

**CSE 484 / CSE M 584: Computer Security and Privacy**

**Cryptography:**  
**Hash Functions, MACs (finish)**  
**Asymmetric Cryptography (start)**

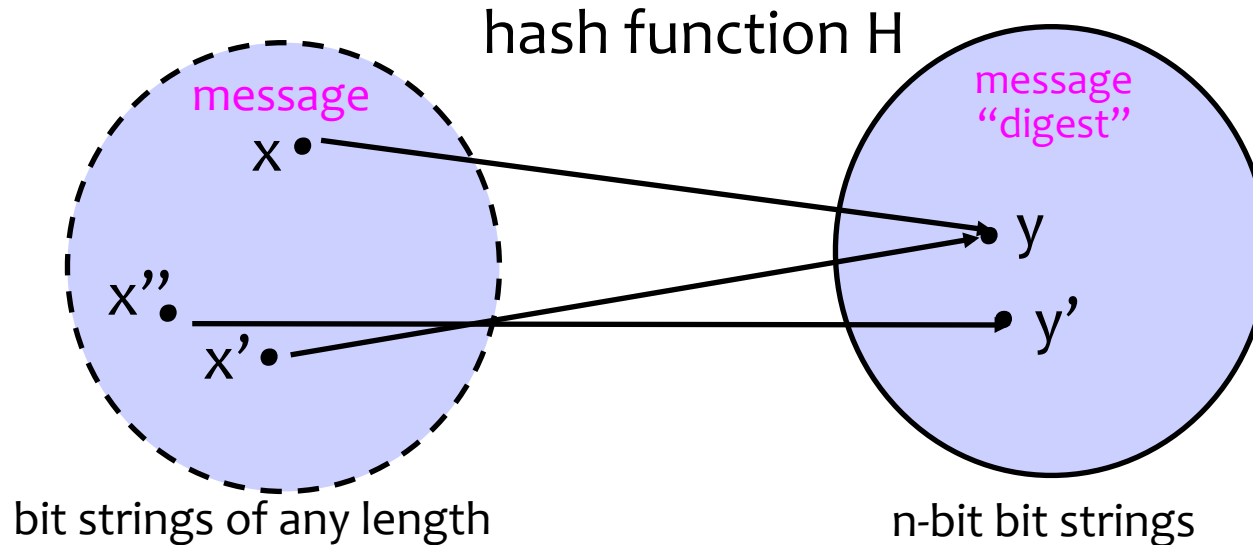
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# Hash Functions

# Hash Functions: Main Idea



- Hash function H is a lossy compression function
  - Collision:  $h(x)=h(x')$  for distinct inputs  $x, x'$
- $H(x)$  should look “random”
  - Every bit (almost) equally likely to be 0 or 1
- Cryptographic hash function needs a few properties...

# Property 1: One-Way

- Intuition: hash should be hard to invert
  - “Preimage resistance”
  - Let  $h(x') = y \in \{0,1\}^n$  for a random  $x'$
  - Given  $y$ , it should be hard to find any  $x$  such that  $h(x)=y$
- How hard?
  - Brute-force: try every possible  $x$ , see if  $h(x)=y$
  - SHA-1 (common hash function) has 160-bit output
    - Expect to try  $2^{159}$  inputs before finding one that hashes to  $y$ .

# Property 2: Collision Resistance

- Should be hard to find  $x \neq x'$  such that  $h(x) = h(x')$
- Birthday paradox means that brute-force collision search is **only  $O(2^{n/2})$ , not  $O(2^n)$** 
  - For SHA-1, this means  $O(2^{80})$  vs.  $O(2^{160})$

# One-Way vs. Collision Resistance

- One-wayness does not imply collision resistance
  - Suppose  $g$  is one-way
  - Define  $h(x)$  as  $g(x')$  where  $x'$  is  $x$  except the last bit
    - $h$  is one-way (to invert  $h$ , must invert  $g$ )
    - Collisions for  $h$  are easy to find: for any  $x$ ,  $h(x0)=h(x1)$
- Collision resistance does not imply one-wayness
  - Suppose  $g$  is collision-resistant
  - Define  $y=h(x)$  to be  $0x$  if  $x$  is  $n$ -bit long,  $1g(x)$  otherwise
    - Collisions for  $h$  are hard to find: if  $y$  starts with  $0$ , then there are no collisions, if  $y$  starts with  $1$ , then must find collisions in  $g$
    - $h$  is not one way: half of all  $y$ 's (those whose first bit is  $0$ ) are easy to invert (how?); random  $y$  is invertible with probab.  $\frac{1}{2}$

# Property 3: Weak Collision Resistance

- Given randomly chosen  $x$ , hard to find  $x'$  such that  $h(x)=h(x')$ 
  - Attacker must find collision for a specific  $x$ . By contrast, to break collision resistance it is enough to find any collision.
  - Brute-force attack requires  $O(2^n)$  time
- Weak collision resistance does not imply collision resistance.

# Hashing vs. Encryption

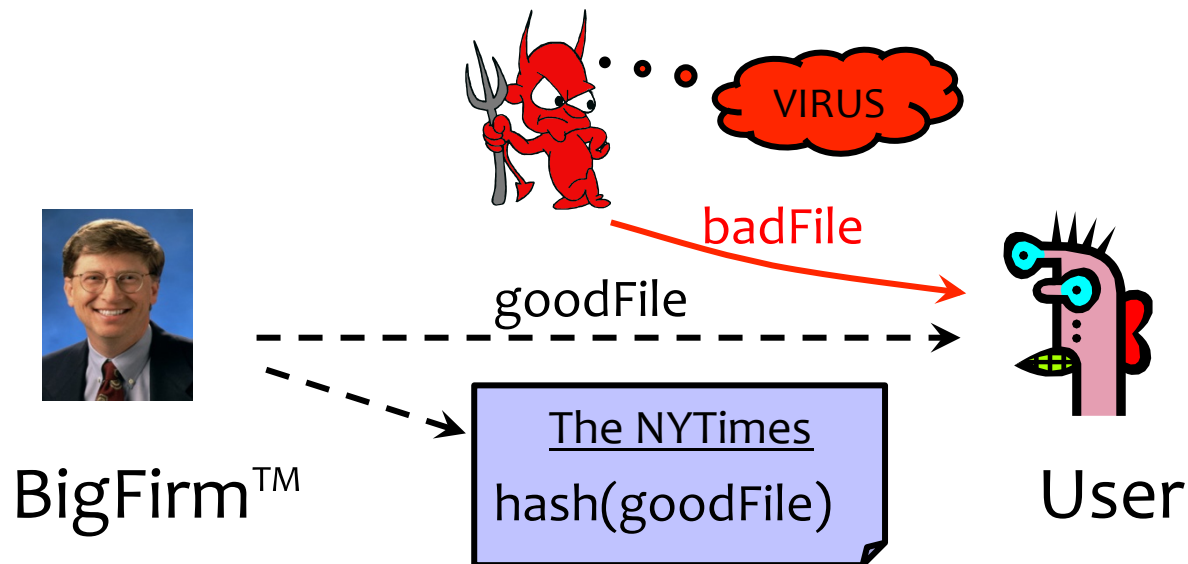
- Hashing is one-way. There is no “un-hashing”
  - A ciphertext can be decrypted with a decryption key... hashes have no equivalent of “decryption”
- Hash(x) looks “random” but can be compared for equality with Hash(x’)
  - Hash the same input twice → same hash value
  - Encrypt the same input twice → different ciphertexts
- Cryptographic hashes are also known as “cryptographic checksums” or “message digests”



# Application: Password Hashing

- Instead of user password, store `hash(password)`
- When user enters a password, compute its hash and compare with the entry in the password file
  - System does not store actual passwords!
  - Cannot go from hash to password!
- Why is hashing better than encryption here?
- Does hashing protect weak, easily guessable passwords?

# Application: Software Integrity



Goal: Software manufacturer wants to ensure file is received by users without modification.

Idea: given goodFile and  $\text{hash}(\text{goodFile})$ , very hard to find badFile such that  $\text{hash}(\text{goodFile}) = \text{hash}(\text{badFile})$

# Which Property Do We Need?

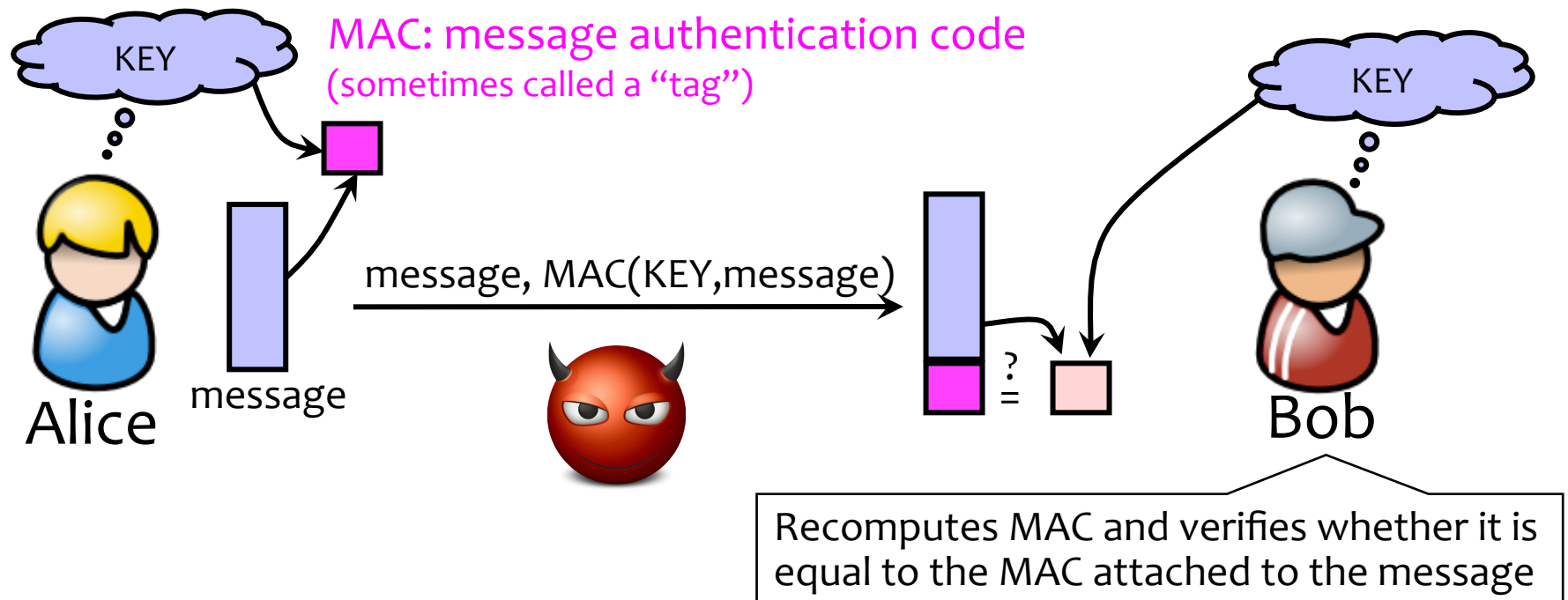
- UNIX passwords stored as  $\text{hash}(\text{password})$ 
  - One-wayness: hard to recover the/a valid password
- Integrity of software distribution
  - Weak collision resistance
  - But software images are not really random... may need full collision resistance if considering malicious developers
- Auction bidding
  - Alice wants to bid  $B$ , sends  $H(B)$ , later reveals  $B$
  - One-wayness: rival bidders should not recover  $B$  (this may mean that she needs to hash some randomness with  $B$  too)
  - Collision resistance: Alice should not be able to change her mind to bid  $B'$  such that  $H(B)=H(B')$

# Common Hash Functions

- MD5
  - 128-bit output
  - Designed by Ron Rivest, used very widely
  - Collision-resistance broken (summer of 2004)
- RIPEMD-160
  - 160-bit variant of MD5
- SHA-1 (Secure Hash Algorithm)
  - 160-bit output
  - US government (NIST) standard as of 1993-95
  - Also recently broken! (Theoretically -- not practical.)
- SHA-256, SHA-512, SHA-224, SHA-384
- SHA-3: standard released by NIST in August 2015

# Recall: Achieving Integrity

Message authentication schemes: A tool for protecting integrity.



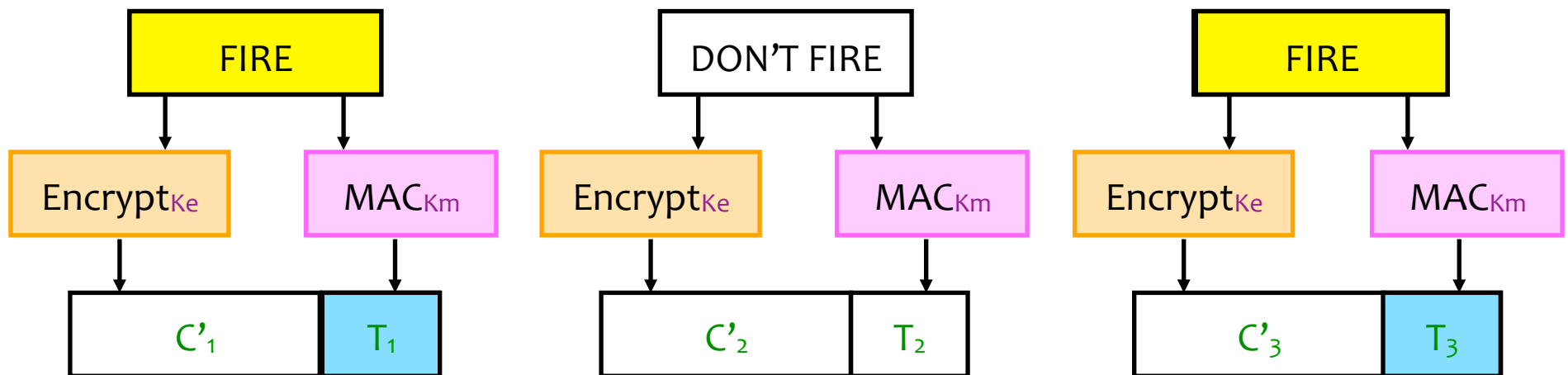
**Integrity and authentication:** only someone who knows KEY can compute correct MAC for a given message.

# HMAC

- Construct MAC from a cryptographic hash function
  - Invented by Bellare, Canetti, and Krawczyk (1996)
  - Used in SSL/TLS, mandatory for IPsec
- Why not encryption?
  - Hashing is faster than block ciphers in software
  - Can easily replace one hash function with another
  - There used to be US export restrictions on encryption

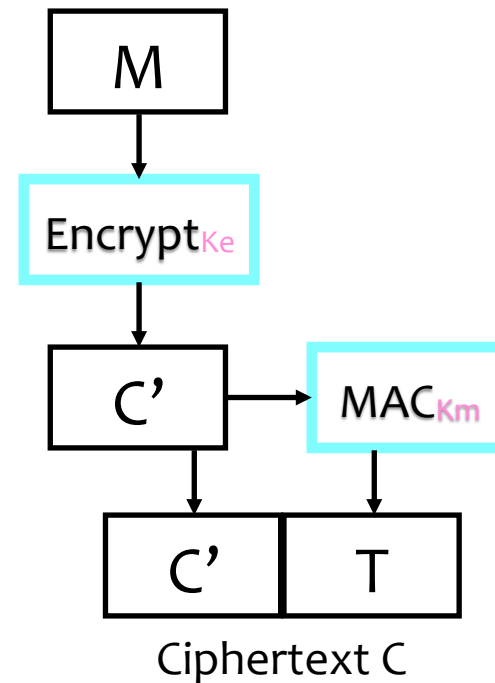
# Authenticated Encryption

- What if we want both privacy and integrity?
- Natural approach: combine encryption scheme and a MAC.
- **But be careful!**
  - Obvious approach: Encrypt-and-MAC
  - Problem: MAC is deterministic! same plaintext  $\rightarrow$  same MAC



# Authenticated Encryption

- Instead:  
*Encrypt then MAC.*
- (Not as good:  
MAC-then-Encrypt)



**Encrypt-then-MAC**

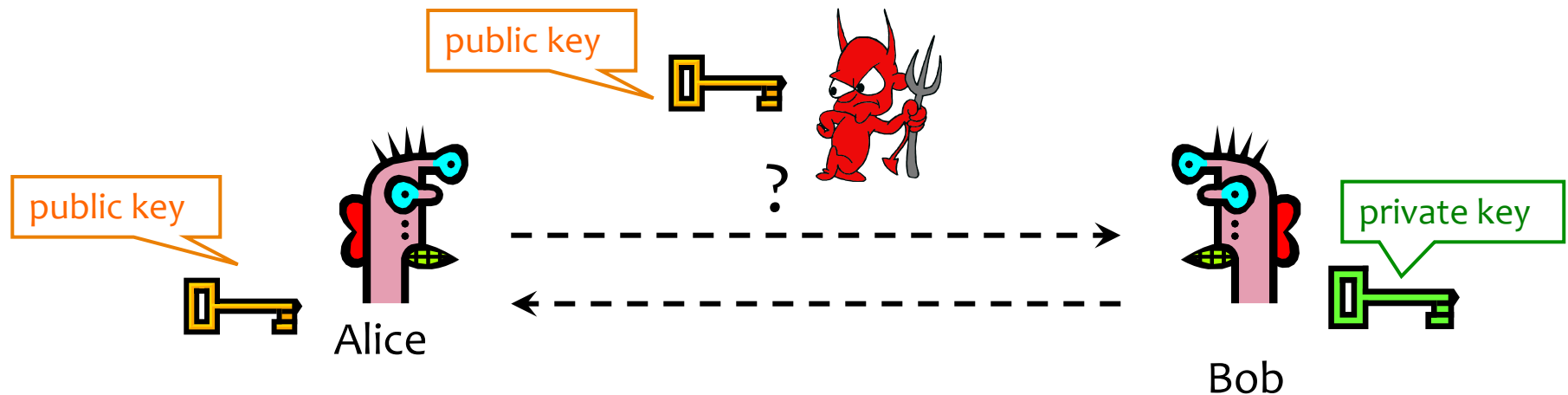


# Asymmetric (Public Key) Cryptography

# Reminder: Symmetric Cryptography

- **1 secret key (or 2 or ...)**, shared between sender/receiver
- Repeat fast and simple operations lots of times (rounds) to mix up key and ciphertext
- **Why do we think it is secure?** (simplistic)
  - Lots of heuristic arguments
    - If we do lots and lots and lots of mixing, no simple formula (and reversible) describing the whole process (cryptographic weakness).
    - Mix in ways we think it's hard to short-circuit all the rounds. Especially non-linear mixing, e.g., S-boxes.
  - Some math gives us confidence in these assumptions

# Public Key Crypto: Basic Problem



Given: Everybody knows Bob's **public key**  
Only Bob knows the corresponding **private key**

Goals: 1. Alice wants to send a secret message to Bob  
2. Bob wants to authenticate himself

# Public Key Cryptography

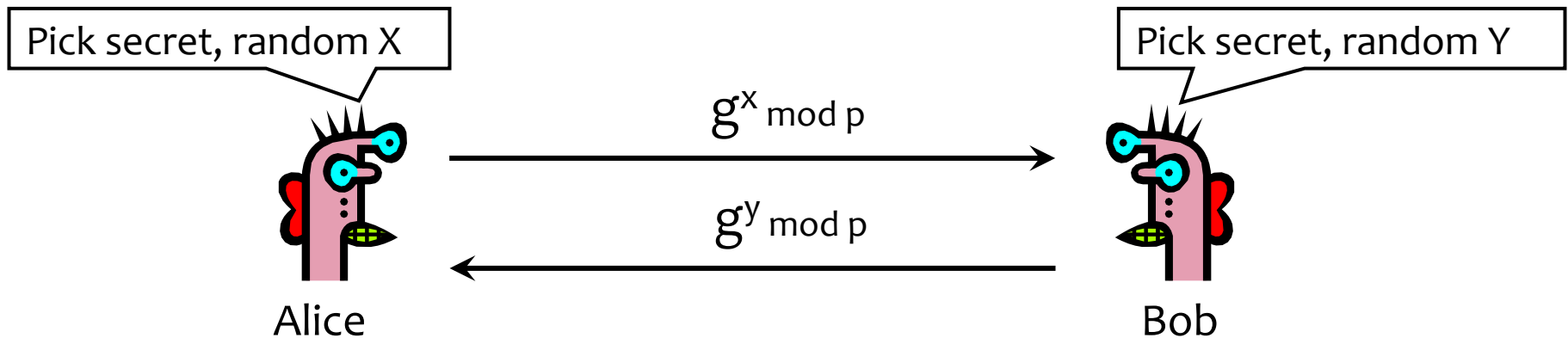
- Everyone has **1 private key and 1 public key**
  - Or 2 private and 2 public, when considering both encryption and authentication
- Mathematical relationship between private and public keys
- **Why do we think it is secure?** (simplistic)
  - Relies entirely on **problems we believe are “hard”**

# Applications of Public Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (or at least different)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can “sign” a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

# Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info:  $p$  and  $g$ 
  - $p$  is a large prime number,  $g$  is a generator of  $Z_p^*$ 
    - $Z_p^* = \{1, 2 \dots p-1\}$ ;  $\forall a \in Z_p^* \exists i$  such that  $a = g^i \pmod p$
    - Modular arithmetic: numbers “wrap around” after they reach  $p$



Compute  $k = (g^y)^x = g^{xy} \pmod p$

Compute  $k = (g^x)^y = g^{xy} \pmod p$

# Why is Diffie-Hellman Secure?

- Discrete Logarithm (DL) problem:  
given  $g^x \bmod p$ , it's hard to extract  $x$ 
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- Computational Diffie-Hellman (CDH) problem:  
given  $g^x$  and  $g^y$ , it's hard to compute  $g^{xy} \bmod p$ 
  - ... unless you know  $x$  or  $y$ , in which case it's easy
- Decisional Diffie-Hellman (DDH) problem:  
given  $g^x$  and  $g^y$ , it's hard to tell the difference between  $g^{xy} \bmod p$  and  $g^r \bmod p$  where  $r$  is random

# Properties of Diffie-Hellman

- Assuming DDH problem is hard (depends on choice of parameters!), Diffie-Hellman protocol is a secure key establishment protocol against passive attackers
  - Eavesdropper can't tell the difference between the established key and a random value
  - Can use the new key for symmetric cryptography
- Diffie-Hellman protocol (by itself) does not provide authentication



# Requirements for Public Key Encryption

- **Key generation:** computationally easy to generate a pair (public key  $PK$ , private key  $SK$ )
- **Encryption:** given plaintext  $M$  and public key  $PK$ , easy to compute ciphertext  $C = E_{PK}(M)$
- **Decryption:** given ciphertext  $C = E_{PK}(M)$  and private key  $SK$ , easy to compute plaintext  $M$ 
  - Infeasible to learn anything about  $M$  from  $C$  without  $SK$
  - Trapdoor function:  $Decrypt(SK, Encrypt(PK, M)) = M$