CSE 484 and CSE M 584 (Winter 2009)

Networks (missed material)
Public key cryptography

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Intrusion Detection

Intrusion Detection Systems

- Advantage: can recognize new attacks and new versions of old attacks
- Disadvantages
  - High false positive rate
  - Must be trained on known good data
    - Training is hard because network traffic is very diverse
  - Protocols are finite-state machines, but current state of a connection is difficult to see from the network
  - Definition of “normal” constantly evolves
    - What’s the difference between a flash crowd and a denial of service attack?

Intrusion Detection Problems

- Lack of training data with real attacks
  - But lots of "normal" network traffic, system call data
- Data drift
  - Statistical methods detect changes in behavior
  - Attacker can attack gradually and incrementally
- Main characteristics not well understood
  - By many measures, attack may be within bounds of "normal" range of activities
- False identifications are very costly
  - Sysadm will spend many hours examining evidence

Intrusion Detection Errors

- False negatives: attack is not detected
  - Big problem in signature-based misuse detection
- False positives: harmless behavior is classified as an attack
  - Big problem in statistical anomaly detection
- Both types of IDS suffer from both error types
- Which is a bigger problem?
  - Attacks are fairly rare events

Conditional Probability

- Suppose two events A and B occur with probability Pr(A) and Pr(B), respectively
- Let Pr(AB) be probability that both A and B occur
- What is the conditional probability that A occurs assuming B has occurred?

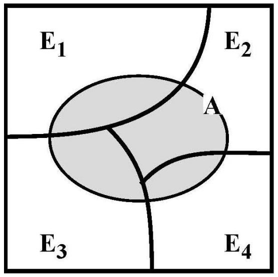
$$\Pr(A \mid B) = \frac{\Pr(AB)}{\Pr(B)}$$

Bayes' Theorem

- Suppose mutually exclusive events $E_1, \ldots, E_n$ together cover the entire set of possibilities
- Then probability of any event A occurring is

$$\Pr(A) = \Sigma_{1 \le i \le n} \Pr(A \mid E_i) \bullet \Pr(E_i)$$

  - Intuition: since $E_1, \ldots, E_n$ cover entire probability space, whenever A occurs, some event E_i must have occurred



- Can rewrite this formula as

$$\Pr(E_i \mid A) = \frac{\Pr(A \mid E_i) \bullet \Pr(E_i)}{\Pr(A)}$$

Base-Rate Fallacy

- 1% of traffic is SYN floods; IDS accuracy is 90%
  - IDS classifies a SYN flood as attack with prob. 90%, classifies a valid connection as attack with prob. 10%
- What is the probability that a connection flagged by IDS as a SYN flood is actually valid traffic?

$$\Pr(\text{valid} \mid \text{alarm}) = \frac{\Pr(\text{alarm} \mid \text{valid}) \cdot \Pr(\text{valid})}{\Pr(\text{alarm})}$$

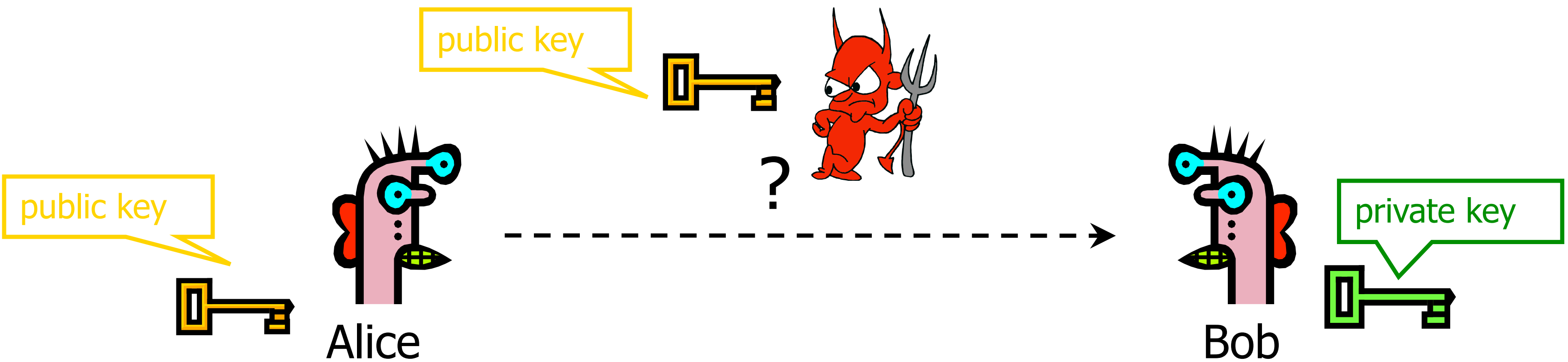
$$= \frac{\Pr(\text{alarm} \mid \text{valid}) \cdot \Pr(\text{valid})}{\Pr(\text{alarm} \mid \text{valid}) \cdot \Pr(\text{valid}) + \Pr(\text{alarm} \mid \text{SYN flood}) \cdot \Pr(\text{SYN flood})}$$

$$= \frac{0.10 \cdot 0.99}{0.10 \cdot 0.99 + 0.90 \cdot 0.01}$$

= 92% chance raised alarm is false!!!

Public Key Cryptography

Basic Problem



Given: Everybody knows Bob's public key
Only Bob knows the corresponding private key

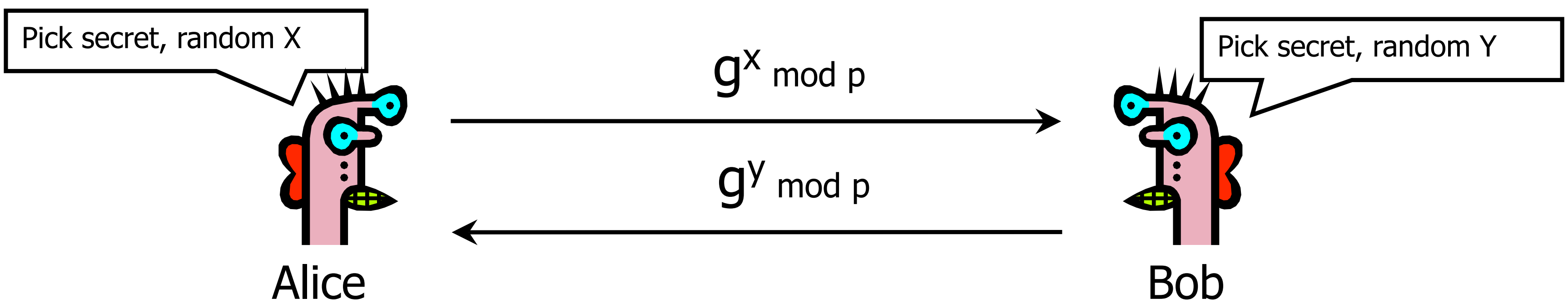
Goals: 1. Alice wants to send a secret message to Bob
2. Bob wants to authenticate himself

Applications of Public-Key Crypto

- Encryption for confidentiality
  - Anyone can encrypt a message
    - With symmetric crypto, must know secret key to encrypt
  - Only someone who knows private key can decrypt
  - Key management is simpler (maybe)
    - Secret is stored only at one site: good for open environments
- Digital signatures for authentication
  - Can "sign" a message with your private key
- Session key establishment
  - Exchange messages to create a secret session key
  - Then switch to symmetric cryptography (why?)

Diffie-Hellman Protocol (1976)

- Alice and Bob never met and share no secrets
- Public info: p and g
  - p is a large prime number, g is a generator of Z_p^*
    - $Z_p^*=\{1, 2 \ldots p-1\}$; $\forall a \in Z_p^*\ \exists i$ such that $a=g^i \bmod p$
    - Modular arithmetic: numbers “wrap around” after they reach p



Compute $k=(g^y)^x=g^{xy} \bmod p$

Compute $k=(g^x)^y=g^{xy} \bmod p$

Why Is Diffie-Hellman Secure?

- **Discrete Logarithm (DL) problem:**

  given g^x mod p, it's hard to extract x
  - There is no known efficient algorithm for doing this
  - This is not enough for Diffie-Hellman to be secure!
- **Computational Diffie-Hellman (CDH) problem:**

  given g^x and g^y, it's hard to compute g^{xy} mod p
  - … unless you know x or y, in which case it's easy
- **Decisional Diffie-Hellman (DDH) problem:**

  given g^x and g^y, it's hard to tell the difference between g^{xy} mod p and g^r mod p where r is random

Properties of Diffie-Hellman

- Assuming DDH problem is hard, Diffie-Hellman protocol is a secure key establishment protocol against <u>passive</u> attackers
  - Eavesdropper can't tell the difference between established key and a random value
  - Can use new key for symmetric cryptography
    - Approx. 1000 times faster than modular exponentiation
- Diffie-Hellman protocol (by itself) does not provide authentication
- DDH: not true for integers mod p, but true for other groups

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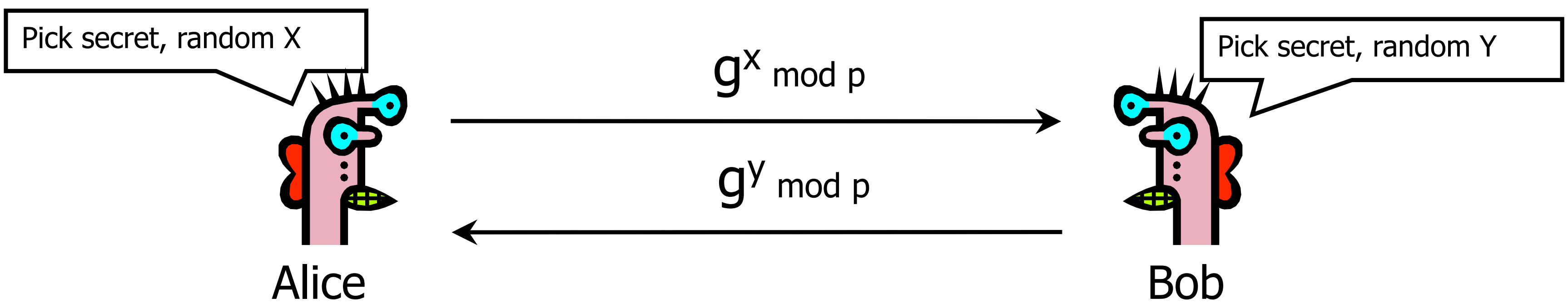


Compute $k=H((g^{yx}))=H(g^{xy}) \bmod p$

Compute $k=H((g^x)^y)=H(g^{xy}) \bmod p$

Requirements for Public-Key Crypto

- **Key generation:** computationally easy to generate a pair (public key PK, private key SK)
  - Computationally infeasible to determine private key SK given only public key PK
- **Encryption:** given plaintext M and public key PK, easy to compute ciphertext $C=E_{PK}(M)$
- **Decryption:** given ciphertext $C=E_{PK}(M)$ and private key SK, easy to compute plaintext M
  - Infeasible to compute M from C without SK
  - Even infeasible to learn partial information about M
  - Trapdoor function: Decrypt(SK,Encrypt(PK,M))=M

Some Number Theory Facts

- Euler totient function $\varphi(n)$ where $n \geq 1$ is the number of integers in the $[1,n]$ interval that are relatively prime to n
  - Two numbers are relatively prime if their greatest common divisor (gcd) is 1
- Euler's theorem:

  if $a \in Z_n^*$, then $a^{\varphi(n)} = 1 \bmod n$
- Special case: Fermat's Little Theorem

  if p is prime and $\gcd(a,p)=1$, then $a^{p-1} = 1 \bmod p$

RSA Cryptosystem

[Rivest, Shamir, Adleman 1977]

- **Key generation:**
  - Generate large primes p, q
    - Say, 1024 bits each (need primality testing, too)
  - Compute $n=pq$ and $\varphi(n)=(p-1)(q-1)$
  - Choose small e, relatively prime to $\varphi(n)$
    - Typically, $e=3$ or $e=2^{16}+1=65537$ (why?)
  - Compute unique d such that $ed = 1 \bmod \varphi(n)$
  - Public key = (e,n); private key = (d,n)
- **Encryption** of m: $c = m^e \bmod n$
  - Modular exponentiation by repeated squaring
- **Decryption** of c: $c^d \bmod n = (m^e)^d \bmod n = m$

Why RSA Decryption Works

- $e \cdot d = 1 \bmod \varphi(n)$
- Thus $e \cdot d = 1 + k \cdot \varphi(n) = 1 + k(p-1)(q-1)$ for some k

- Let m be any integer in Z_n
- If $\gcd(m,p)=1$, then $m^{ed} = m \bmod p$
  - By Fermat's Little Theorem, $m^{p-1} = 1 \bmod p$
  - Raise both sides to the power $k(q-1)$ and multiply by m
  - $m^{1+k(p-1)(q-1)} = m \bmod p$, thus $m^{ed} = m \bmod p$
  - By the same argument, $m^{ed} = m \bmod q$
- Since p and q are distinct primes and $p \cdot q = n$,

  $m^{ed} = m \bmod n$

Why Is RSA Secure?

- RSA problem: given $n=pq$, e such that $\gcd(e,(p-1)(q-1))=1$ and c, find m such that $m^e=c \bmod n$
  - i.e., recover m from ciphertext c and public key (n,e) by taking e^{th} root of c
  - There is no known efficient algorithm for doing this
- Factoring problem: given positive integer n, find primes $p_1, \ldots, p_k$ such that $n=p_1^{e_1}p_2^{e_2}\ldots p_k^{e_k}$
- If factoring is easy, then RSA problem is easy, but there is no known reduction from factoring to RSA
  - It may be possible to break RSA without factoring n

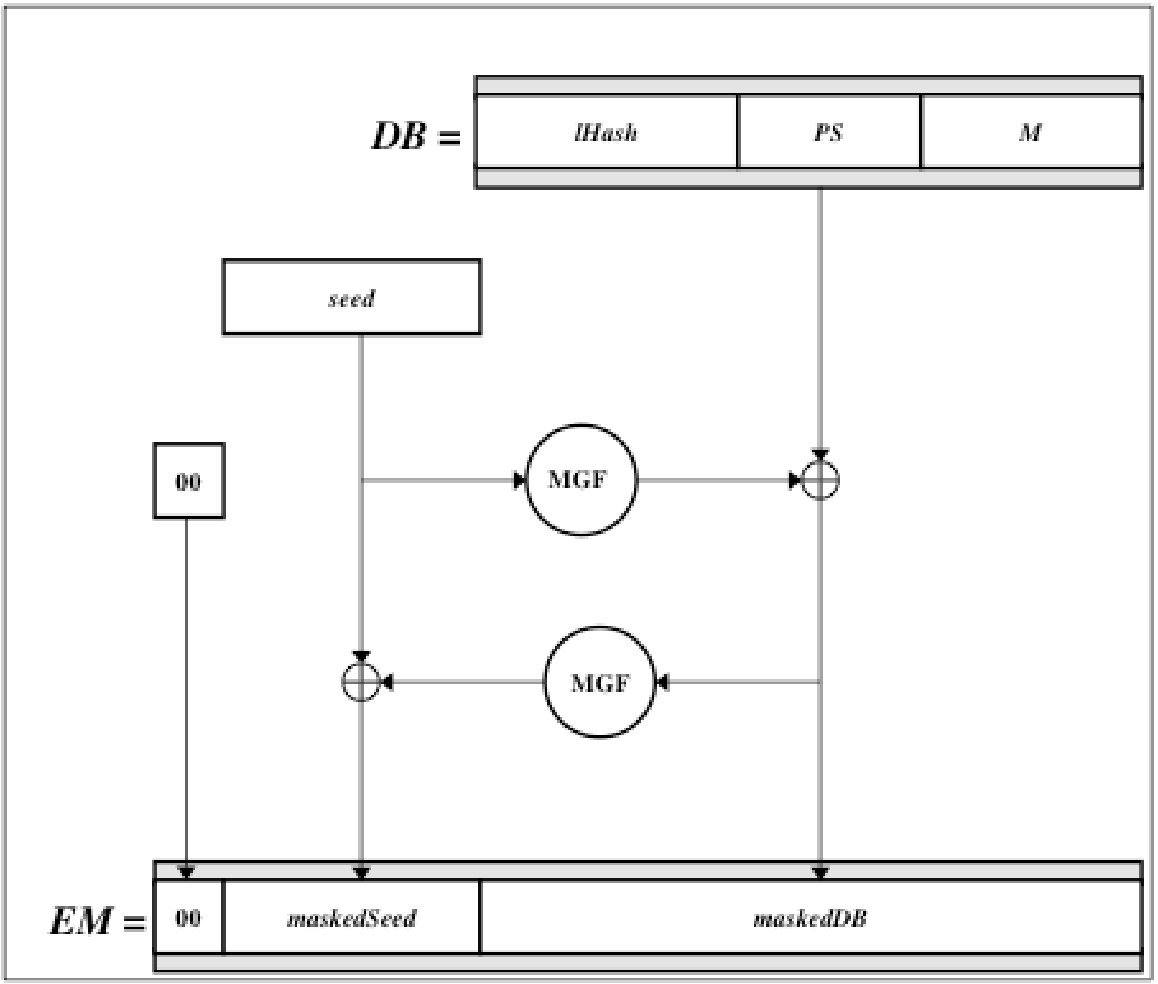
Caveats

- e =3 is a common exponent
  - If $m < n^{1/3}$, then $c = m^3 < n$ and can just take the cube root of c to recover m
    - Even problems if “pad” m in some ways [Hastad]
  - Let $c_i = m^3 \bmod n_i$ - same message is encrypted to three people
    - Adversary can compute $m^3 \bmod n_1 n_2 n_3$ (using CRT)
    - Then take ordinary cube root to recover m
- Don’t use RSA directly for privacy!

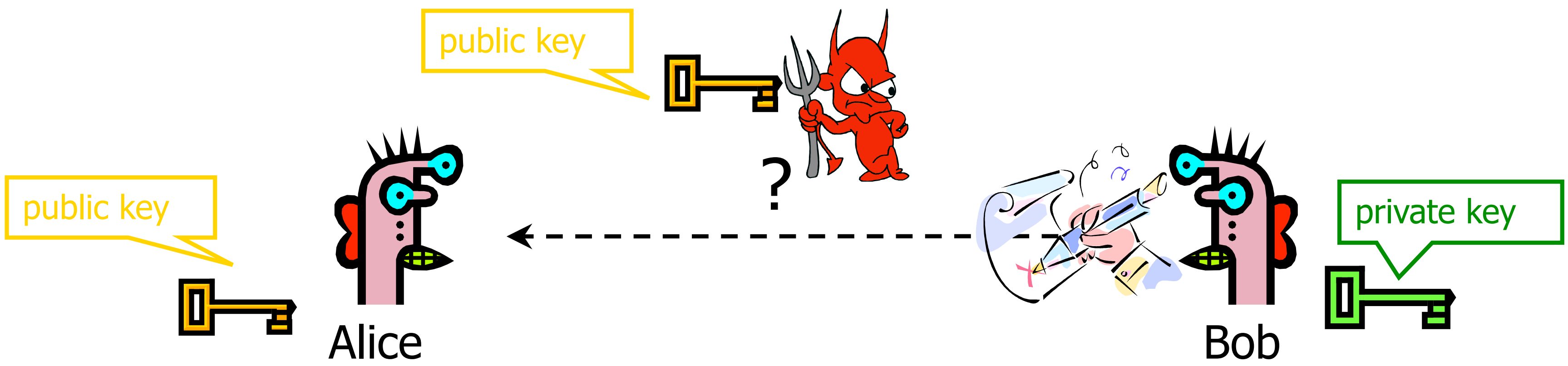
Integrity in RSA Encryption

- Plain RSA does not provide integrity
  - Given encryptions of m_1 and m_2, attacker can create encryption of $m_1 \cdot m_2$
    - $(m_1^e) \cdot (m_2^e) \bmod n = (m_1 \cdot m_2)^e \bmod n$
  - Attacker can convert m into m^k without decrypting
    - $(m_1^e)^k \bmod n = (m^k)^e \bmod n$
- In practice, OAEP is used: instead of encrypting M, encrypt $M \oplus G(r) \,;\, r \oplus H(M \oplus G(r))$
  - r is random and fresh, G and H are hash functions
  - Resulting encryption is plaintext-aware: infeasible to compute a valid encryption without knowing plaintext
    - … if hash functions are "good" and RSA problem is hard

OAEP (image from PKCS #1 v2.1)



Digital Signatures: Basic Idea



<u>Given</u>: Everybody knows Bob's public key
Only Bob knows the corresponding private key

<u>Goal</u>: Bob sends a "digitally signed" message

1. To compute a signature, must know the private key
2. To verify a signature, enough to know the public key

RSA Signatures

- Public key is (n,e), private key is d
- To sign message m: $s = m^d \bmod n$
  - Signing and decryption are the same **underlying** operation in RSA
  - It's infeasible to compute s on m if you don't know d
- To verify signature s on message m:

  $s^e \bmod n = (m^d)^e \bmod n = m$
  - Just like encryption
  - Anyone who knows n and e (public key) can verify signatures produced with d (private key)
- In practice, also need padding & hashing

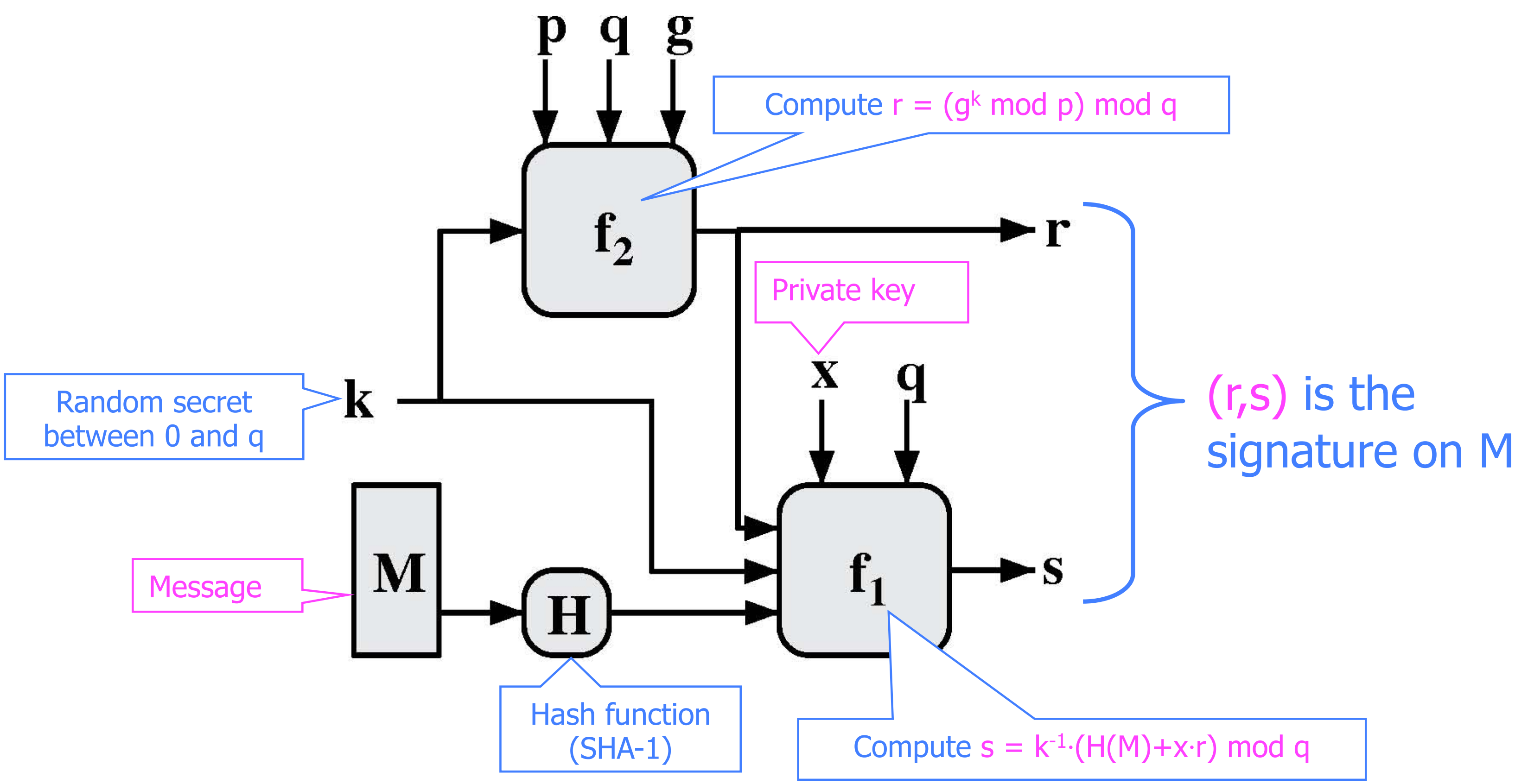
Encryption and Signatures

- Often people think: Encryption and decryption are inverses.
- That's a common view
  - True for the RSA **primitive (underlying component)**
- But not one we'll take
  - To really use RSA, we need padding
  - And there are many other decryption methods

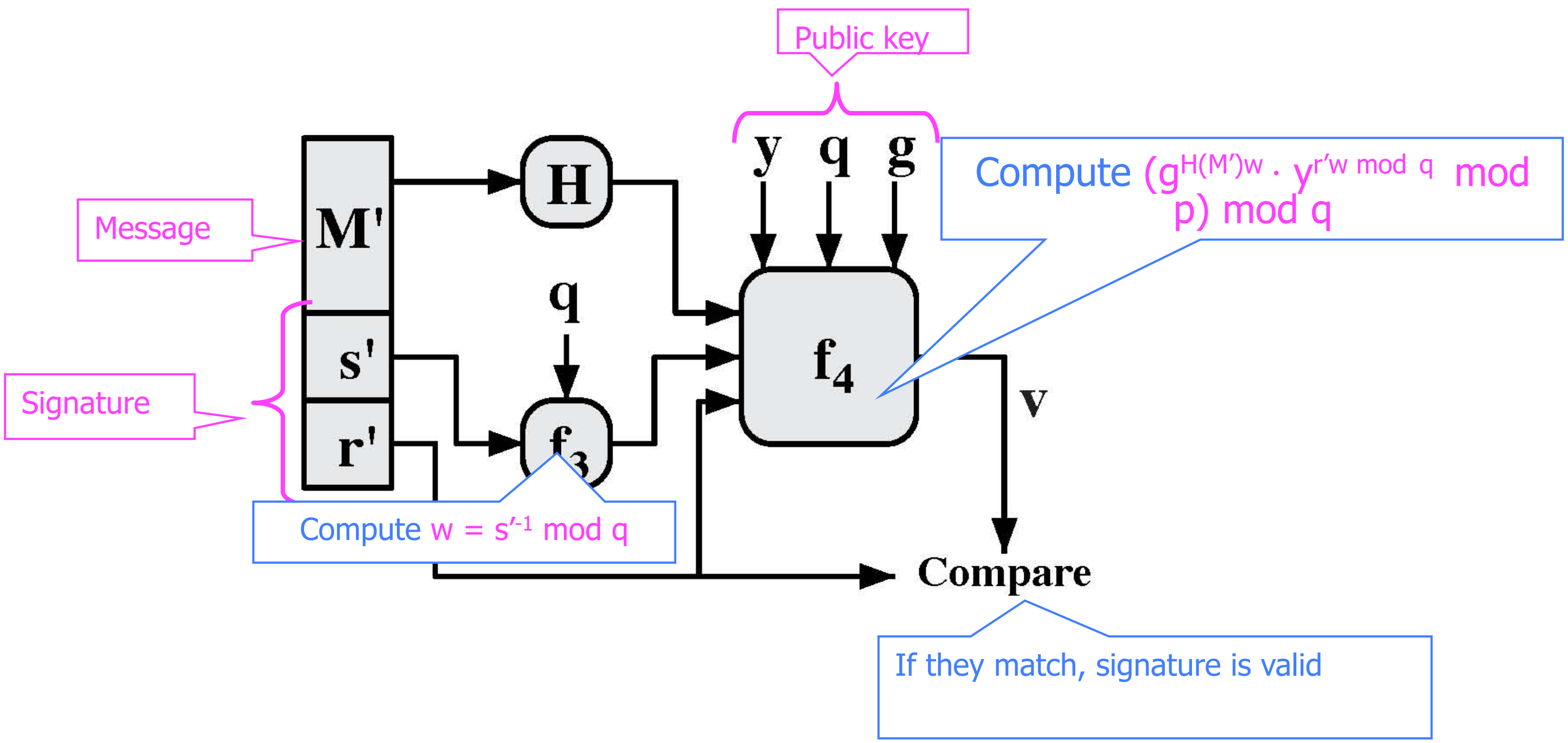
Digital Signature Standard (DSS)

- U.S. government standard (1991-94)
  - Modification of the ElGamal signature scheme (1985)
- Key generation:
  - Generate large primes p, q such that q divides p-1
    - $2^{159} < q < 2^{160}$, $2^{511+64t} < p < 2^{512+64t}$ where $0 \le t \le 8$
  - Select $h \in Z_p^*$ and compute $g = h^{(p-1)/q} \bmod p$
  - Select random x such $1 \le x \le q-1$, compute $y = g^x \bmod p$
- Public key: $(p, q, g, y = g^x \bmod p)$, private key: x
- Security of DSS requires hardness of discrete log
  - If could solve discrete logarithm problem, would extract x (private key) from $g^x \bmod p$ (public key)

DSS: Signing a Message



DSS: Verifying a Signature



Why DSS Verification Works

- If (r,s) is a legitimate signature, then
  $r = (g^k \bmod p) \bmod q$; $s = k^{-1} \cdot (H(M)+x \cdot r) \bmod q$
- Thus $H(M) = -x \cdot r + k \cdot s \bmod q$
  - Multiply both sides by $w = s^{-1} \bmod q$
- $H(M) \cdot w + x \cdot r \cdot w = k \bmod q$
  - Exponentiate g to both sides
- $(g^{H(M) \cdot w + x \cdot r \cdot w} = g^k) \bmod p \bmod q$
  - In a valid signature, $g^k \bmod p \bmod q = r$, $g^x \bmod p = y$
- Verify $g^{H(M) \cdot w} \cdot y^{r \cdot w} = r \bmod p \bmod q$

Security of DSS

- Can't create a valid signature without private key
- Given a signature, hard to recover private key
- Can't change or tamper with signed message
- If the same message is signed twice, signatures are different
  - Each signature is based in part on random secret k
- Secret k must be different for each signature!
  - If k is leaked or if two messages re-use the same k, attacker can recover secret key x and forge any signature from then on

Advantages of Public-Key Crypto

- Confidentiality without shared secrets
  - Very useful in open environments
  - No "chicken-and-egg" key establishment problem
    - With symmetric crypto, two parties must share a secret before they can exchange secret messages
    - Caveats to come
- Authentication without shared secrets
  - Use digital signatures to prove the origin of messages
- Reduce protection of information to protection of authenticity of public keys
  - No need to keep public keys secret, but must be sure that Alice's public key is <u>really</u> her true public key

Disadvantages of Public-Key Crypto

- Calculations are 2-3 orders of magnitude slower
  - Modular exponentiation is an expensive computation
  - Typical usage: use public-key cryptography to establish a shared secret, then switch to symmetric crypto
    - We'll see this in IPSec and SSL
- Keys are longer
  - 1024 bits (RSA) rather than 128 bits (AES)
- Relies on unproven number-theoretic assumptions
  - What if factoring is easy?
    - Factoring is <u>believed</u> to be neither P, nor NP-complete
  - (Of course, symmetric crypto also rests on unproven assumptions)

Next Homework

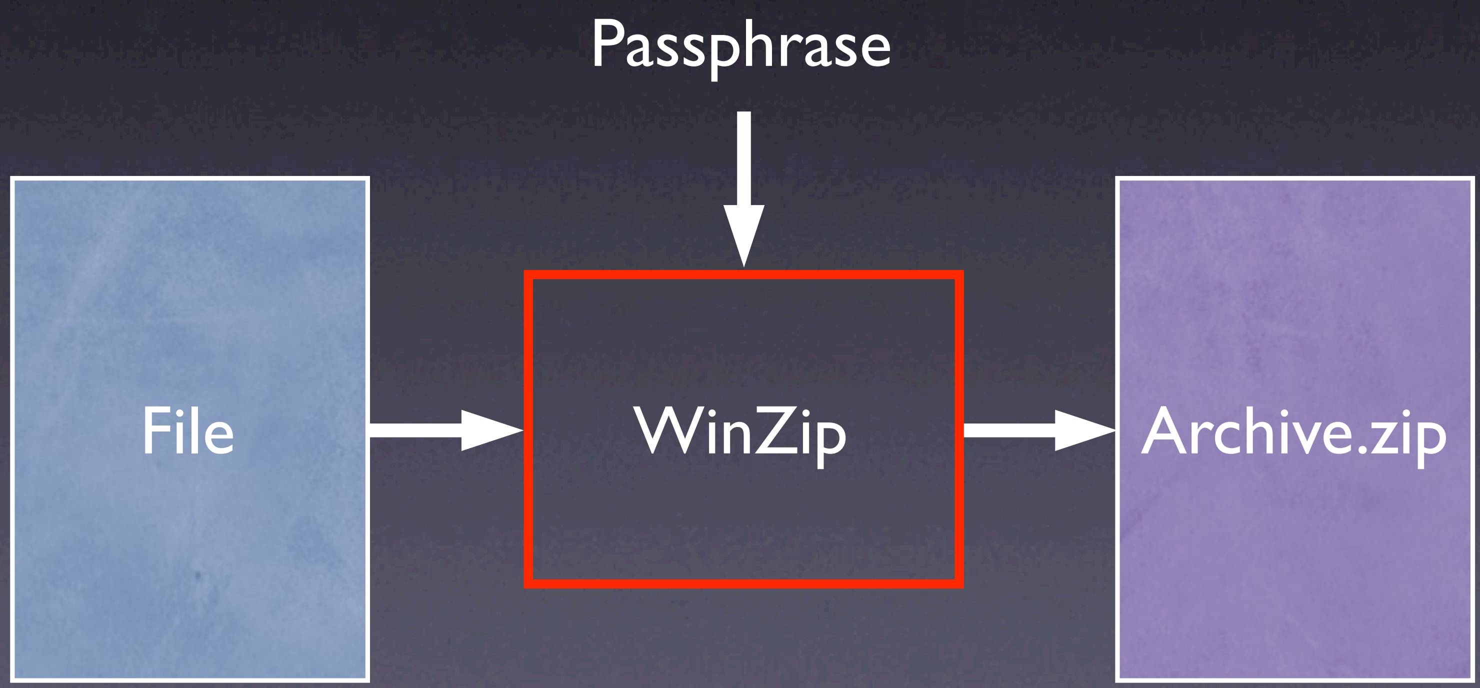
- You'll be looking at WinZip's new AE-2 encryption scheme
  - Based on "Encrypt-then-MAC" (recall a few classes ago --- this is a provably secure mode)
  - But things aren't always that simple
    - Many protocols seem secure but actually have problems
  - Your job: Analyze AE-2

What is WinZip?

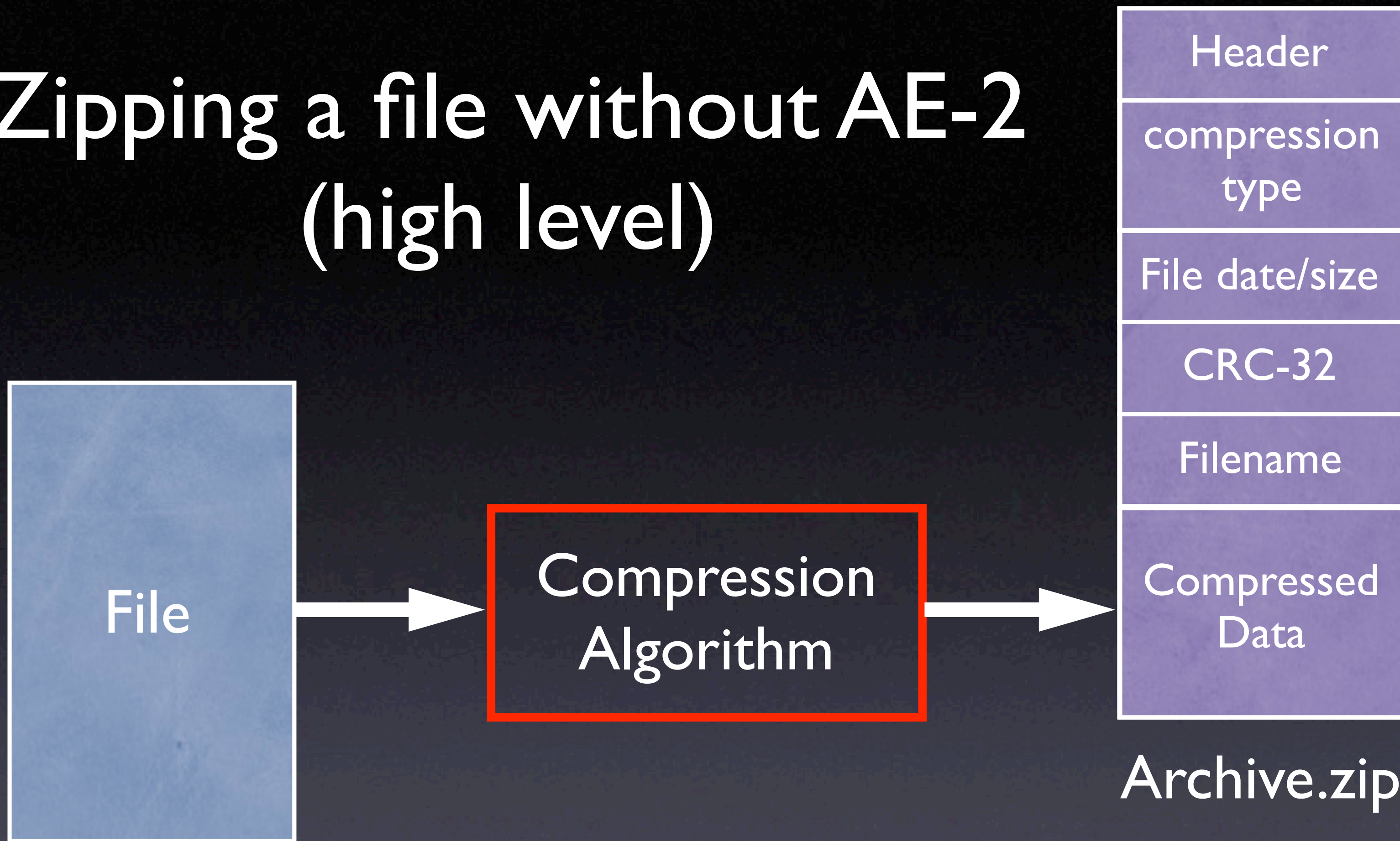
Very popular Windows compression utility. Also an Outlook email plugin. Over 160 million downloads from download.com alone [http://www.winzip.com/empopp.htm].

WinZip encryption

WinZip has the ability to encrypt files. Lots of history, but we'll look at the AE-2 method.



Zipping a file without AE-2 (high level)



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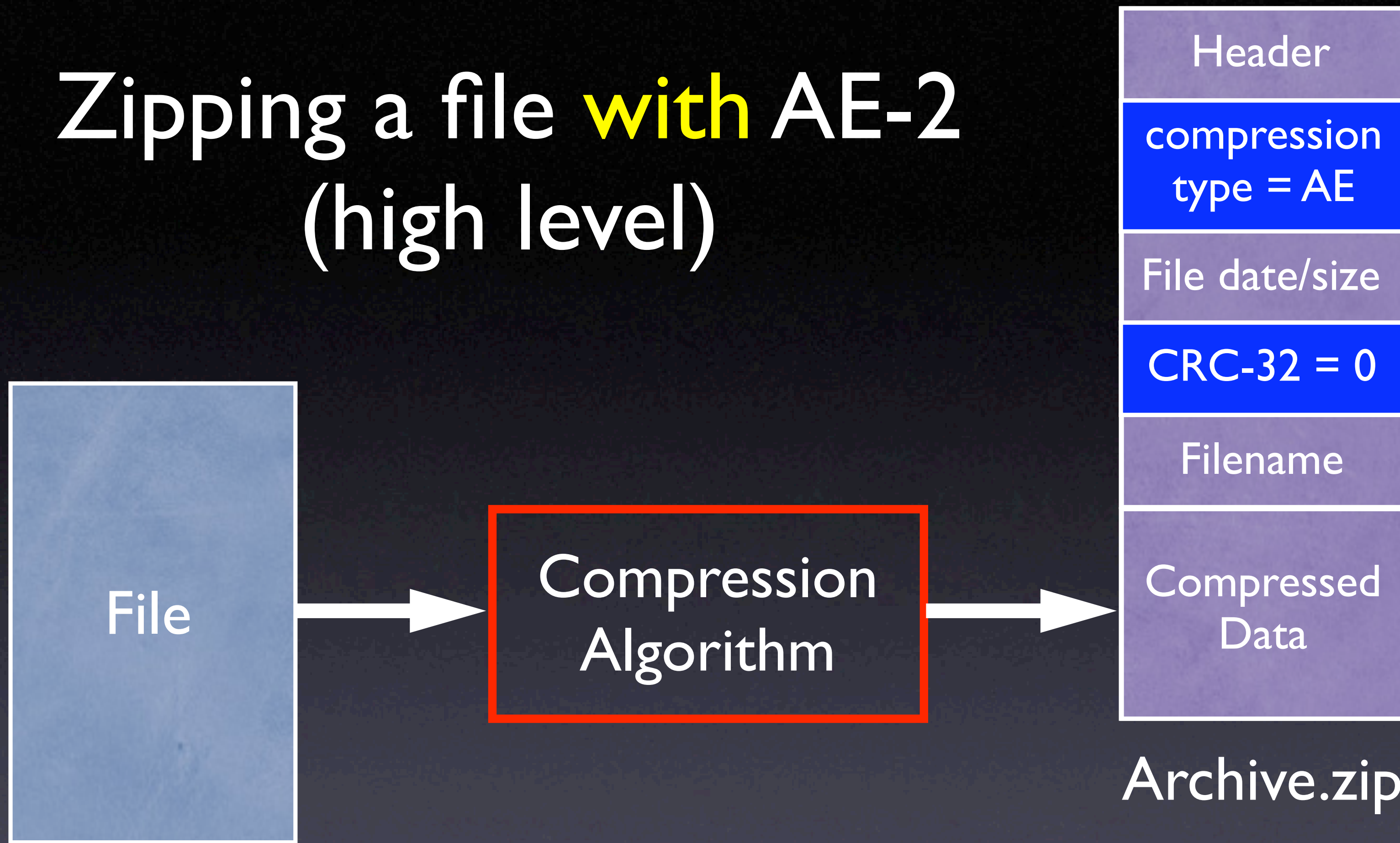

Zipping a file with AE-2 (high level)