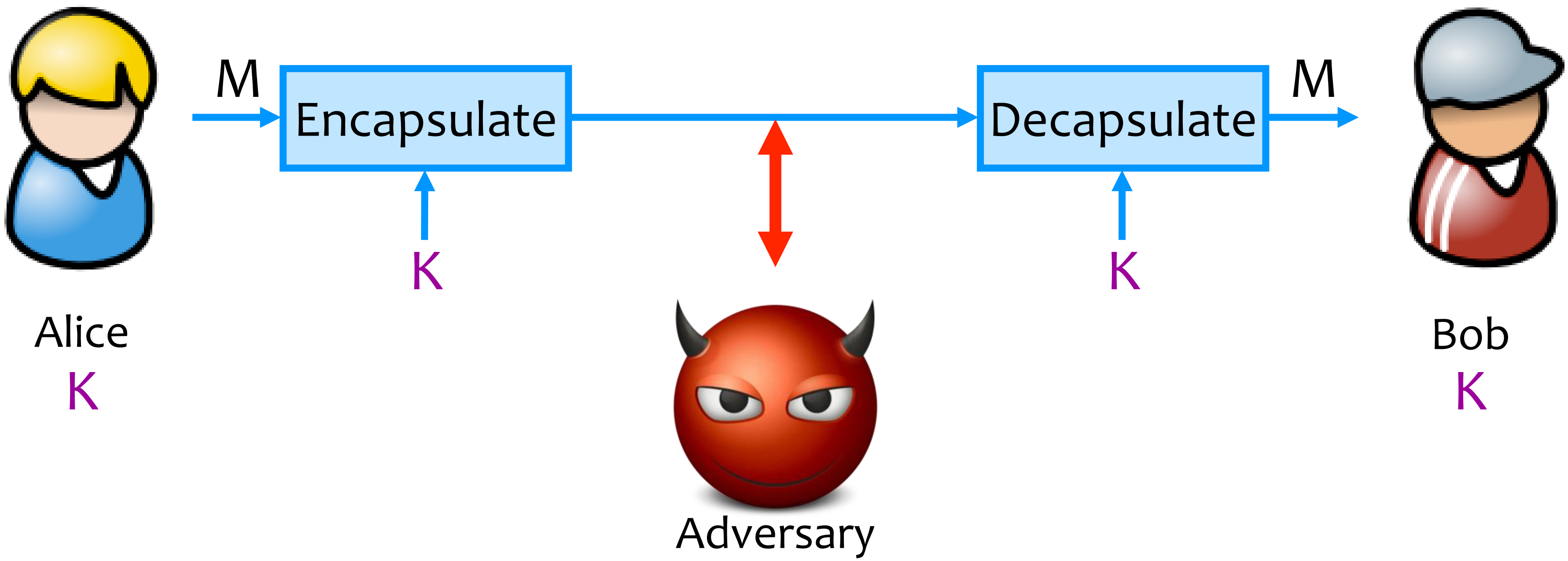
Both compute random bit at bA xor bB

Stepping Back: Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string K, called the key.
- Asymmetric cryptography
  - Each party creates a public key pk and a secret key sk.

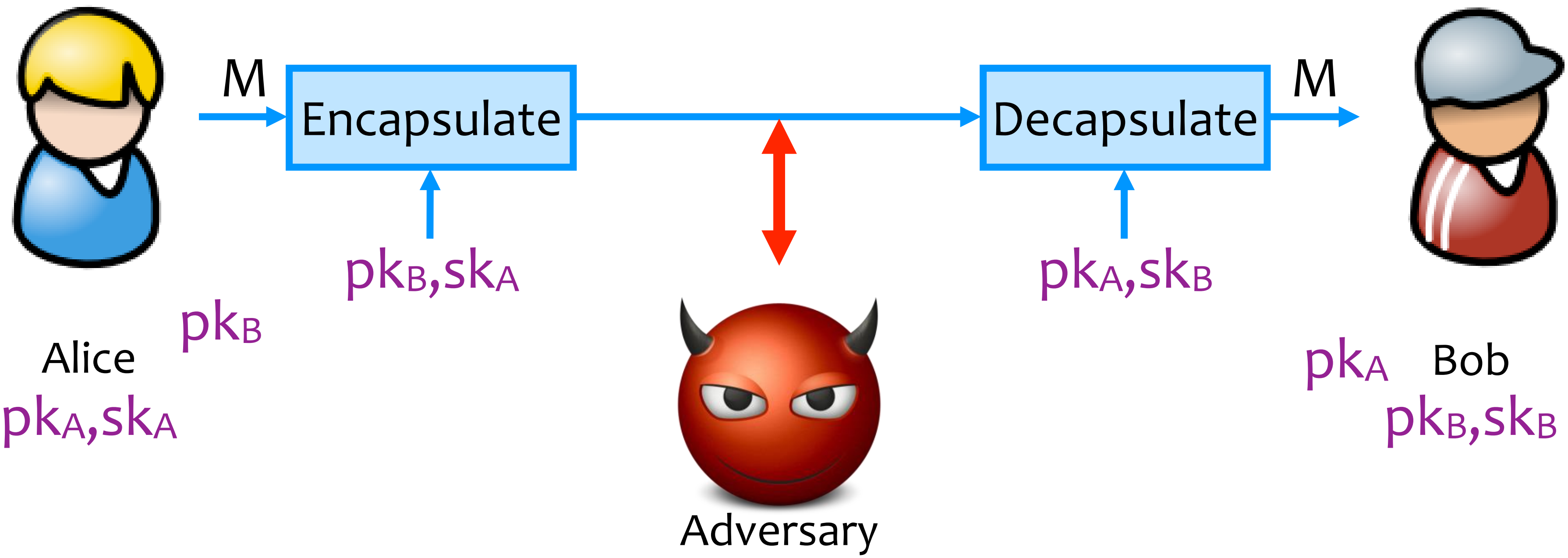
Symmetric Setting

Both communicating parties have access to a shared random string K, called the key.



Asymmetric Setting

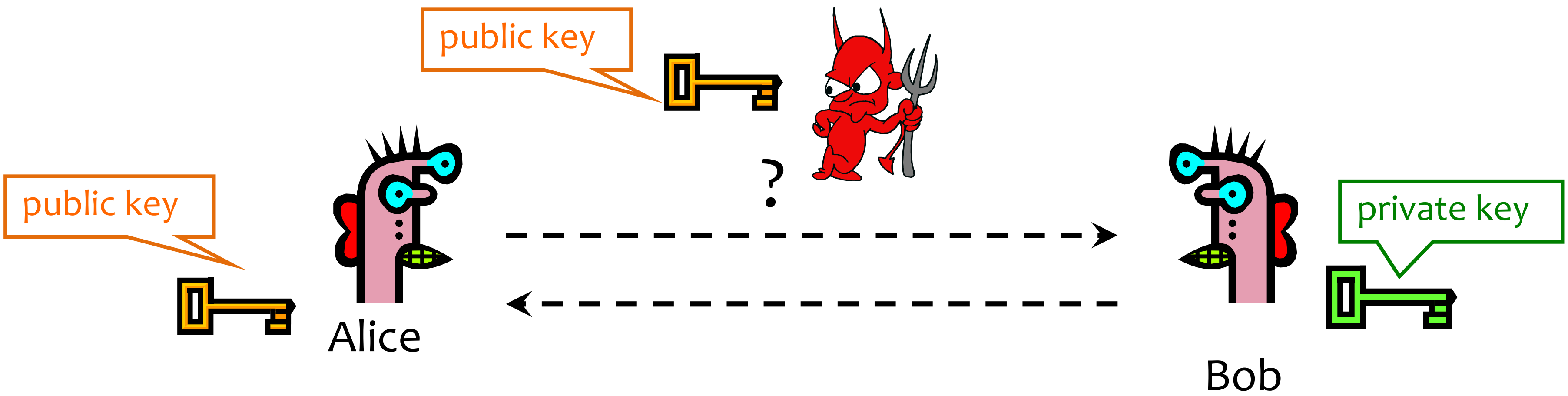
Each party creates a public key pk and a secret key sk.



Flavors of Cryptography

- Symmetric cryptography
  - Both communicating parties have access to a shared random string K, called the key.
  - Challenge: How do you privately share a key?
- Asymmetric cryptography
  - Each party creates a public key pk and a secret key sk.
  - Challenge: How do you validate a public key?

Public Key Crypto: Basic Problem



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate himself

Applications of Public Key Crypto

- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)
- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can "sign" a message with your private key

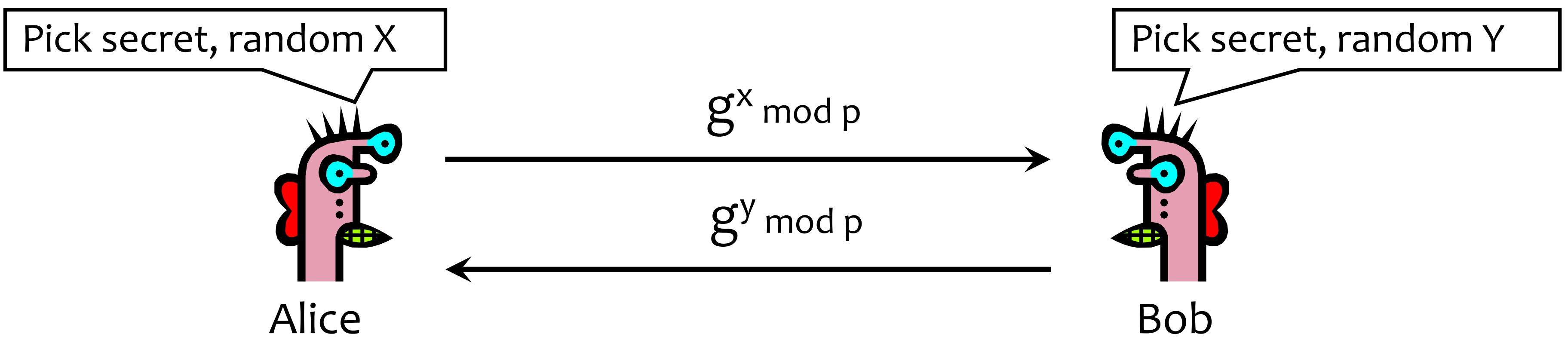
Session Key Establishment

Modular Arithmetic

- Given g and prime p, compute:
  g^1 mod p, g^2 mod p, … g^{100} mod p
  - For p=11, g= 10
    - 10^1 mod 11 = 10, 10^2 mod 11 = 1, 10^3 mod 11 = 10, …
    - Produces cyclic group {10, 1} (order=2)
  - For p=11, g=7
    - 7^1 mod 11 = 7, 7^2 mod 11 = 5, 7^3 mod 11 = 2, …
    - Produces cyclic group {7,5,2,3,10,4,6,9,8,1} (order = 10)
    - g=7 is a "generator" of Z_{11}^*
  - For p=11, g=3
    - 3^1 mod 11 = 3, 3^2 mod 11 = 9, 3^3 mod 11 = 5, …
    - Produces cyclic group {3,9,5,4,1} (order = 5)

Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info: p and g
  - p is a large prime, g is a **generator** of Z_p*
    - $Z_p^* = \{1, 2 \ldots p-1\}$; for all a in Z_p* there exists i s.t. $a=g^i \bmod p$
    - Modular arithmetic: numbers "wrap around" after they reach p



Compute $k=(g^y \bmod p)^x=g^{xy} \bmod p$ Compute $k=(g^x \bmod p)^y=g^{xy} \bmod p$

Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:
  given g^x *mod p*, it's hard to extract x
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:
  given g^x *mod p* and g^y *mod p*, it's hard to compute g^{xy} *mod p*
  - … unless you know x or y, in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:
  given g^x *mod p* and g^y *mod p*, it's hard to tell the difference between g^{xy} *mod p* and g^r *mod p* where r is random

Properties of Diffie-Hellman

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Common recommendation:
    - Choose p=2q+1, where q is also a large prime
    - Choose g that generates a subgroup of order q in Z_p*
  - Eavesdropper can't tell the difference between the established key and a random value
  - Often hash g^{xy} *mod p*, and use the hash as the key
  - Can use the new key for symmetric cryptography
- Diffie-Hellman protocol (by itself) does not provide authentication
  - Party in the middle attack (often called "man in the middle attack")

More on Diffie-Hellman Key Exchange

- **Important Note: We have discussed discrete logs modulo integers.**
- **Significant advantages in using elliptic curve groups – groups with some similar mathematical properties (i.e., are "groups") but have better security and performance (size) properties**

Public Key Encryption

Requirements for Public Key Encryption

- Key generation: computationally easy to generate a pair (public key PK, private key SK)
- Encryption: given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- Decryption: given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to learn anything about M from C without SK
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M