



# Physical Modeling Synthesis of Sound

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## One View of Sound

**Sound is a waveform; we can record it,  
store it, and play it back accurately**

**PCM playback is all we need for  
interactions, movies, games, etc.**

**But, take one visual analogy:**

*“If I take lots of polaroid images, I can flip through  
them real fast and make any image sequence”*

**Interaction? We manipulate lots of PCM**

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## Views of Sound

- **Time Domain**       $x(t)$   
    (from physics, and time's arrow)
- **Frequency Domain**  $X(f)$   
    (from math, and perception)
- **Production**              what caused it
- **Perception**              our "image" of it

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## Views of Sound

- **The Time Domain**  
    is most closely related to  
    **Production**
- **The Frequency Domain**  
    is most closely related to  
    **Perception**

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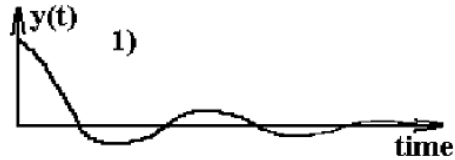


## 2nd Order Linear Diff Eq. Solution

1) Underdamped:

$$y(t) = Y_0 e^{-t/\tau} \cos(\omega t)$$

exp. \* oscillation



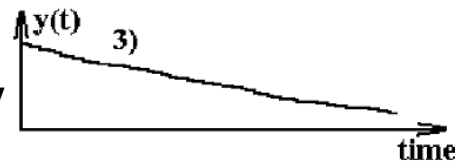
2) Critically damped:

fast exponential decay



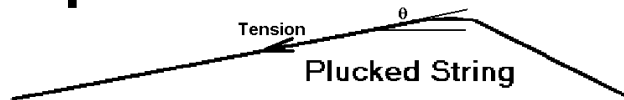
3) Overdamped:

slow exponential decay



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## The Wave Equation



$$df_y = (T \sin\theta)_{x+dx} - (T \sin\theta)_x \quad (\text{for each } dx \text{ of string})$$

$$f(x+dx) = f(x) + \delta f / \delta x \, dx + \dots \quad (\text{Taylor's series in space})$$

assume  $\sin \theta = \theta$  (for small  $\theta$ )

$$F = ma = \rho \, dx \, d^2y/dt^2 \quad (\rho = \text{mass/length})$$

Solution:

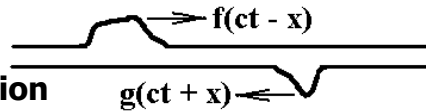
The wave equation  
( $c^2 = T / \rho$ )

$$\frac{d^2 y}{dx^2} = \frac{1}{c^2} \frac{d^2 y}{dt^2}$$

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# Traveling Wave String Solution

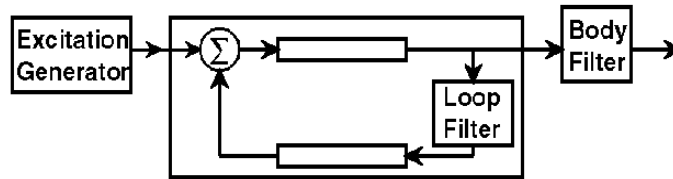
D' Alembert Solution of  
2nd order wave equation  
(left and right going waves)



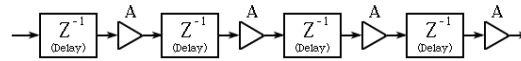
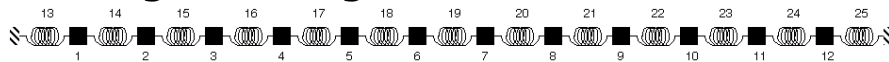
## “Digital Waveguide Filter” Model (Smith)

- Bi-directional delay lines
- Filters for loss, radiation, other

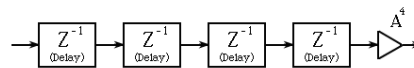
Simple Plucked String  
Physical Model Block Diagram



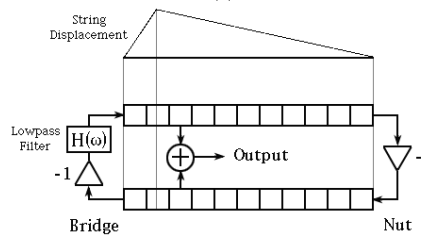
## “Digital Waveguide Filter” Model



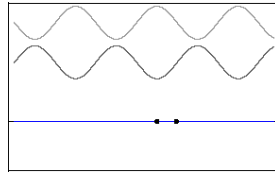
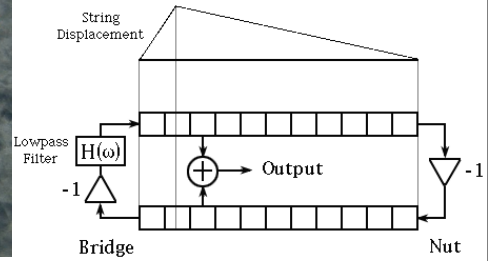
(a)



(b)

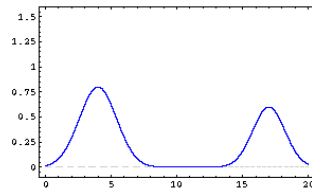
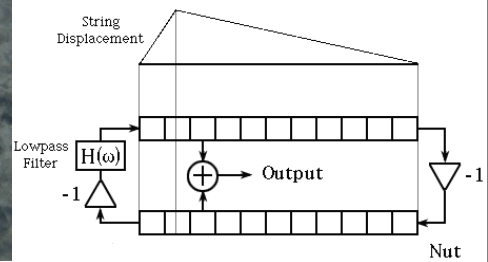


## “Digital Waveguide Filter” Model



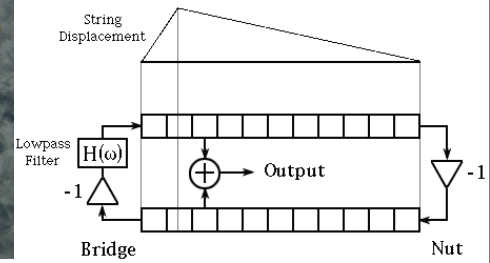
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## “Digital Waveguide Filter” Model



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## “Digital Waveguide Filter” Model



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## Karplus-Strong Model

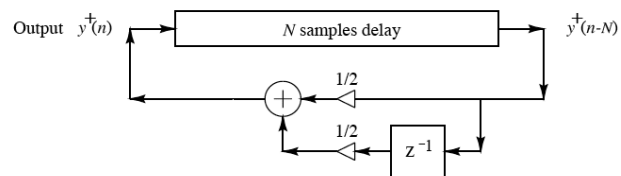
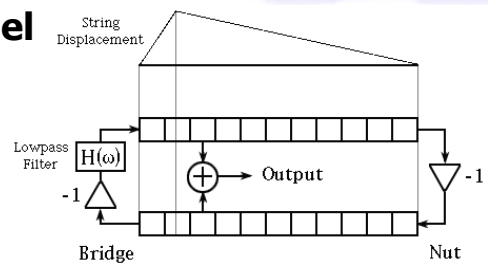


Figure 17. Rigidly terminated string with the simplest frequency-dependent loss filter. All  $N$  loss factors (possibly including losses due to yielding terminations) have been consolidated at a single point and replaced by a one-zero filter approximation.

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## Modal String Solution

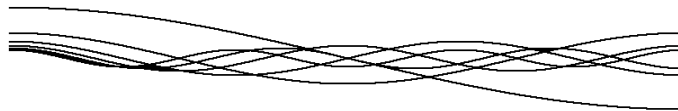


- **Superimposed spatial sine waves**  
(modes derive from spatial “boundary conditions”)
- **Modes result in frequency “partials” (in time)**
- **Harmonic ( $f$ ,  $2f$ ,  $3f$ , etc.) relationship**  
(speed of sound  $c = \text{constant}$ )
- **Stiffness can cause minor stretching of harmonic frequencies (  $c(f)$  )**

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## Modal Solution for Bars

- **Bars are often free at one or both ends**



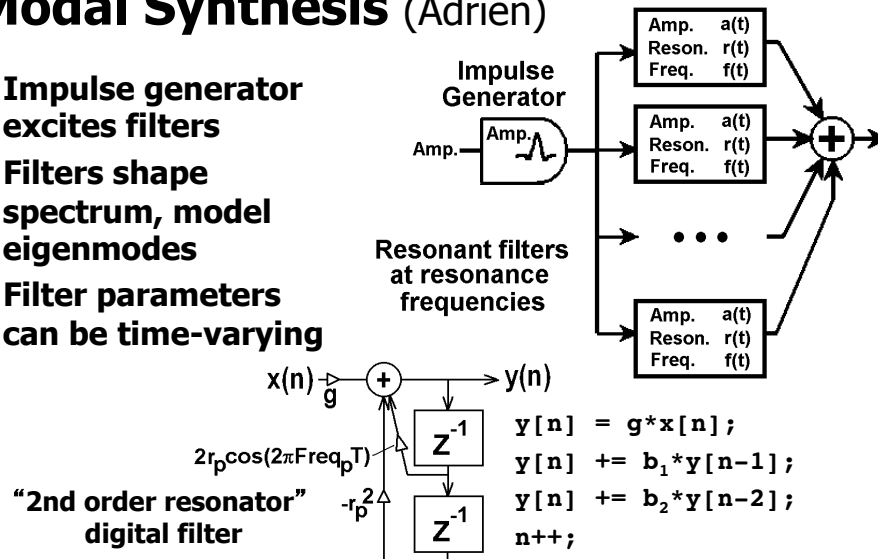
- **Spatial modal solution still holds**
- **Modes no longer harmonic. Stiffness of rigid bars “stretches” frequencies.**
- **Modes:  $f$ ,  $2.765f$ ,  $5.404f$ ,  $8.933f$ , etc.**

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## Modal Synthesis (Adrien)

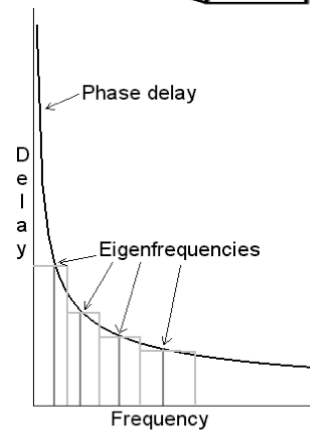
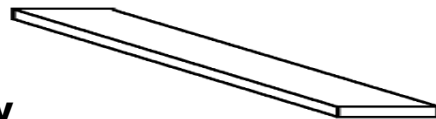
- Impulse generator excites filters
- Filters shape spectrum, model eigenmodes
- Filter parameters can be time-varying



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## Stiffness in Bars

- Stiffness makes wave propagation frequency dependent ( $c(f)$ )
- Models:
  - Modal partials
  - Use all-pass phase filter to “stretch” waveguide harmonics
  - Merge waveguide with modal by modeling each mode with filter and delay

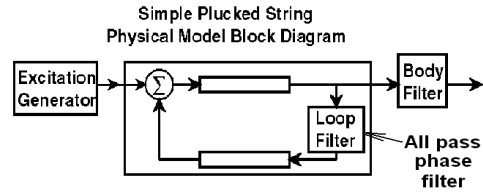


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# Stiffness

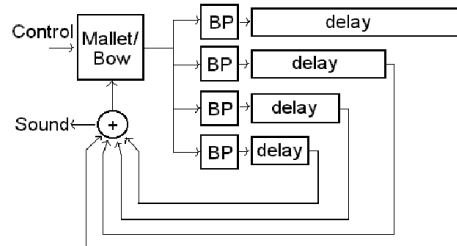
## All-pass waveguide (Smith & Jaffe)

- *Acoustics View: Frequency dependent propagation*
- *Filter View: Stretch comb filter harmonics*



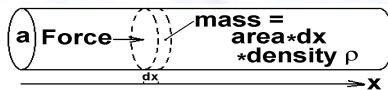
## Banded waveguides (Essl)

- *Acoustics View: Wave train closures*
- *Filter View: Comb filters with one resonance each*

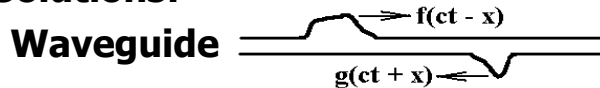


Or a purely modal model (lacks space and time)

# Tubes



- **Open or closed at either end**
- **Wave equation solution same as strings**
- **Modes always harmonic because speed of sound is constant with frequency**
- **Solutions:**

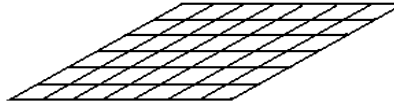


or Modal



Open + Closed: odd 1/4 wavelengths

## Two and Higher Dimensions

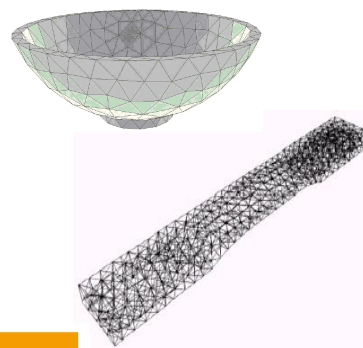
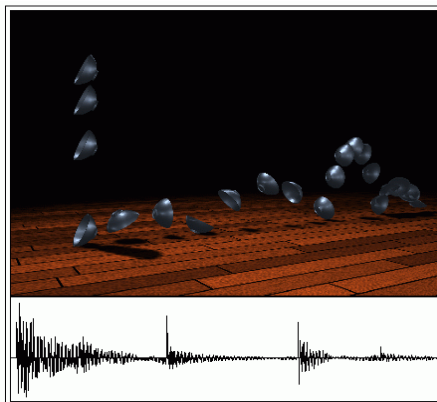


- **2 (N) Dimensional Waveguide Meshes**
- **or Finite Elements and Finite Differences**
  - Discretize objects into cells (elements)
  - Express interactions between them
  - Express differential equation for system
  - Solve by discrete steps in space and time
- **or Modal Solution**

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## Finite Elements

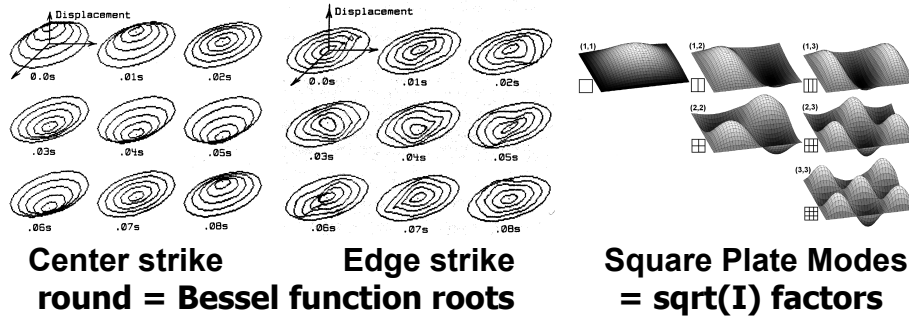
(with O' Brien and Essl)



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## Hi-D Modal Solutions

### Modes of Plates are inharmonic



**Modes in higher dimensions are problematic  
(impossible analytically except in very simple cases)**

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## Where Are We So Far?

- **Physical descriptions (equations)**
- **Give rise to solutions:**
  1. **Traveling Waves**
  2. **Spatial/Frequency Modes**
- **We can solve the equations directly using**
  3. **Finite Elements/Meshes**
- **How to choose? Are there more?**

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## Waveguides

- **Strengths:**
  - Cheap in both computation and memory
  - Parametrically meaningful, extensible for more realism
- **Weaknesses:**
  - Little in the real world looks, behaves, or sounds exactly like a plucked string, flute, etc.
  - Each family needs a different model
  - No general blind signal model

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## Modal Modeling

- **Strengths:**
  - Generic, flexible, cheap if only a few modes
  - Great for modeling struck objects of metal, glass, wood
- **Weaknesses:**
  - No inherent spatial sampling
  - No (meaningful) phase delay
  - Hard to interact directly and continuously (rubbing, damping, etc).
  - No general blind signal model (closest)

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## Meshes, Finite Elements

- **Strengths**
  - (somewhat) arbitrary geometries
  - Fewer assumptions than parametric forms
  - Can strike, damp, rub, introduce non-linearities at arbitrary points
- **Weaknesses:**
  - Expensive
  - Don't know all the computational solutions
  - Sampling in space/time (high Q problems)
  - Dispersion is strange (diagonals vs. not)
  - No general blind signal model

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## Sound Views: Frequency Domain

- Many physical systems have modes (damped oscillations)
- Wave equation (2nd order) or Bar equation (4th order) need 2 or 4 “boundary conditions” for solution
- Once boundary conditions are set solutions are sums of exponentially damped sinusoidal modes

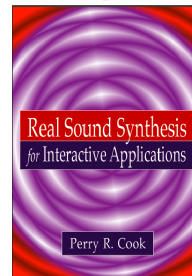
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## References and Resources

### Synthesis ToolKit in C++ (STK)

- STK: a set of classes in C++ for rapid experimentation with sound synthesis. Available for free (source, multi-platform)
  - <http://www.cs.princeton.edu/~prc>
  - <http://www-ccrma.stanford.edu/~gary>
  - <http://www-ccrma.stanford.edu/software/stk>
- Based on "Unit Generators," the classical computer music/sound building blocks:
- Oscillators, Filters, Delay Lines, etc.
- Build your own algorithms from these

### Book on interactive sound synthesis



Many examples and figures from these notes

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