CSE 484 (Winter 2010)

Asymmetric Cryptography

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Thanks to Dan Boneh, Dieter Gollmann, John Manferdelli, John Mitchell, Vitaly Shmatikov, Bennet Yee, and many others for sample slides and materials ...

Goals for Today

- Asymmetric Cryptography

Last Time

RSA Cryptosystem

[Rivest, Shamir, Adleman 1977]

- Key generation:
  - Generate large primes p, q
    - Say, 1024 bits each (need primality testing, too)
  - Compute $n=pq$ and $\varphi(n)=(p-1)(q-1)$
  - Choose small e, relatively prime to $\varphi(n)$
    - Typically, $e=3$ or $e=2^{16}+1=65537$ (why?)
  - Compute unique d such that $ed = 1 \bmod \varphi(n)$
  - Public key = (e,n); private key = (d,n)
- Encryption of m: $c = m^e \bmod n$
  - Modular exponentiation by repeated squaring
- Decryption of c: $c^d \bmod n = (m^e)^d \bmod n = m$

On PK encryption

- Encrypted message needs to be in interpreted as an integer less than n
  - Reason: Otherwise can't decrypt.
  - Message is very often a symmetric encryption key.

Last Time

Caveats

- e =3 is a common exponent
  - If $m < n^{1/3}$, then $c = m^3 < n$ and can just take the cube root of c to recover m
    - Even problems if “pad” m in some ways [Hastad]
  - Let $c_i = m^3 \bmod n_i$ - same message is encrypted to three people
    - Adversary can compute $m^3 \bmod n_1 n_2 n_3$ (using CRT)
    - Then take ordinary cube root to recover m
- Don’t use RSA directly for privacy!

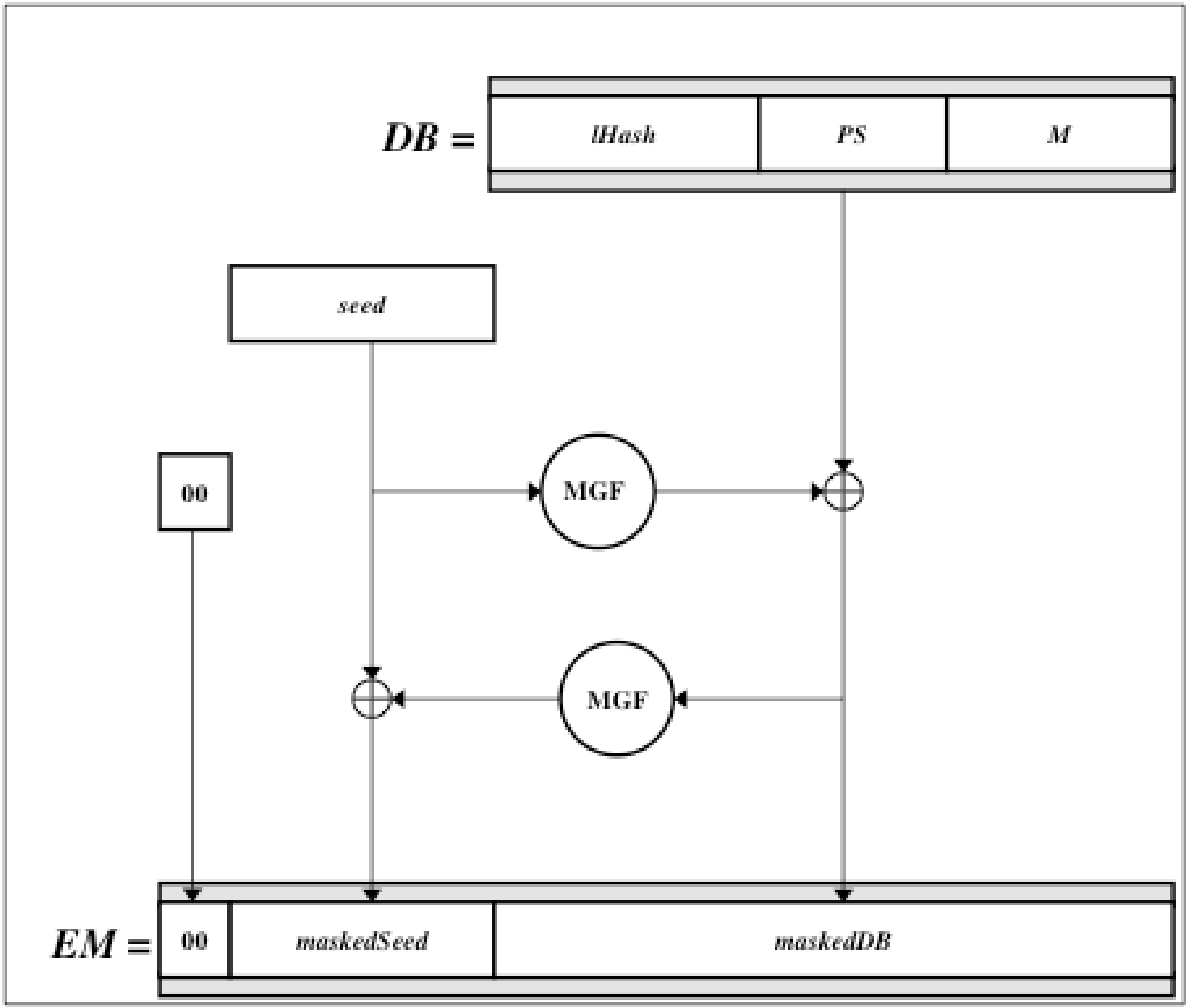
Last Time

Integrity in RSA Encryption

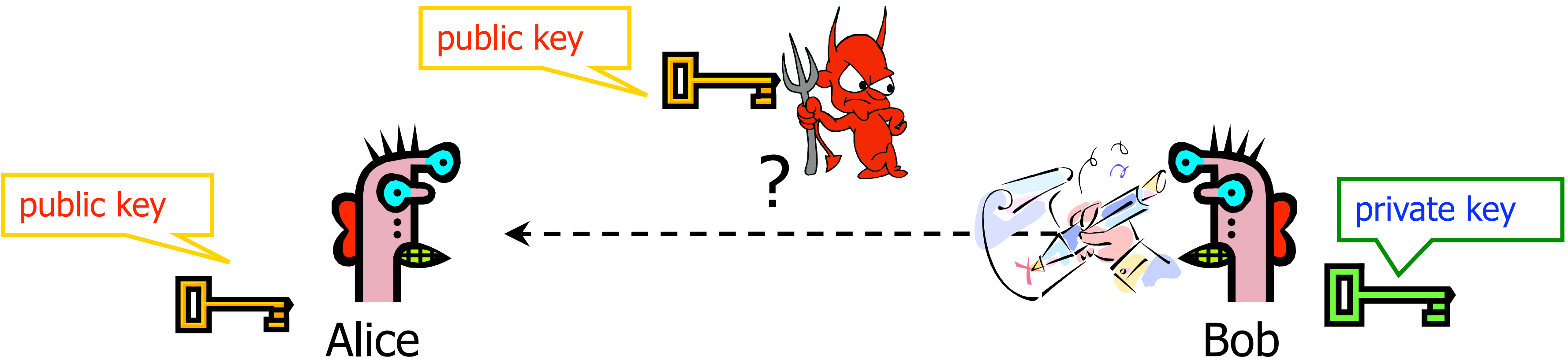
- Plain RSA does not provide integrity
  - Given encryptions of m_1 and m_2, attacker can create encryption of $m_1 \cdot m_2$
    - $(m_1^e) \cdot (m_2^e) \bmod n = (m_1 \cdot m_2)^e \bmod n$
  - Attacker can convert m into m^k without decrypting
    - $(m_1^e)^k \bmod n = (m^k)^e \bmod n$
- In practice, OAEP is used: instead of encrypting M, encrypt $M \oplus G(r) \,;\, r \oplus H(M \oplus G(r))$
  - r is random and fresh, G and H are hash functions
  - Resulting encryption is plaintext-aware: infeasible to compute a valid encryption without knowing plaintext
    - … if hash functions are "good" and RSA problem is hard

Last Time

OAEP (image from PKCS #1 v2.1)



Digital Signatures: Basic Idea



Given: Everybody knows Bob's public key

Only Bob knows the corresponding private key

Goal: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, enough to know the public key

RSA Signatures

- Public key is (n,e), private key is d
- To sign message m: $s = m^d \bmod n$
  - Signing and decryption are the same **underlying** operation in RSA
  - It's infeasible to compute s on m if you don't know d
- To verify signature s on message m:

  $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption
  - Anyone who knows n and e (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing
  - Standard padding/hashing schemes exist for RSA signatures

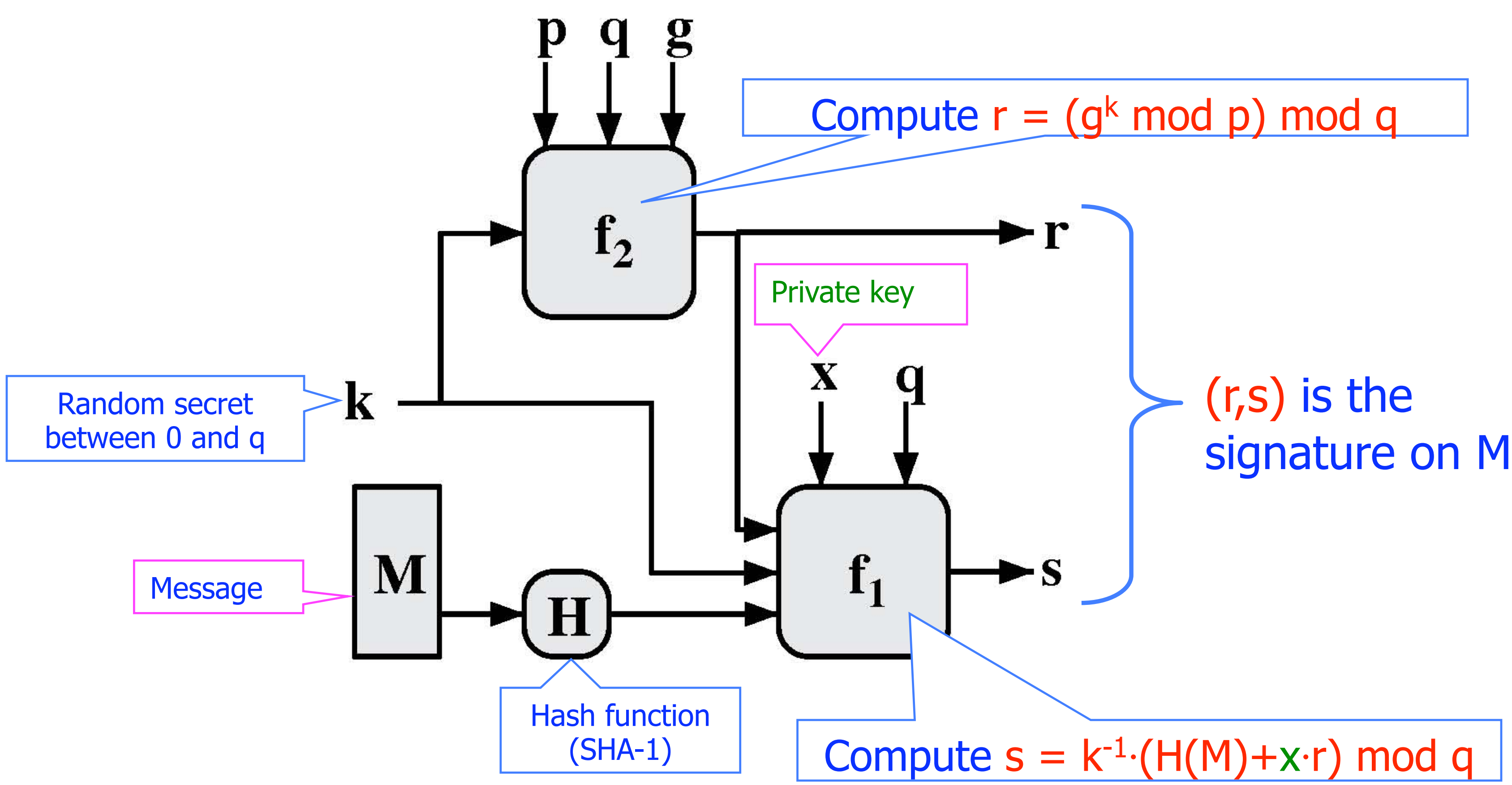
Encryption and Signatures

- Often people think: Encryption and decryption are inverses.
- That's a common view
  - True for the RSA **primitive (underlying component)**
- But not one we'll take
  - To really use RSA, we need padding
  - And there are many other decryption methods

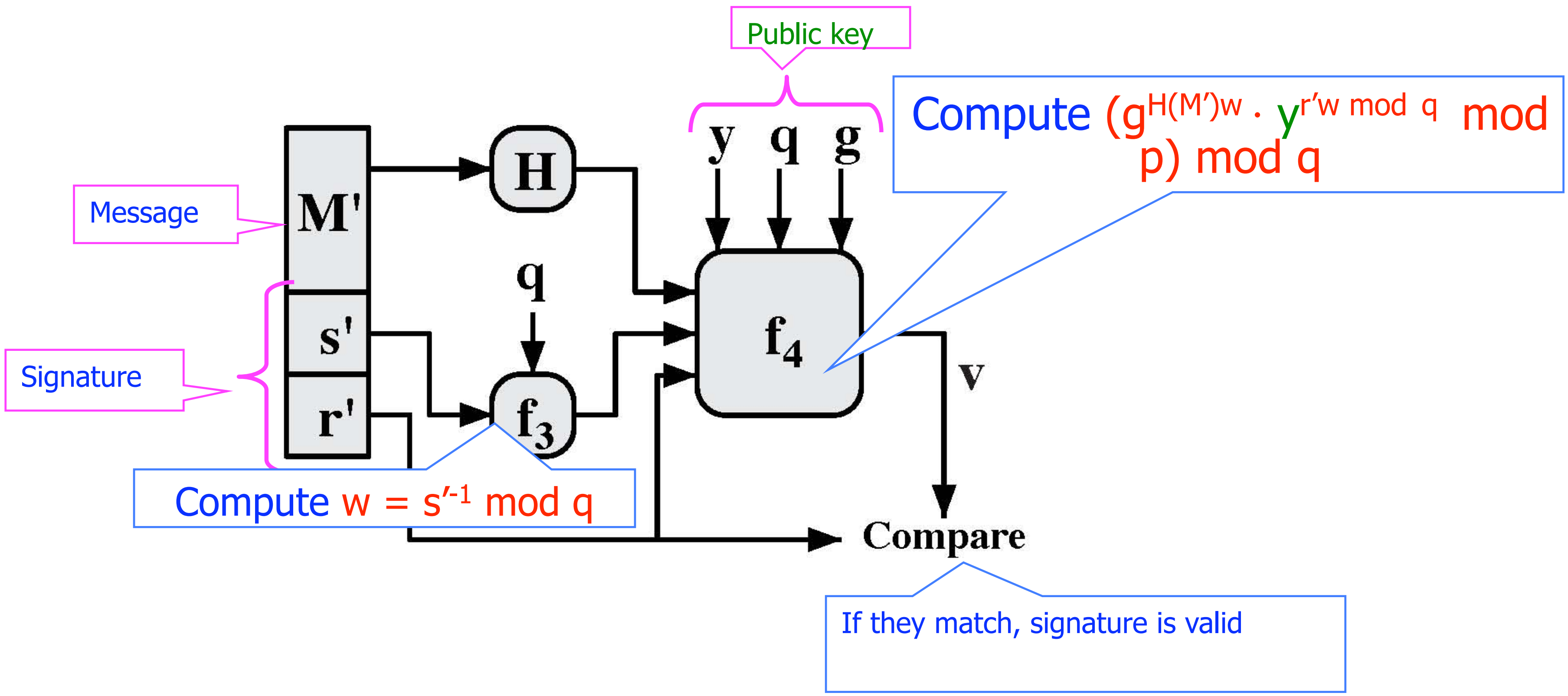
Digital Signature Standard (DSS)

- U.S. government standard (1991-94)
  - Modification of the ElGamal signature scheme (1985)
- Key generation:
  - Generate large primes p, q such that q divides p-1
    - $2^{159} < q < 2^{160}$, $2^{511+64t} < p < 2^{512+64t}$ where $0 \leq t \leq 8$
  - Select $h \in Z_p^*$ and compute $g = h^{(p-1)/q} \bmod p$
  - Select random x such $1 \leq x \leq q-1$, compute $y = g^x \bmod p$
- Public key: $(p, q, g, y = g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from $g^x \bmod p$ (public key)

DSS: Signing a Message (Skim)



DSS: Verifying a Signature (Skim)



Why DSS Verification Works (Skim)

- If (r,s) is a legitimate signature, then

  $r = (g^k \text{ mod } p) \text{ mod } q$; $s = k^{-1} \cdot (H(M)+x \cdot r) \text{ mod } q$

- Thus $H(M) = -x \cdot r + k \cdot s \text{ mod } q$
  - Multiply both sides by $w = s^{-1} \text{ mod } q$
- $H(M) \cdot w + x \cdot r \cdot w = k \text{ mod } q$
  - Exponentiate g to both sides
- $(g^{H(M) \cdot w + x \cdot r \cdot w} = g^k) \text{ mod } p \text{ mod } q$
  - In a valid signature, $g^k \text{ mod } p \text{ mod } q = r$, $g^x \text{ mod } p = y$
- Verify $g^{H(M) \cdot w} \cdot y^{r \cdot w} = r \text{ mod } p \text{ mod } q$

Security of DSS

- Can’t create a valid signature without private key
- Given a signature, hard to recover private key
- Can’t change or tamper with signed message
- If the same message is signed twice, signatures are different
  - Each signature is based in part on random secret k
- Secret k must be different for each signature!
  - If k is leaked or if two messages re-use the same k, attacker can recover secret key x and forge any signature from then on
  - Example problem scenario: rebooted VMs; restarted embedded machines

Advantages of Public-Key Crypto

- Confidentiality without shared secrets
  - Very useful in open environments
  - No "chicken-and-egg" key establishment problem
    - With symmetric crypto, two parties must share a secret before they can exchange secret messages
    - Caveats to come
- Authentication without shared secrets
  - Use digital signatures to prove the origin of messages
- Reduce protection of information to protection of authenticity of public keys
  - No need to keep public keys secret, but must be sure that Alice's public key is really her true public key

Disadvantages of Public-Key Crypto

- Calculations are 2-3 orders of magnitude slower
  - Modular exponentiation is an expensive computation
  - Typical usage: use public-key cryptography to establish a shared secret, then switch to symmetric crypto
    - E.g., IPsec, SSL, SSH, ...
- Keys are longer
  - 1024+ bits (RSA) rather than 128 bits (AES)
- Relies on unproven number-theoretic assumptions
  - What if factoring is easy?
    - Factoring is believed to be neither P, nor NP-complete
  - (Of course, symmetric crypto also rests on unproven assumptions)

Exponentiation

- How to compute $M^x \bmod N$?
- Say, x = 13
- Sums of power of 2, $x = 8+4+1 = 2^3+2^2+2^0$
- Can also write x in binary, e.g., x = 1101
- Can solve by repeated squaring
  - y = 1;
  - $y = y^2 * M \bmod N$ // y = M
  - $y = y^2 * M \bmod N$ // $y = M^2 * M = M^{2+1} = M^3$
  - $y = y^2 \bmod N$ // $y = (M^3)^2 = M^6$
  - $y = y^2 * M \bmod N$ // $y = (M^6)^2 * M = M^{12+1} = M^{13} = M^x$

Timing attacks

Collect timings for exponentiation with a bunch of messages M1, M2, ... (e.g., RSA signing operations with a private exponent)

Assume (inductively) know $b_3=1$, $b_2=1$, guess $b_1=1$

i	$b_i = 0$	$b_i = 1$	Comp	Meas
3	$y = y^2 \bmod N$	$y = y^2 * M1 \bmod N$		
2	$y = y^2 \bmod N$	$y = y^2 * M1 \bmod N$		
1	$y = y^2 \bmod N$	$y = y^2 * M1 \bmod N$	X1 secs	
0	$y = y^2 \bmod N$	$y = y^2 * M1 \bmod N$		Y1 secs

i	$b_i = 0$	$b_i = 1$	Comp	Meas
3	$y = y^2 \bmod N$	$y = y^2 * M2 \bmod N$		
2	$y = y^2 \bmod N$	$y = y^2 * M2 \bmod N$		
1	$y = y^2 \bmod N$	$y = y^2 * M2 \bmod N$	X2 secs	
0	$y = y^2 \bmod N$	$y = y^2 * M2 \bmod N$		Y2 secs

Timing attacks

- If b_1 = 1, then set of { Yj - Xj | j in {1,2, ..} } has distribution with "small" variance (due to time for final step, i=0)
  - "Guess" was correct when we computed X1, X2, ...
- If b_1 = 0, then set of { Yj - Xj | j in {1,2, ..} } has distribution with "large" variance (due to time for final step, i=0, and incorrect guess for b_1)
  - "Guess" was incorrect when we computed X1, X2, ...
  - So time computation wrong (Xj computed as large, but really small, ...)
- Strategy: Force user to sign large number of messages M1, M2, ... Record timings for signing.
- Iteratively learn bits of key by using above property.