

# Cryptography (cont.)

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Daniel Halperin  
Tadayoshi Kohno

Thanks to Dan Boneh, Dieter Gollmann, John Manferdelli, John Mitchell, Vitaly Shmatikov, Bennet Yee, and many others for sample slides and materials ...

# Updates Oct. 19th

- **Lab 1** is due *Friday*
  - TA office hours Fri before class  
(12-2:20, CSE 002)
  - My office hours today after class  
(CSE 210)

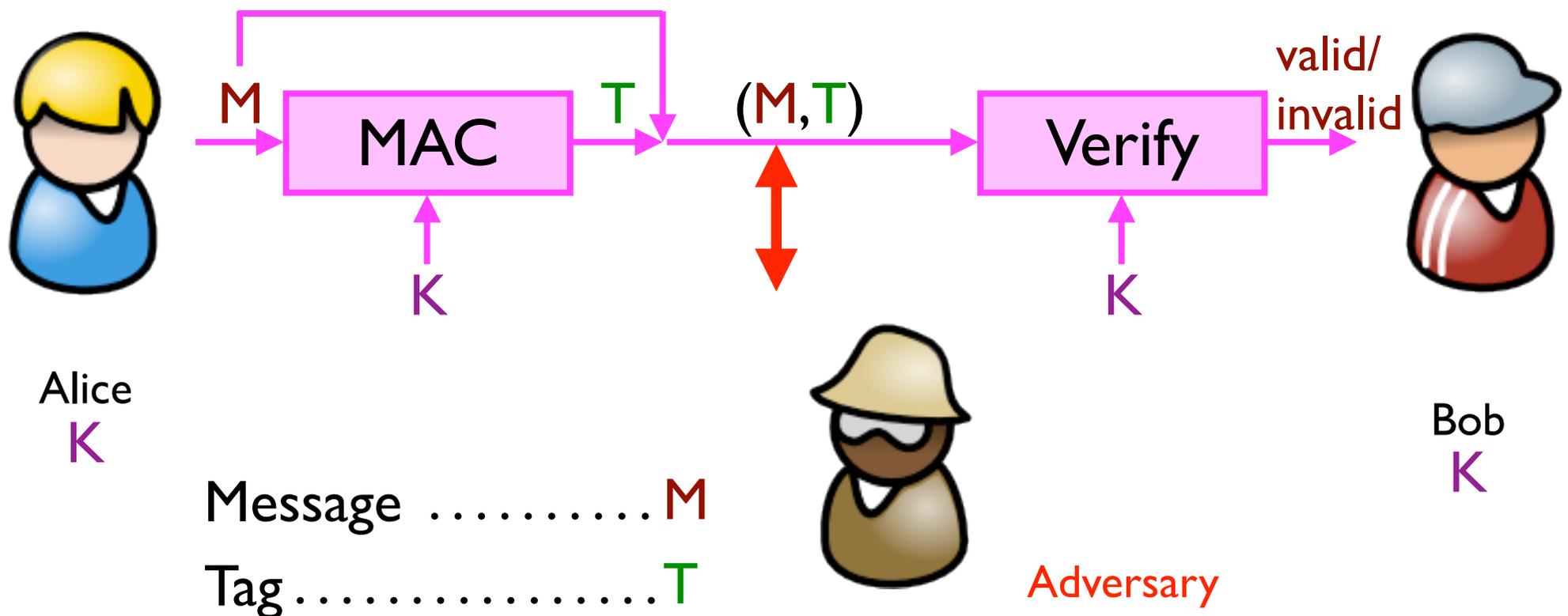
# Today

- Integrity for symmetric crypto
- More generally, hash functions

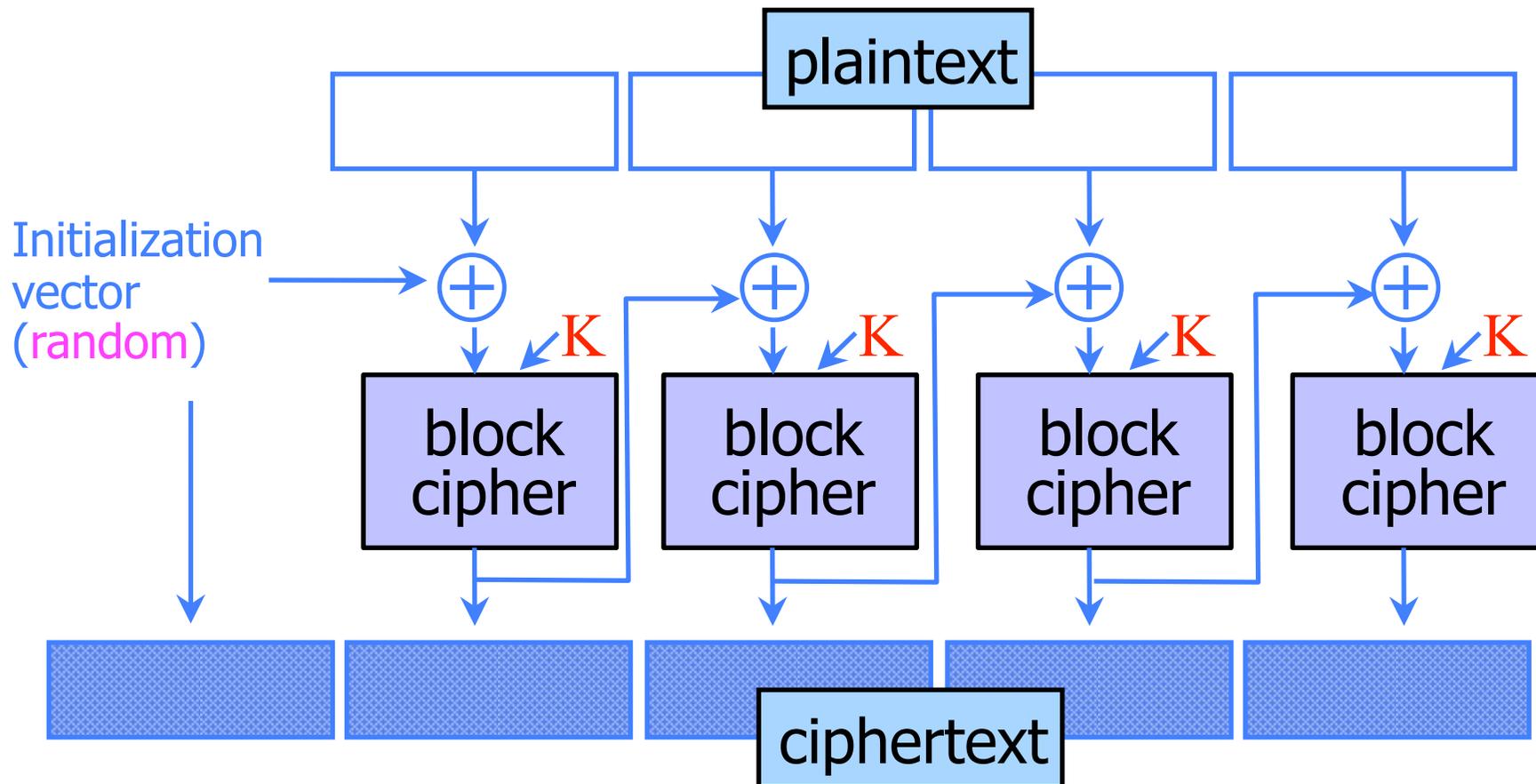
# Achieving Integrity (Symmetric)

Message authentication schemes: A tool for protecting integrity.

(Also called message authentication codes or MACs.)

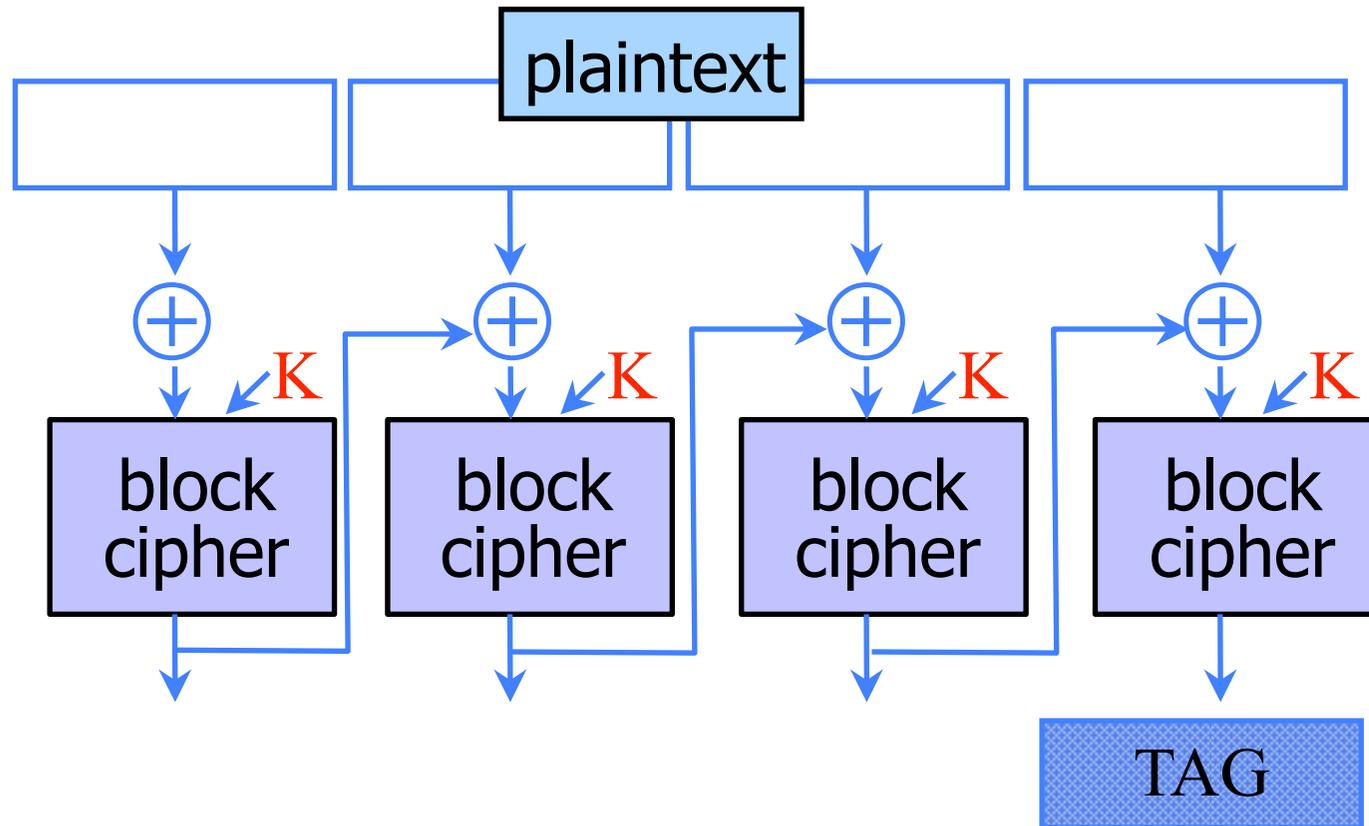


# CBC Mode: Encryption



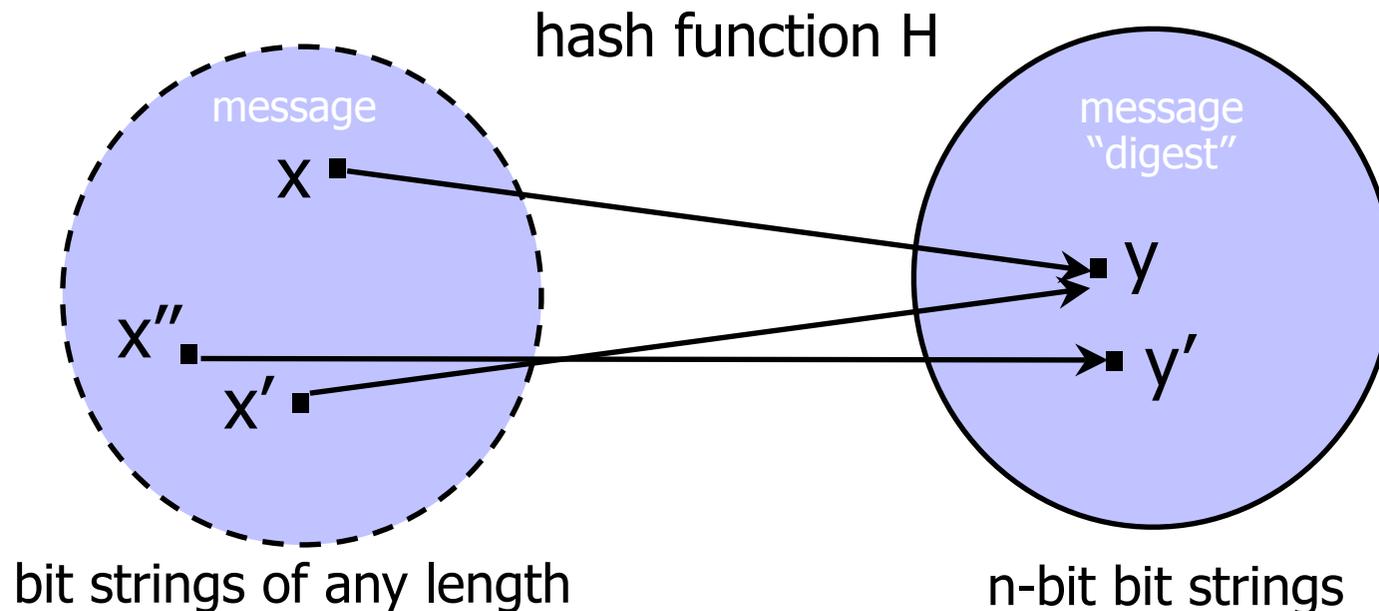
- ◆ Identical blocks of plaintext encrypted differently
- ◆ Last cipherblock depends on entire plaintext
  - Still does not guarantee integrity

# CBC-MAC



- ◆ Not secure when system may MAC messages of different lengths.
  - NIST recommends a derivative called CMAC (not required)

# Broad Class of Hash Functions



- ◆ H is a lossy compression function
  - Collisions:  $h(x)=h(x')$  for distinct inputs  $x, x'$
  - Result of hashing should "look random" (make this precise later)
    - Intuition: half of digest bits are "1"; any bit in digest is "1" half the time
- ◆ Cryptographic hash function needs a few properties...

# One-Way

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- ◆ Intuition: hash should be hard to invert
  - “Preimage resistance”
  - Let  $h(x')=y \in \{0,1\}^n$  for a random  $x'$
  - Given  $y$ , it should be hard to find any  $x$  such that  $h(x)=y$
- ◆ How hard?
  - Brute-force: try every possible  $x$ , see if  $h(x)=y$
  - SHA-1 (common hash function) has 160-bit output
    - Expect to try  $2^{159}$  inputs before finding one that hashes to  $y$ .

# Collision Resistance

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- ◆ Should be hard to find distinct  $x, x'$  such that  $h(x)=h(x')$ 
  - Brute-force collision search is only  $O(2^{n/2})$ , not  $O(2^n)$
  - For SHA-1, this means  $O(2^{80})$  vs.  $O(2^{160})$
- ◆ Birthday paradox (informal)
  - Let  $t$  be the number of values  $x, x', x'' \dots$  we need to look at before finding the first pair  $x, x'$  s.t.  $h(x)=h(x')$
  - What is probability of collision for each pair  $x, x'$ ?
  - How many pairs would we need to look at before finding the first collision?
  - How many pairs  $x, x'$  total?
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  - What is  $t$ ?  $2^{n/2}$

# One-Way vs. Collision Resistance

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- ◆ One-wayness does not imply collision resistance
  - Suppose  $g$  is one-way
  - Define  $h(x)$  as  $g(x')$  where  $x'$  is  $x$  except the last bit
    - $h$  is one-way (to invert  $h$ , must invert  $g$ )
    - Collisions for  $h$  are easy to find: for any  $x$ ,  $h(x0)=h(x1)$

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- ◆ Collision resistance does not imply one-wayness
  - Suppose  $g$  is collision-resistant
  - Define  $h(x)$  to be  $0x$  if  $x$  is  $n$ -bit long,  $1g(x)$  otherwise
    - Collisions for  $h$  are hard to find: if  $y$  starts with  $0$ , then there are no collisions, if  $y$  starts with  $1$ , then must find collisions in  $g$
    - $h$  is not one way: half of all  $y$ 's (those whose first bit is  $0$ ) are easy to invert (how?); random  $y$  is invertible with probab.  $1/2$

# Weak Collision Resistance

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- ◆ Given randomly chosen  $x$ , hard to find  $x'$  such that  $h(x)=h(x')$ 
  - Attacker must find collision for a specific  $x$ . By contrast, to break collision resistance it is enough to find any collision.
  - Brute-force attack requires  $O(2^n)$  time
  - AKA **second-preimage collision** resistance
- ◆ Weak collision resistance does not imply collision resistance

# Which Property Do We Need?

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- ◆ UNIX passwords stored as  $\text{hash}(\text{password})$ 
  - One-wayness: hard to recover the/a valid password
- ◆ Integrity of software distribution
  - Weak collision resistance (second-preimage resistance)
  - But software images are not really random...
- ◆ Auction bidding
  - Alice wants to bid  $B$ , sends  $H(B)$ , later reveals  $B$
  - One-wayness: rival bidders should not recover  $B$  (this may mean that she needs to hash some randomness with  $B$  too)
  - Collision resistance: Alice should not be able to change her mind to bid  $B'$  such that  $H(B)=H(B')$

# Common Hash Functions

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## ◆ MD5

- 128-bit output
- Designed by Ron Rivest, used very widely
- Collision-resistance broken (summer of 2004)

## ◆ RIPEMD-160

- 160-bit variant of MD5

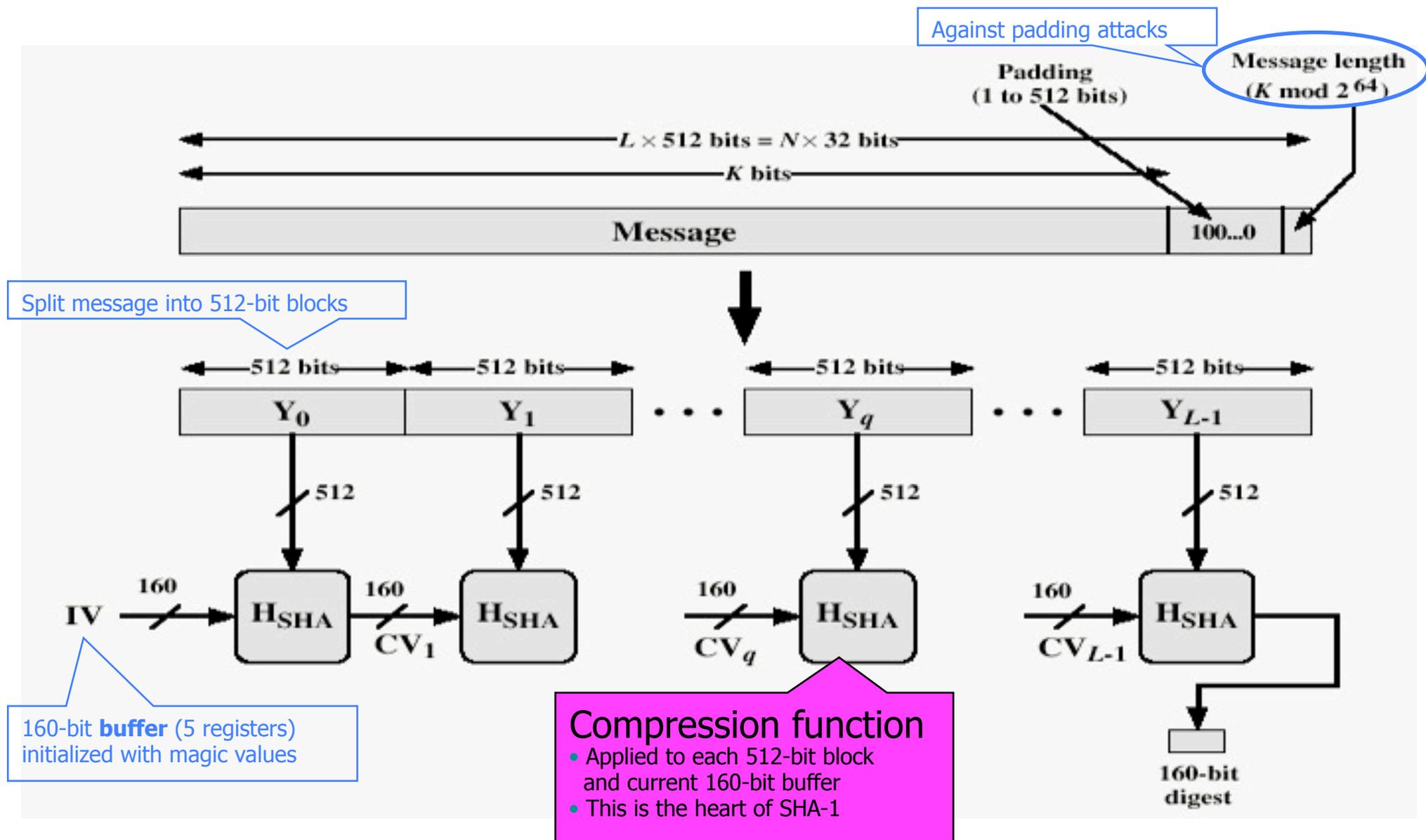
## ◆ SHA-1 (Secure Hash Algorithm)

- 160-bit output
- US government (NIST) standard as of 1993-95
- Also recently broken! (Theoretically -- not practical.)

## ◆ SHA-256, SHA-512, SHA-224, SHA-384

## ◆ SHA-3: Forthcoming.

# Basic Structure of SHA-1 (Not Required)



# How Strong Is SHA-1?

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- ◆ Every bit of output depends on every bit of input
  - Very important property for collision-resistance
- ◆ Brute-force inversion requires  $2^{160}$  ops, birthday attack on collision resistance requires  $2^{80}$  ops
- ◆ Some recent weaknesses (2005)
  - Collisions can be found in  $2^{63}$  ops

# International Criminal Tribunal for Rwanda (Example Application)

- ◆ [http://www.nytimes.com/2009/01/27/science/27arch.html?\\_r=1&ref=science](http://www.nytimes.com/2009/01/27/science/27arch.html?_r=1&ref=science)



**Adama Dieng**

CB44-8847-D68D-8CD2-C2F5  
22FE-177B-2C30-3549-C211



**Angeline Djampou**

EA39-EC39-A5D0-314D-04A6  
5258-572C-9268-8CB7-6404



**Avi Singh**

CD69-2CB5-78CB-D8D7-7D81  
F9B2-9CEA-5B79-DA4F-3806



**Alfred Kwende**

C690-FC5A-8EB7-0B83-B99D  
2593-608A-F421-BEE4-16B2



**Sir Dennis Byron**

CA46-BE7A-B8F6-095A-C706  
1C60-31E7-F9EA-AF96-E2CE



**Everard O'Donnell**

909F-86AB-C1B8-57A7-9CF6  
5BCD-7F5E-F4F6-68CA-70D1

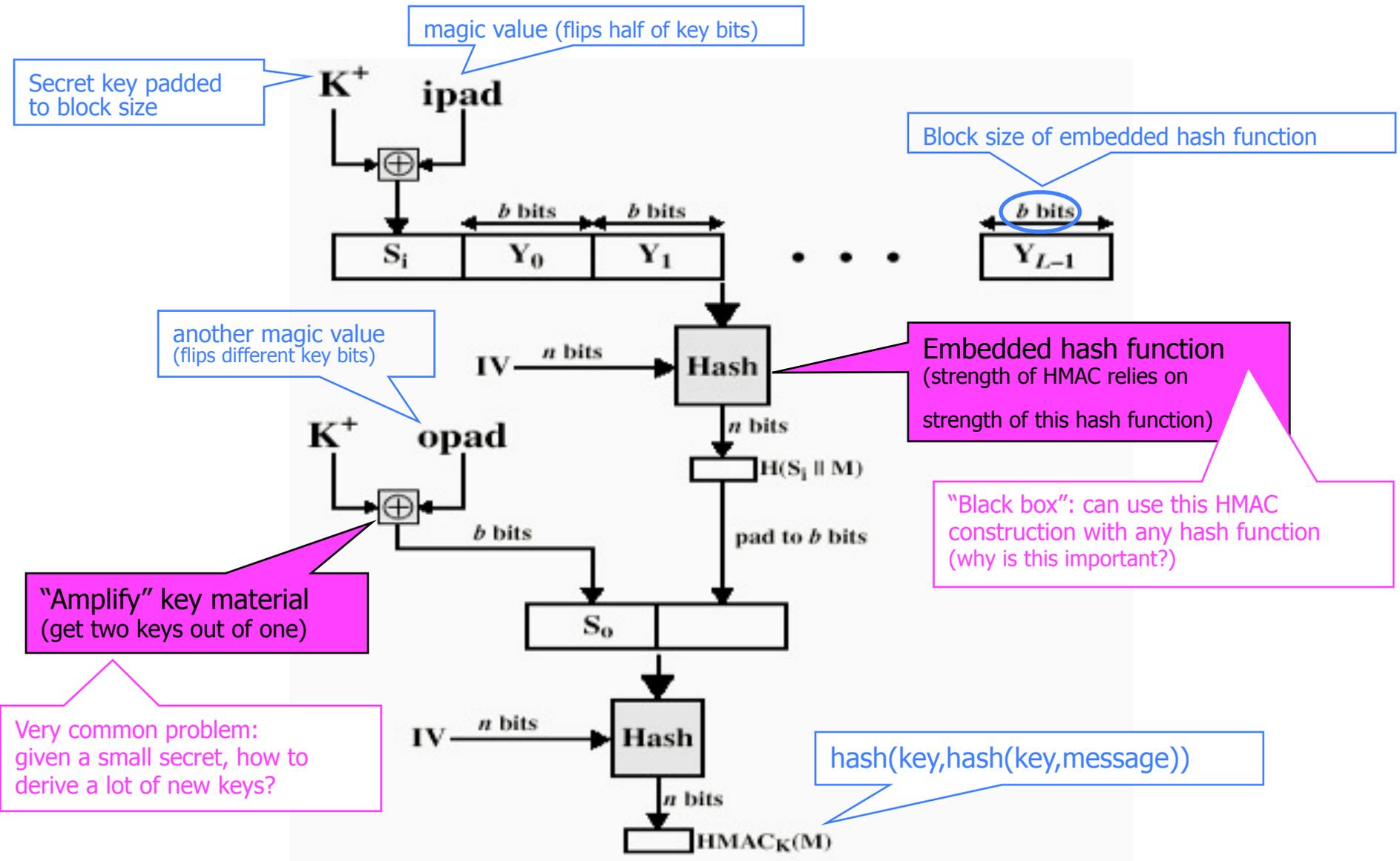
- ◆ Credits: Alexei Czeskis, Karl Koscher, Batya Friedman

# HMAC

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- ◆ Construct MAC by applying a cryptographic hash function to message and key
- ◆ Invented by Bellare, Canetti, and Krawczyk (1996)
- ◆ Mandatory for IP security, also used in SSL/TLS

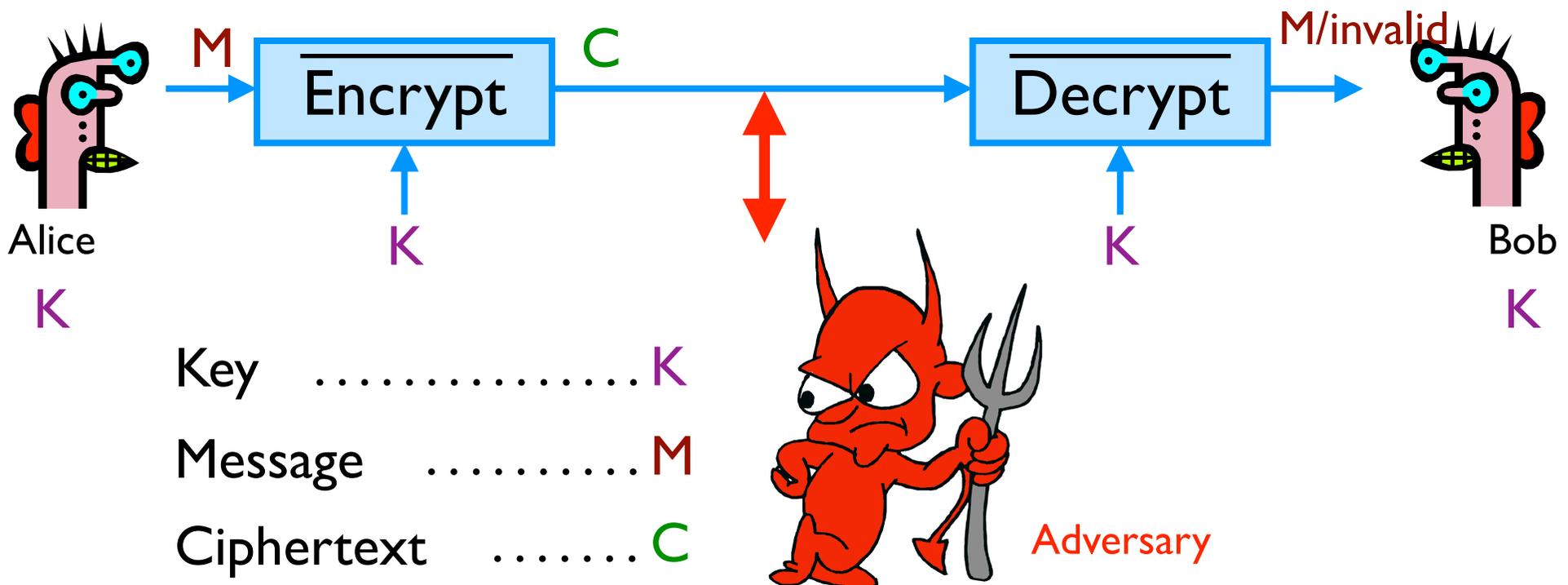
# Structure of HMAC



# Achieving Both Privacy and Integrity

## Authenticated encryption scheme

Recall: Often desire both privacy and integrity. (For SSH, SSL, IPsec, etc.)



# Some subtleties! Encrypt-and-MAC

---

Natural approach for authenticated encryption: Combine an encryption scheme and a MAC.

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$$\bar{E}_{K_e, K_m}$$
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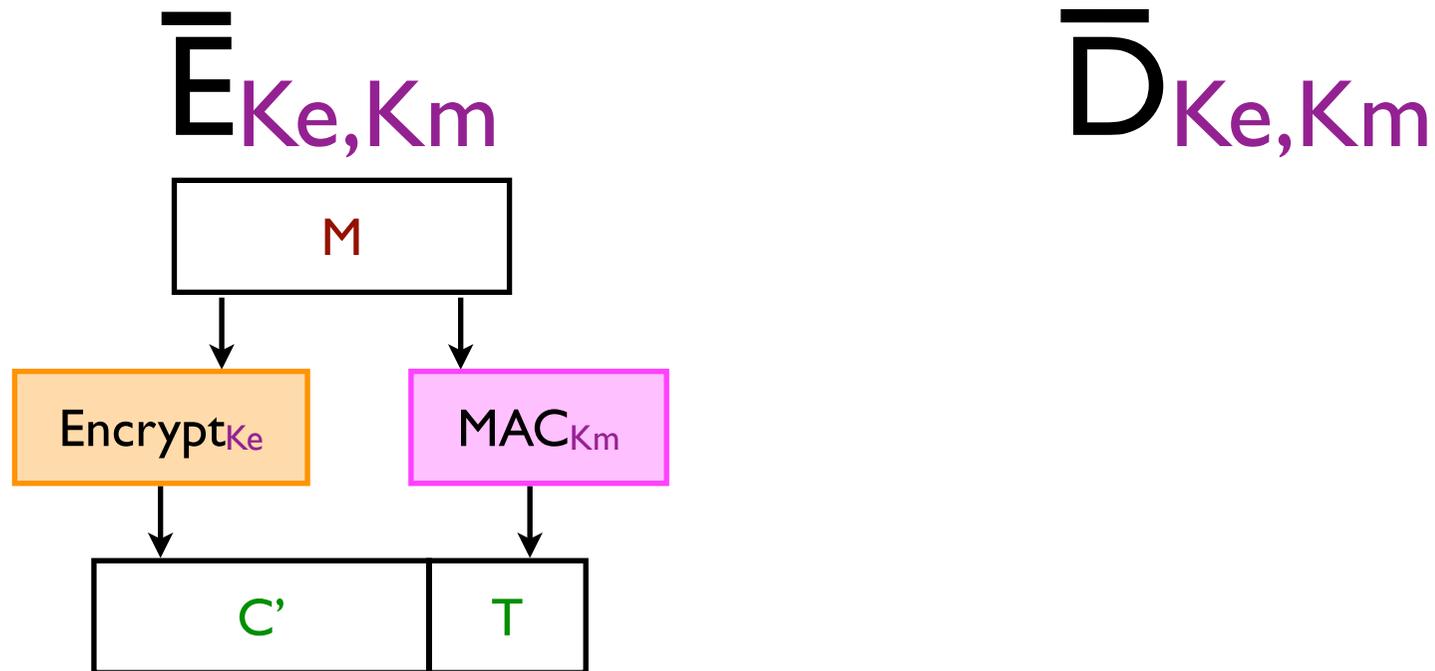
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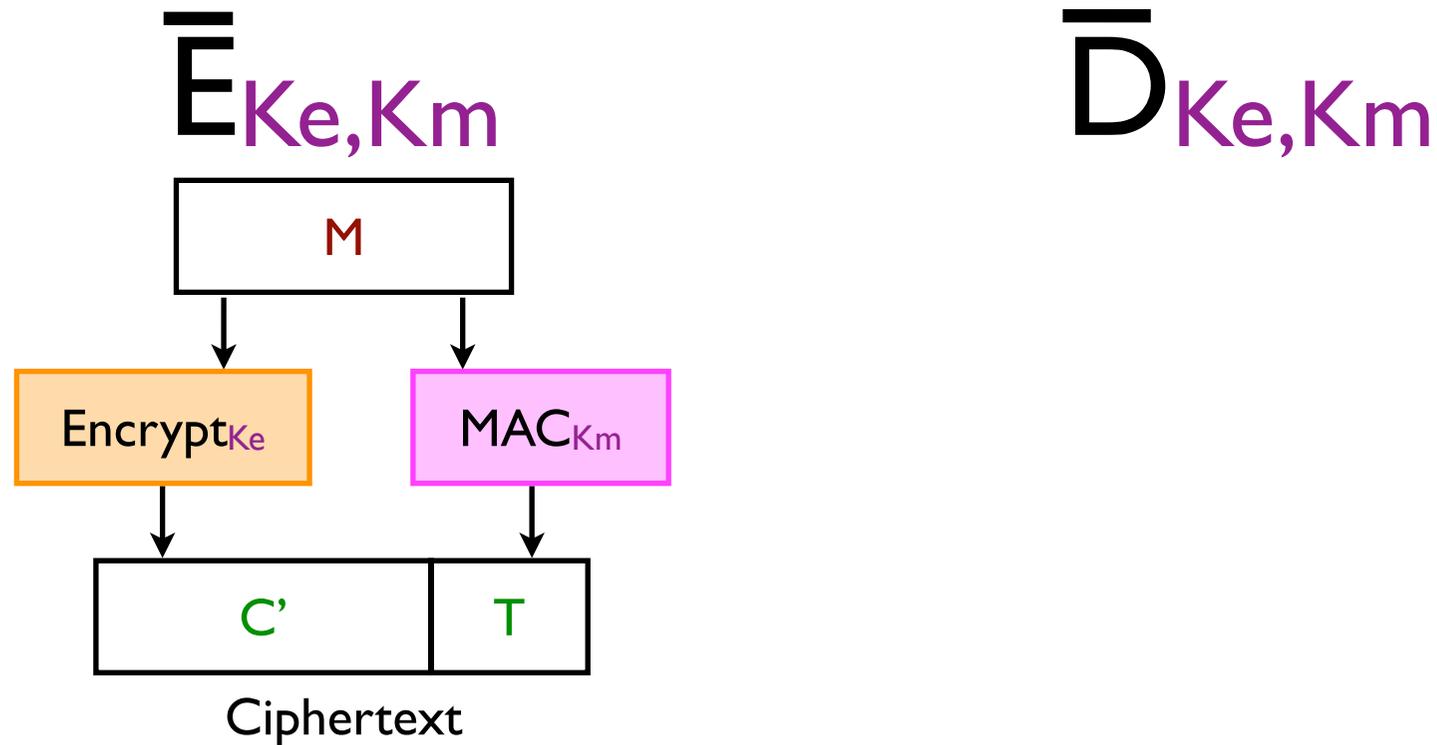
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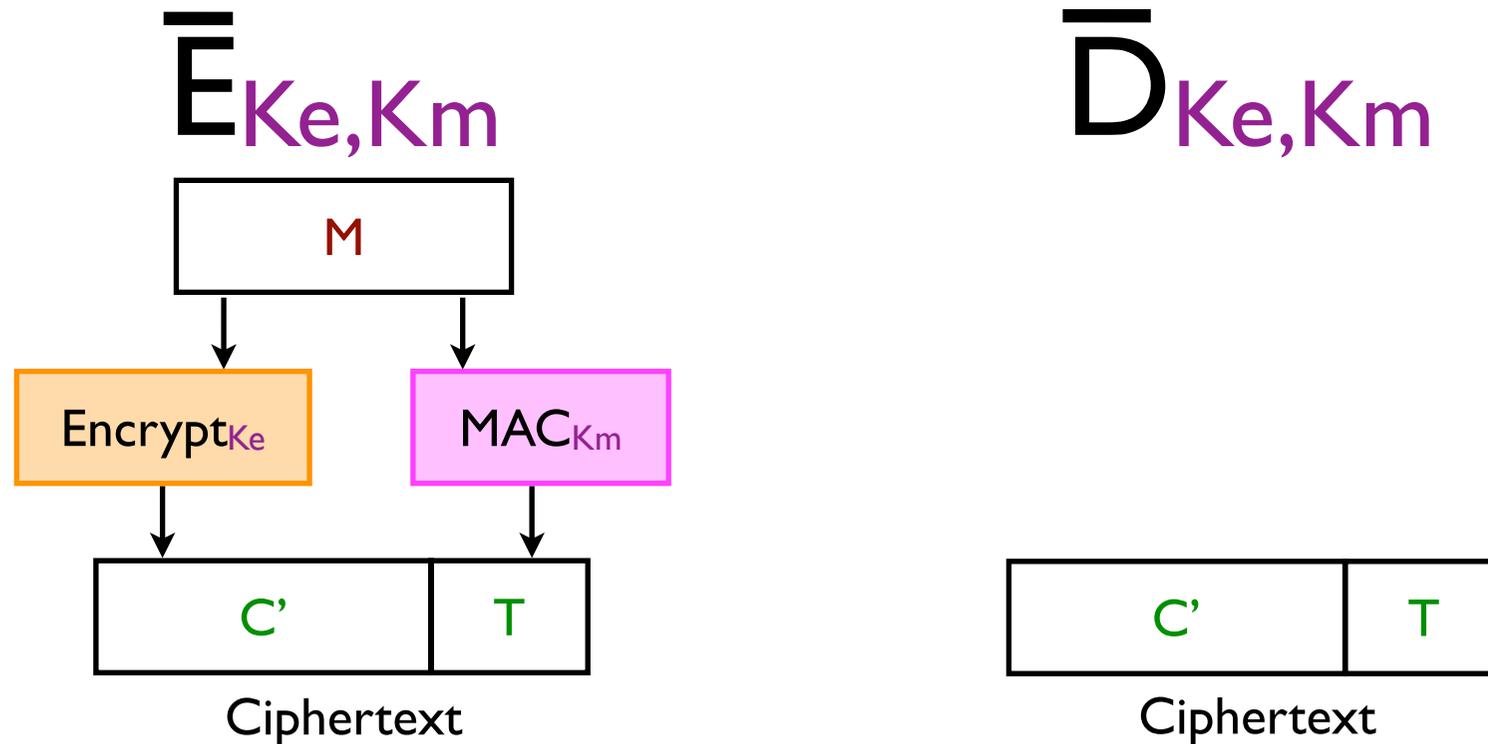
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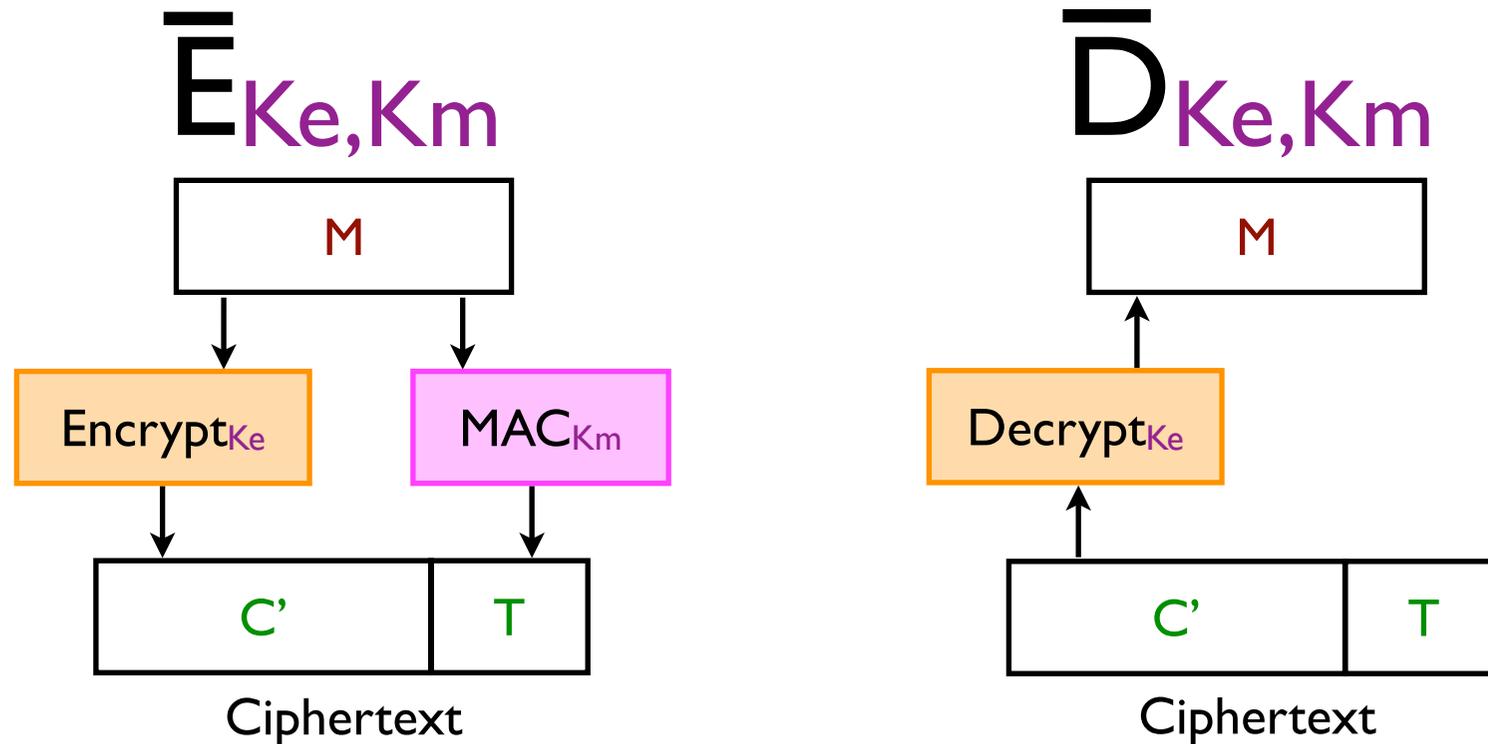
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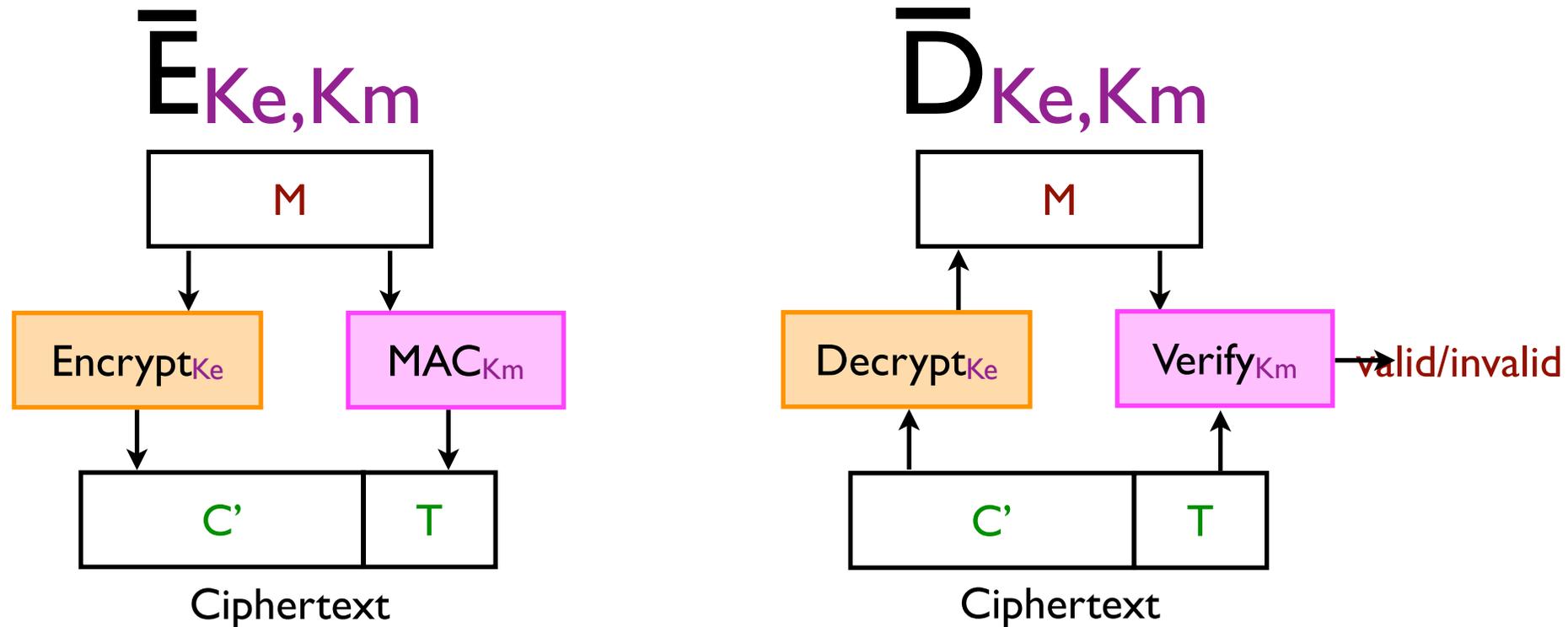
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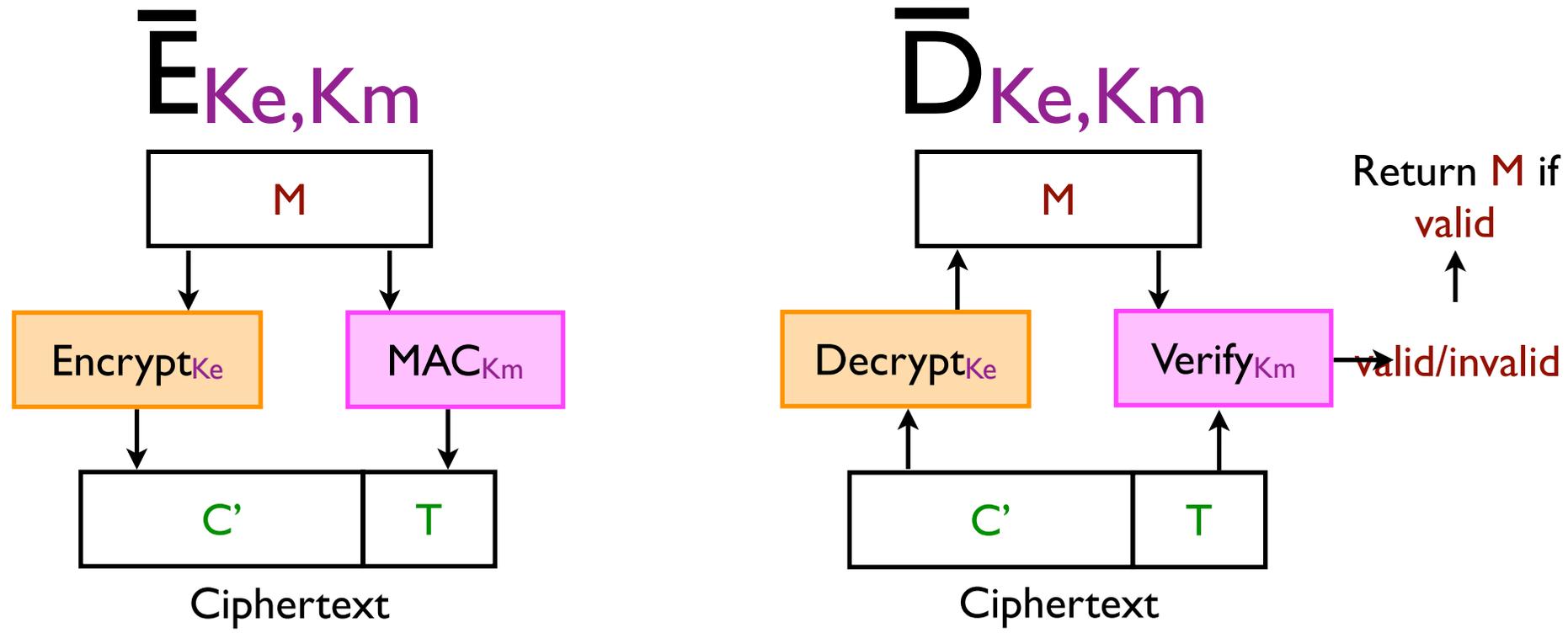
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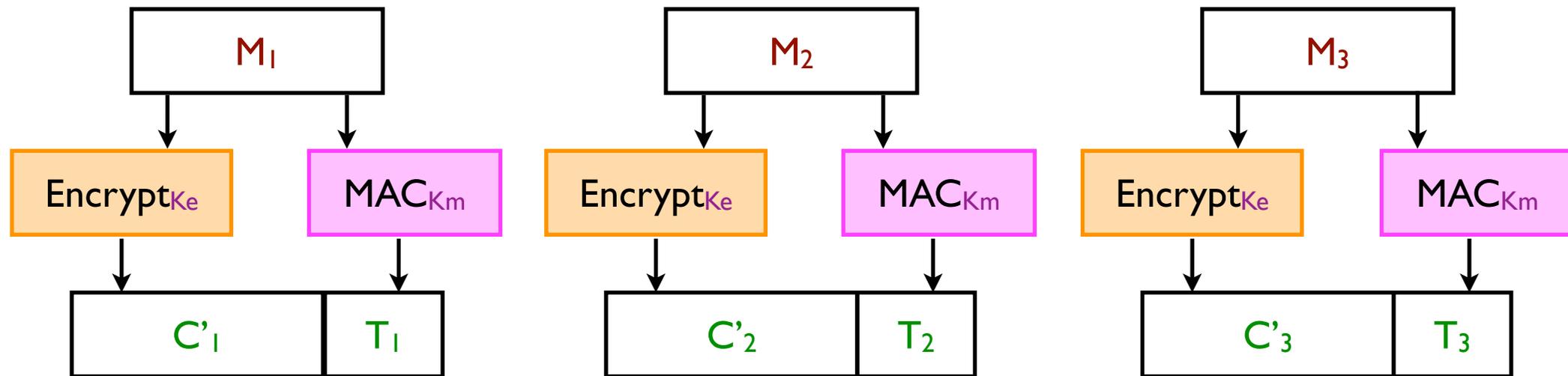
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# But insecure! [BN, Kra]

Assume Alice sends messages:



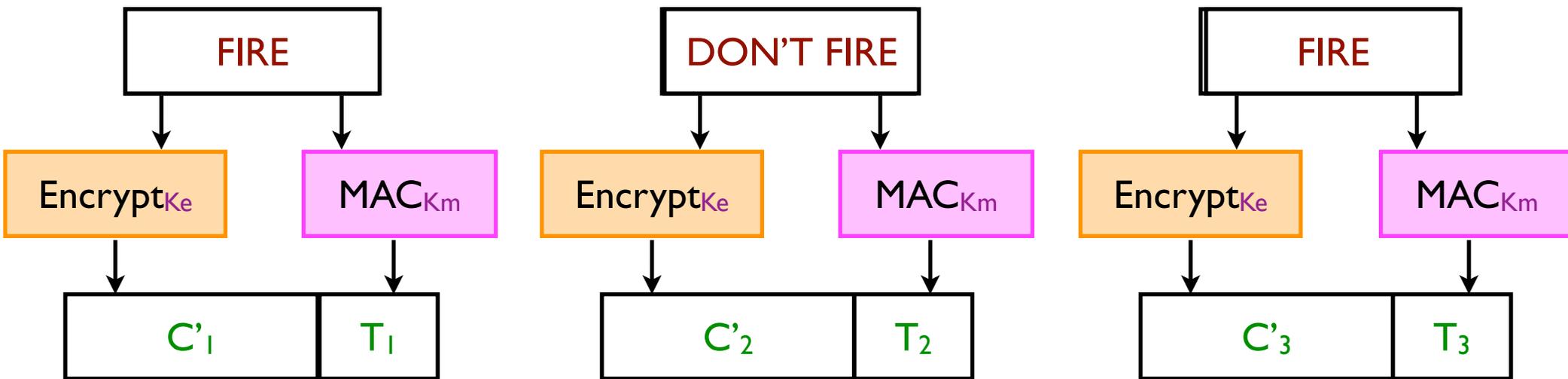
If  $T_i = T_j$  then  $M_i = M_j$

Adversary learns whether two plaintexts are equal.

Especially problematic when  $M_1, M_2, \dots$  take on only a small number of possible values.

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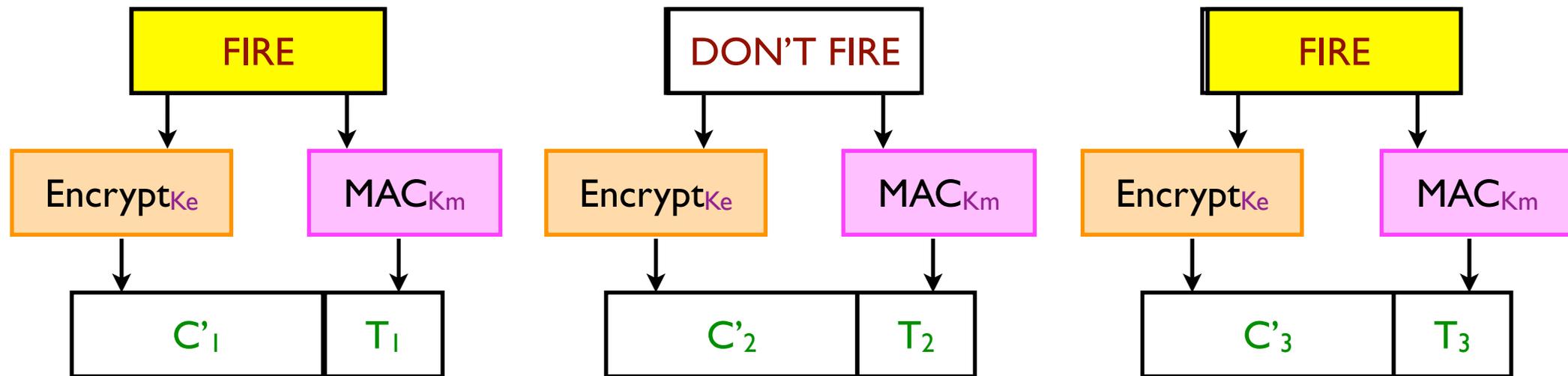
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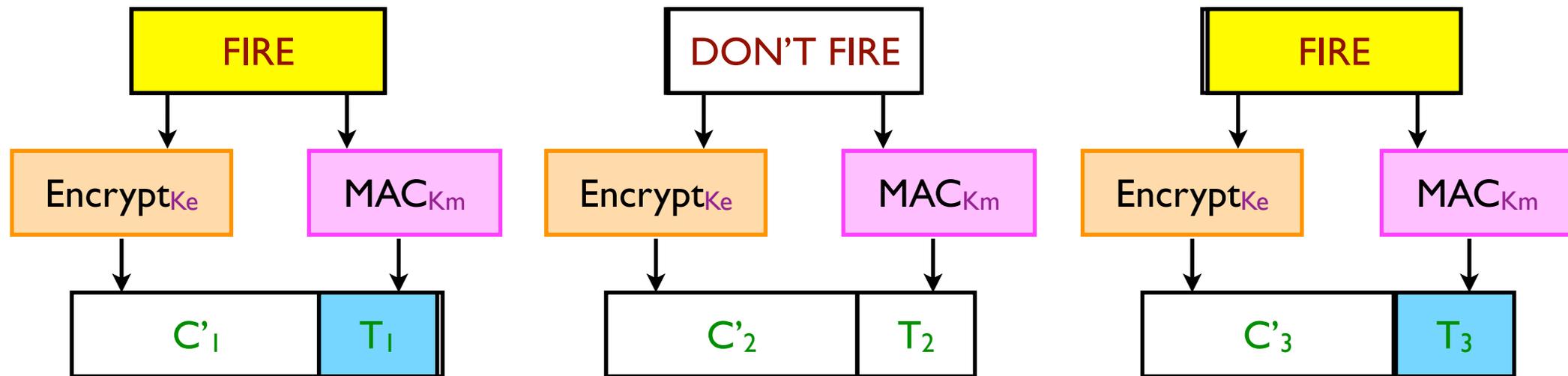
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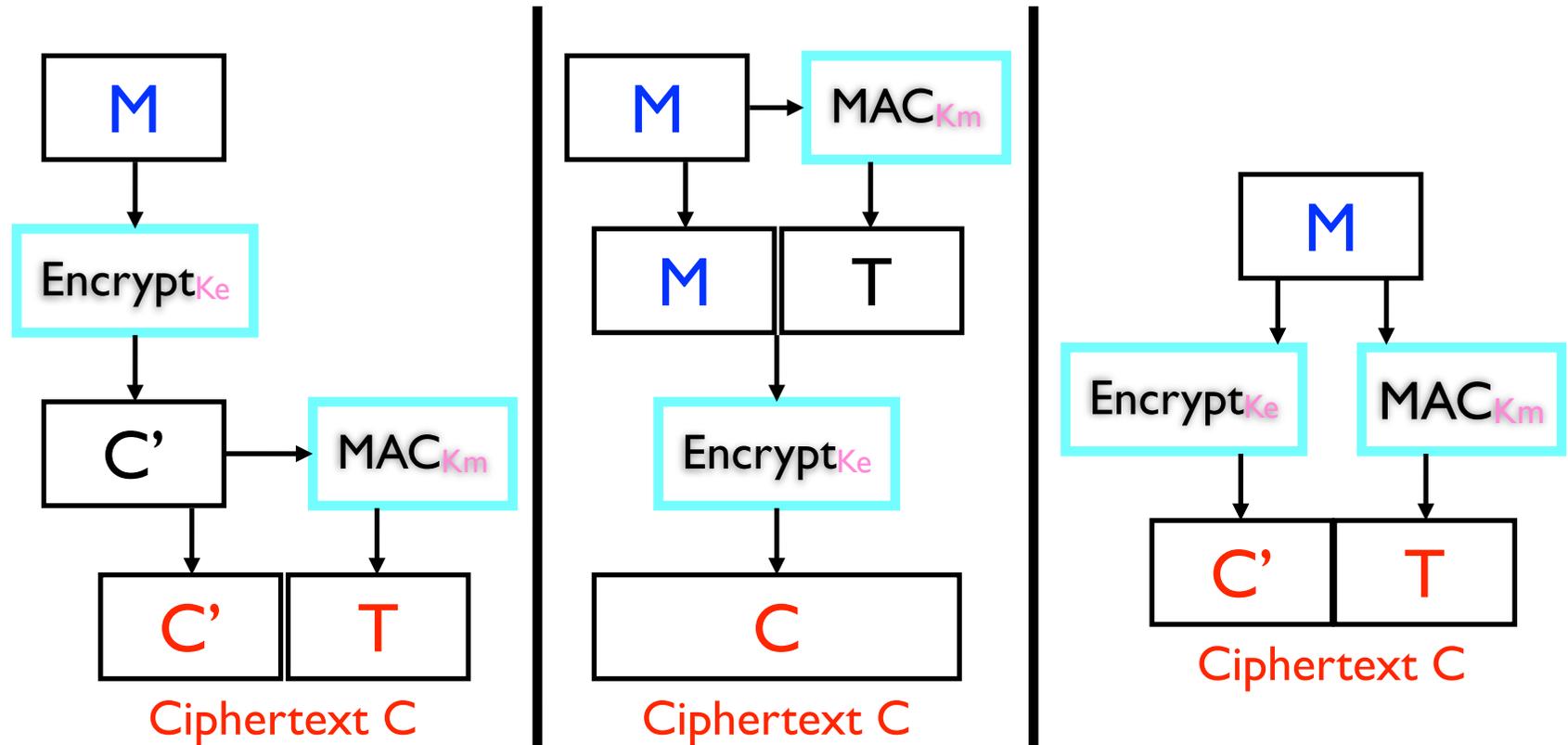


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# Results of [BN00, Kra01]



Encrypt-then-MAC

MAC-then-Encrypt

Encrypt-and-MAC

Privacy

Strong (CCA)

Weak (CPA)

Insecure

Integrity

Strong (CTXT)

Weak (PTXT)

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