CSE 478 Robot Autonomy
Feedback Control

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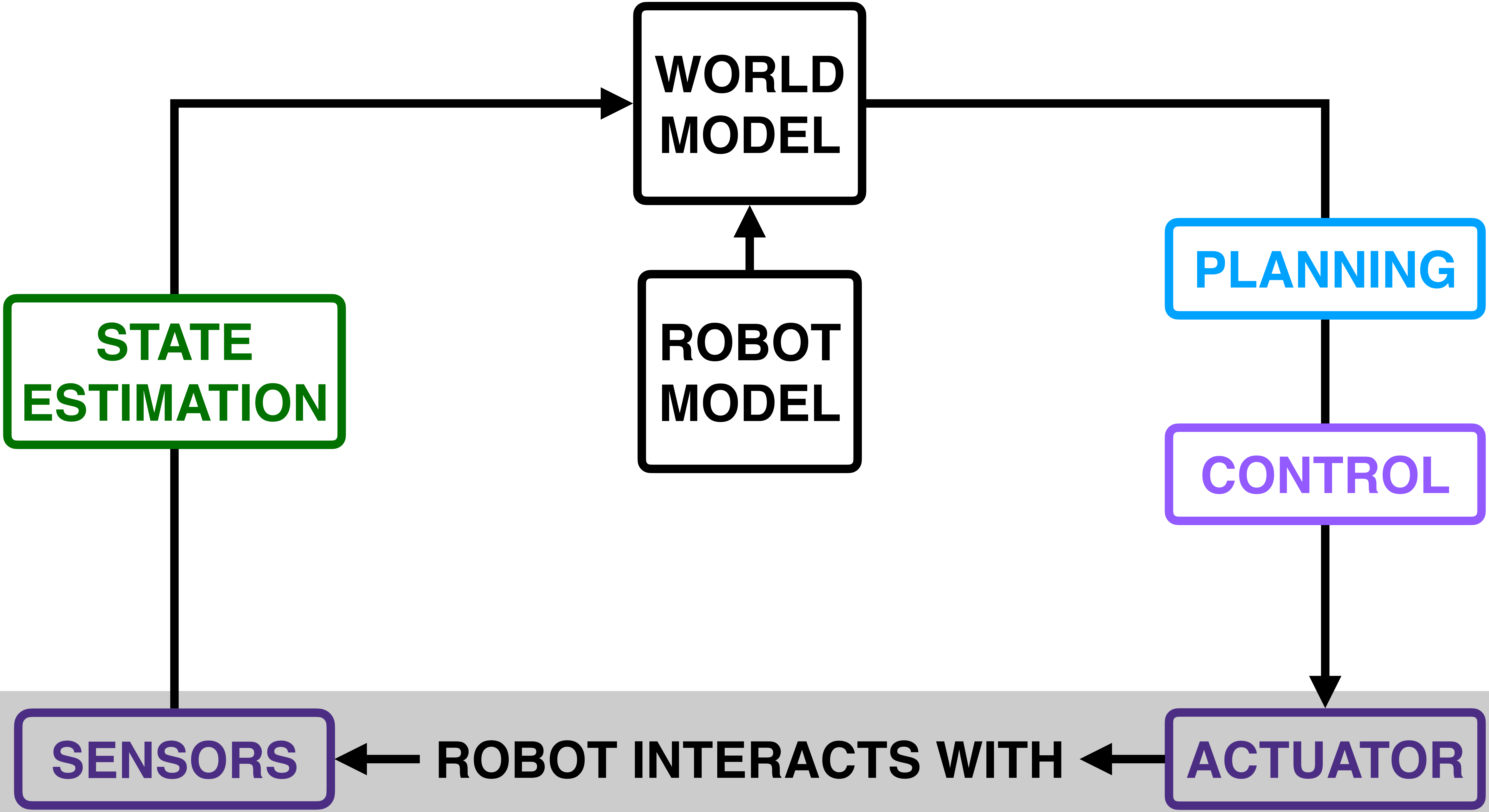
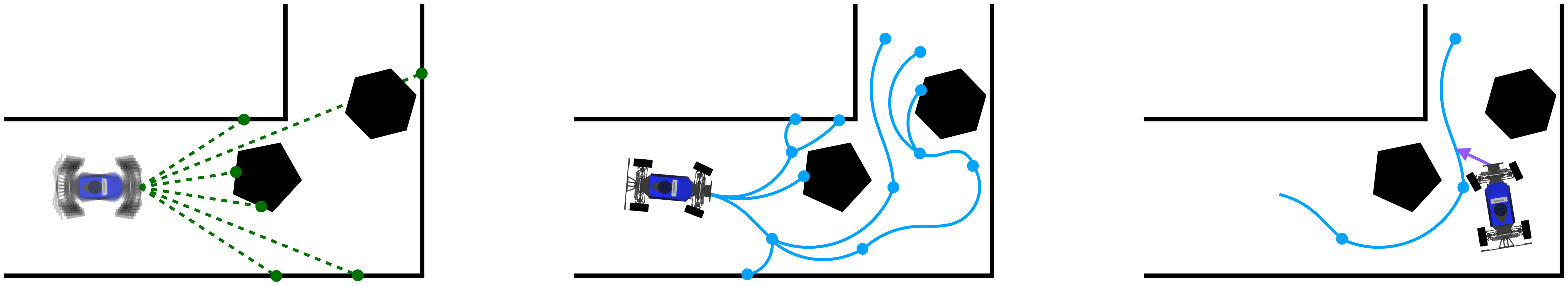

WORLD MODEL
STATE ESTIMATION
ROBOT MODEL
PLANNING
CONTROL
SENSORS
ROBOT INTERACTS WITH
ACTUATOR

The Sense-Plan-Act Paradigm



Estimate robot state

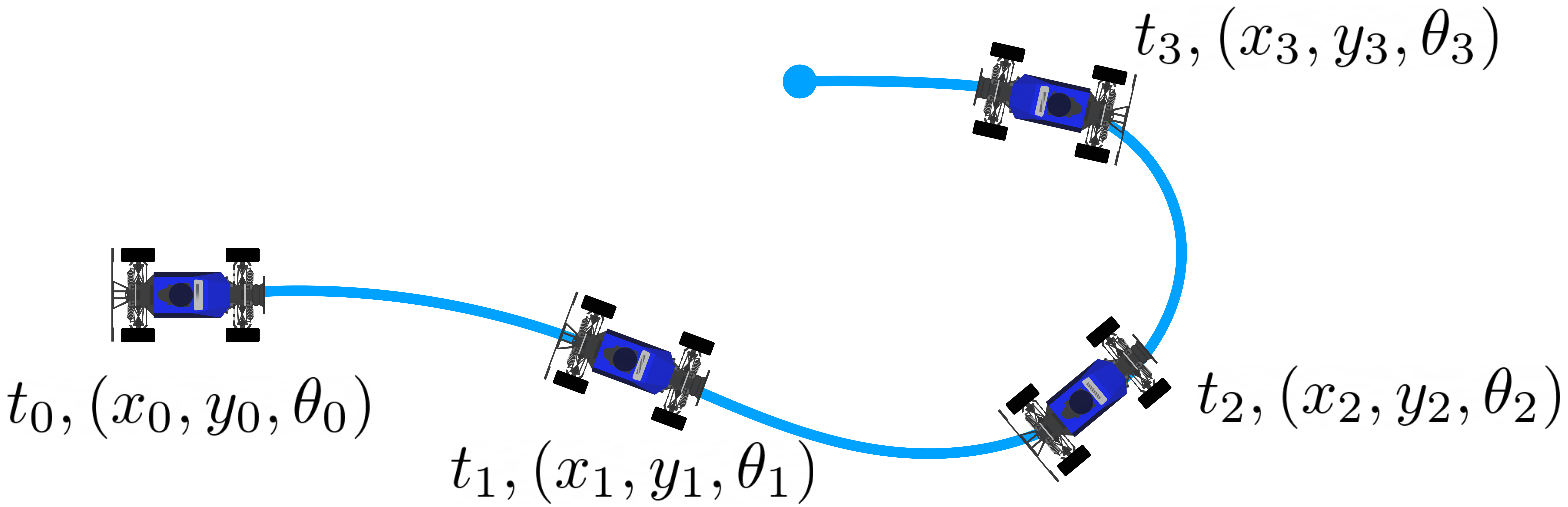
Plan sequence of motions

Control robot to follow plan

When I think about control …

What is Control?

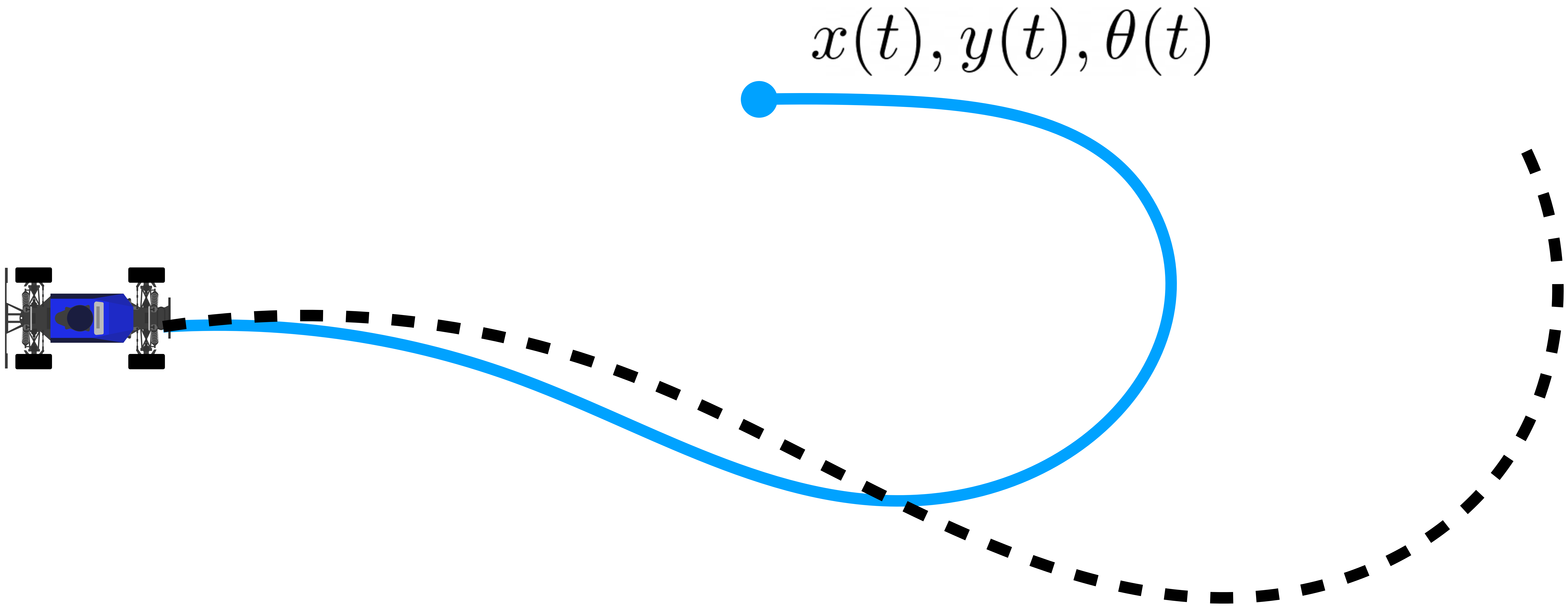
What is a Plan?



Can express this problem as **tracking a reference** trajectory

$$x(t), y(t), \theta(t)$$

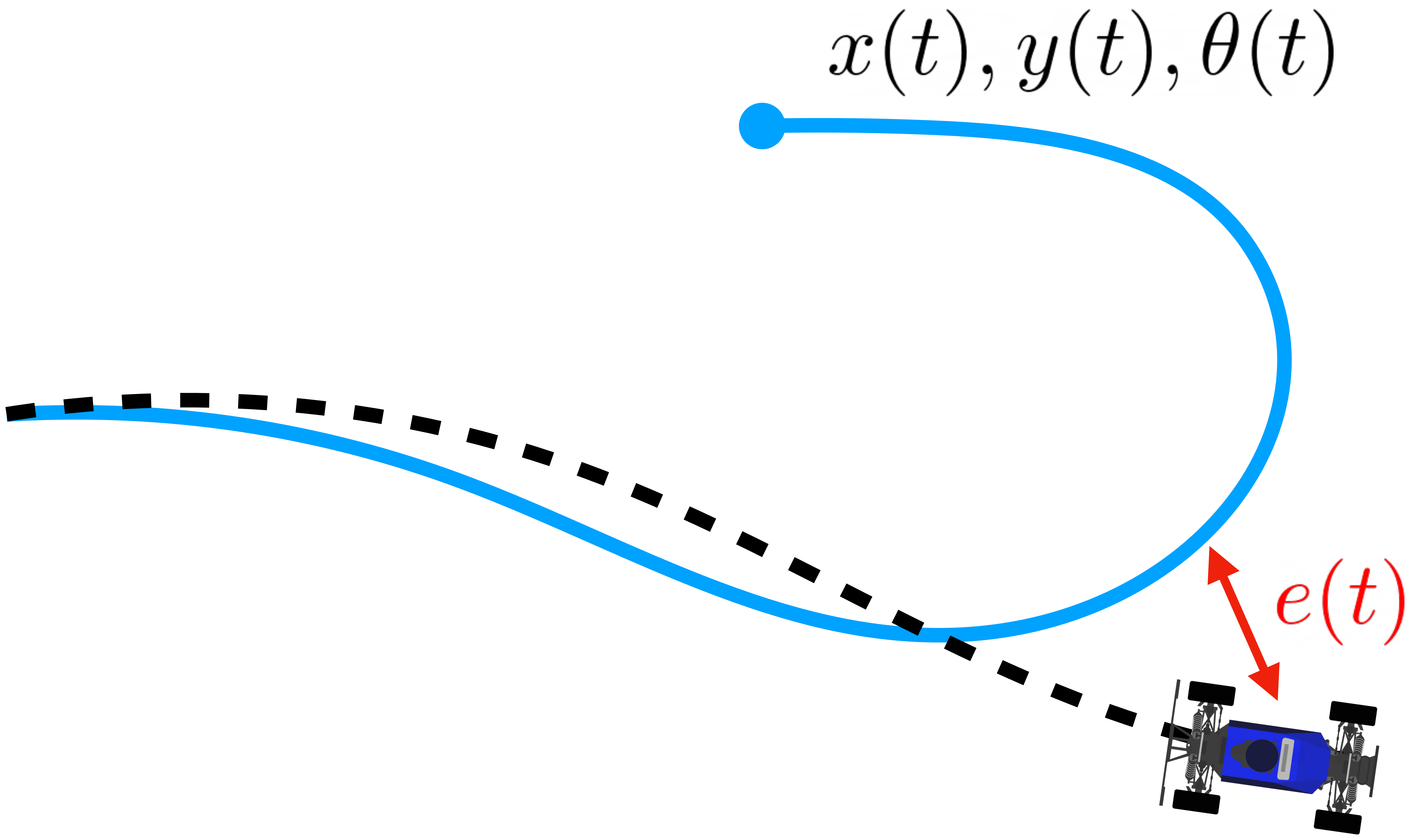
Why Feedback Control?



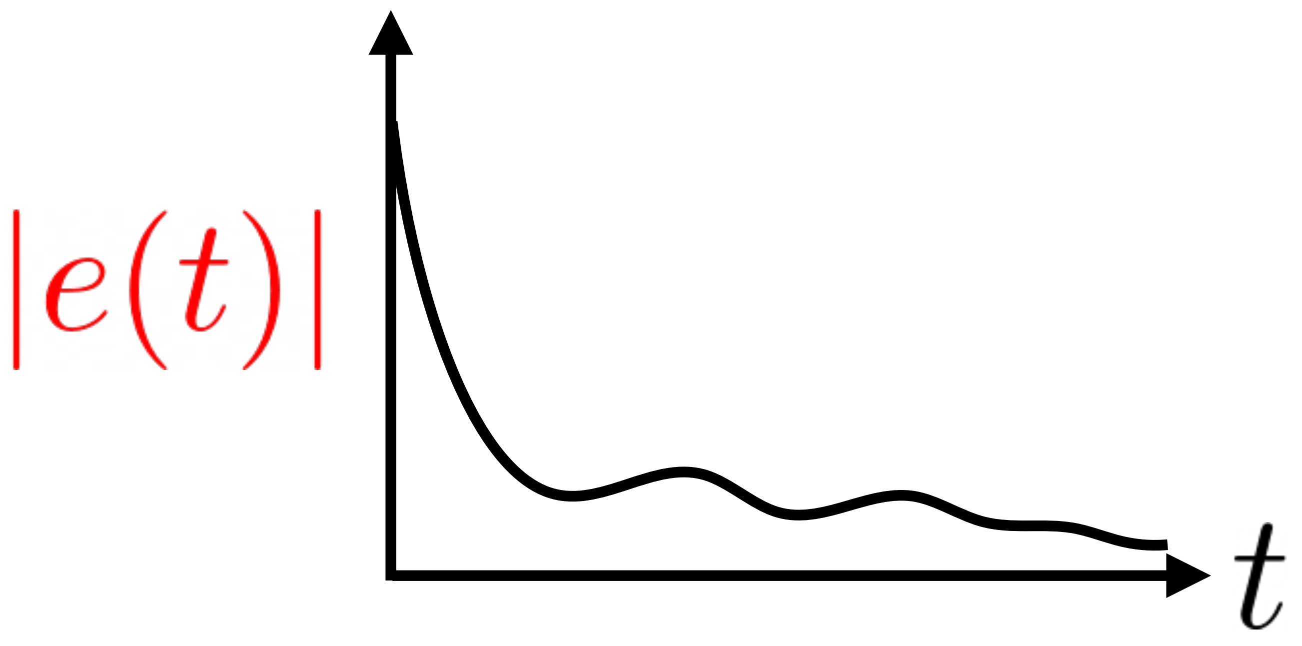
What if we send out controls $u(t)$ from kinematic car model?

Open-loop control leads to **accumulating errors**!

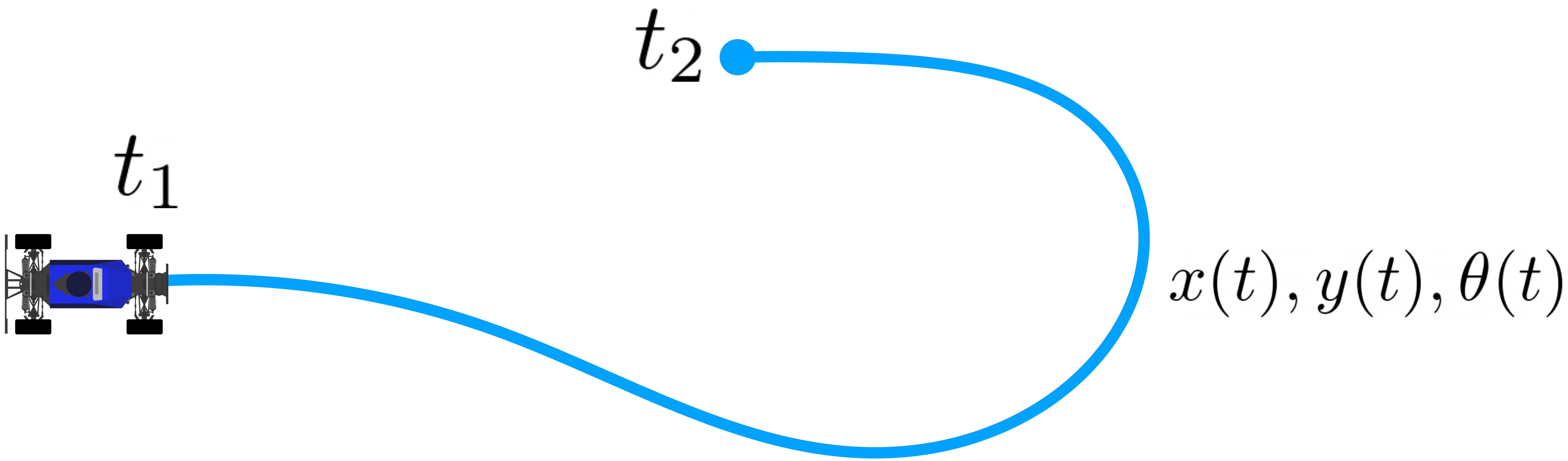
Feedback Control



1. **Measure error** between reference and current state.
2. Take actions to **minimize** this error.



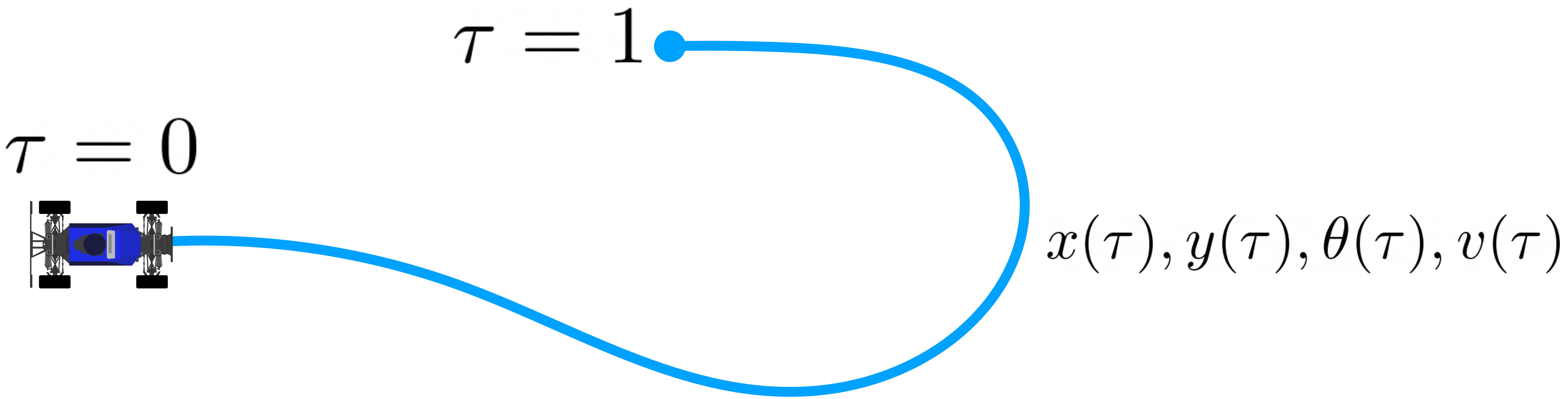
Reference Parameterizations



Option 1: **Time**-parameterized trajectory

Pro: Useful if we want the robot to respect time constraints
Con: Sometimes we only care about deviation from reference

Reference Parameterizations



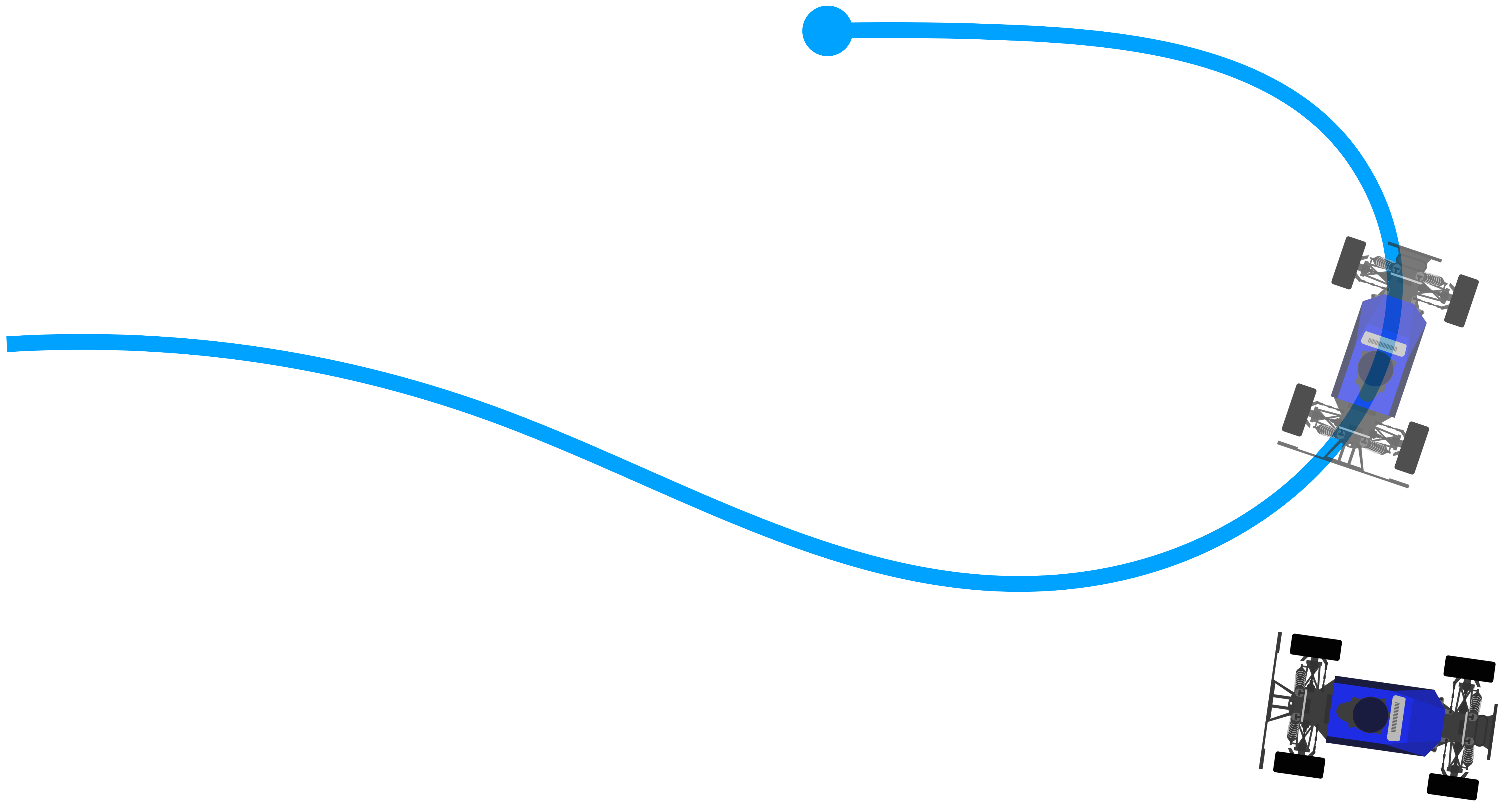
Option 2: **Index**-parameterized geometric path (untimed)

Pro: Useful for conveying shape for the robot to follow
Con: Can’t control when robot will reach a point

Controller Design Decisions

1. Get a reference path/trajectory to track
2. Pick a reference state from the reference path/trajectory
3. Compute error to reference state
4. Compute control law to minimize error

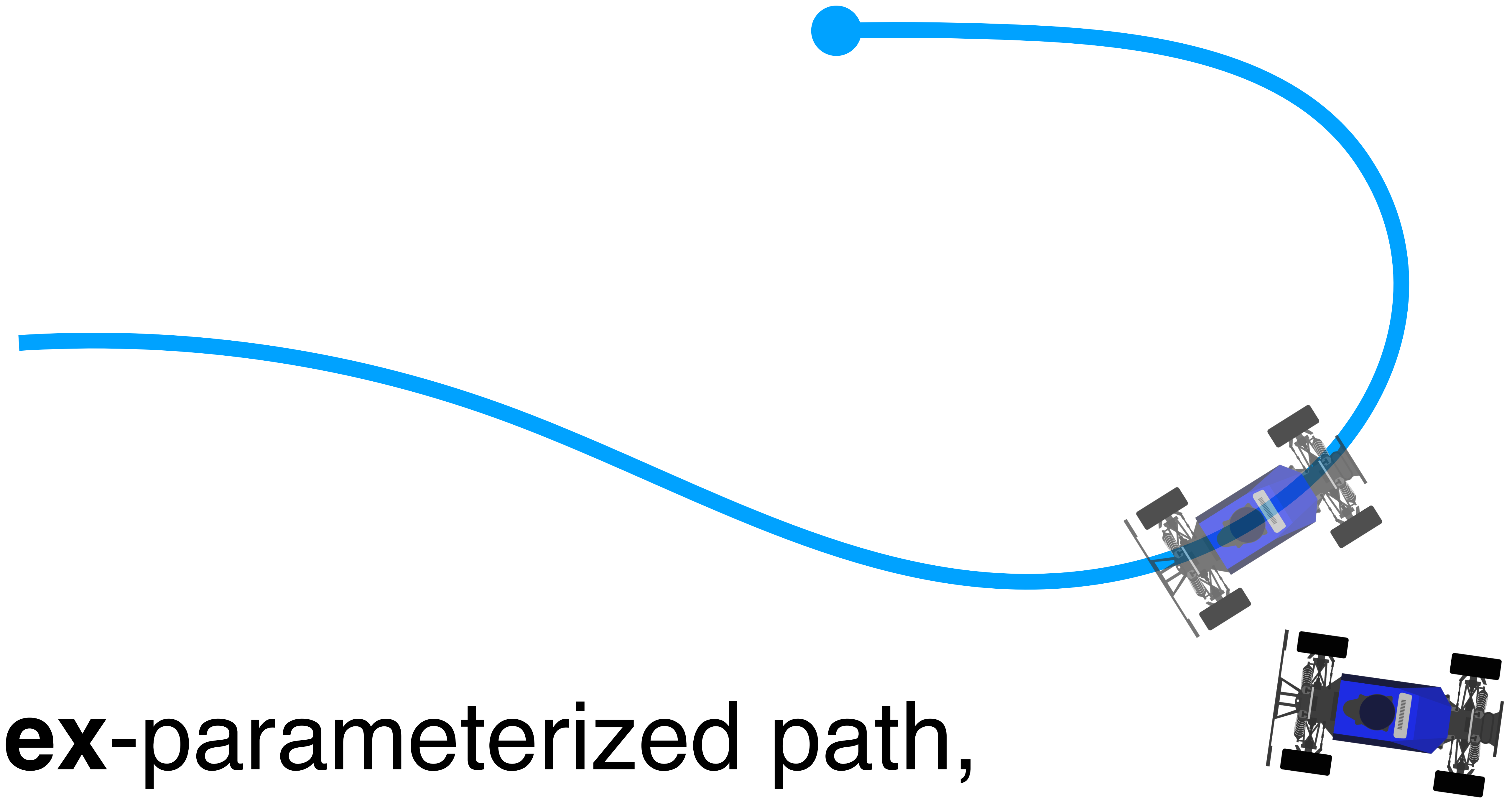
Step 2: Pick a reference (desired) state



$$\begin{bmatrix} x_{\text{ref}} \\ y_{\text{ref}} \\ \theta_{\text{ref}} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

How do we choose a reference state?



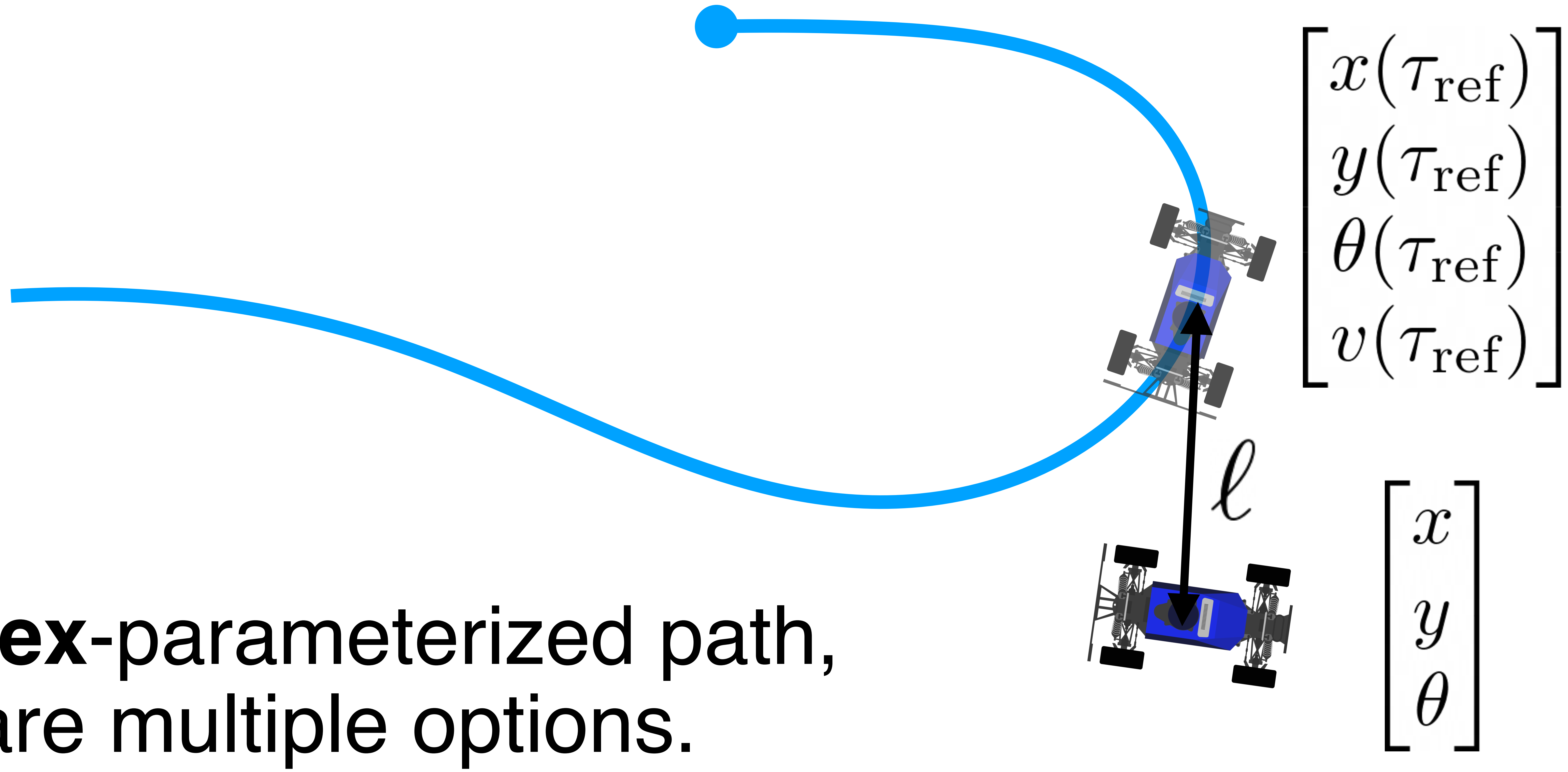
$$\begin{bmatrix} x(\tau_{\text{ref}}) \\ y(\tau_{\text{ref}}) \\ \theta(\tau_{\text{ref}}) \\ v(\tau_{\text{ref}}) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

For an **index**-parameterized path,
there are multiple options.

Closest point $\tau_{\text{ref}} = \arg\min_{\tau} \| \begin{bmatrix} x & y \end{bmatrix}^\top - \begin{bmatrix} x(\tau) & y(\tau) \end{bmatrix}^\top \|$

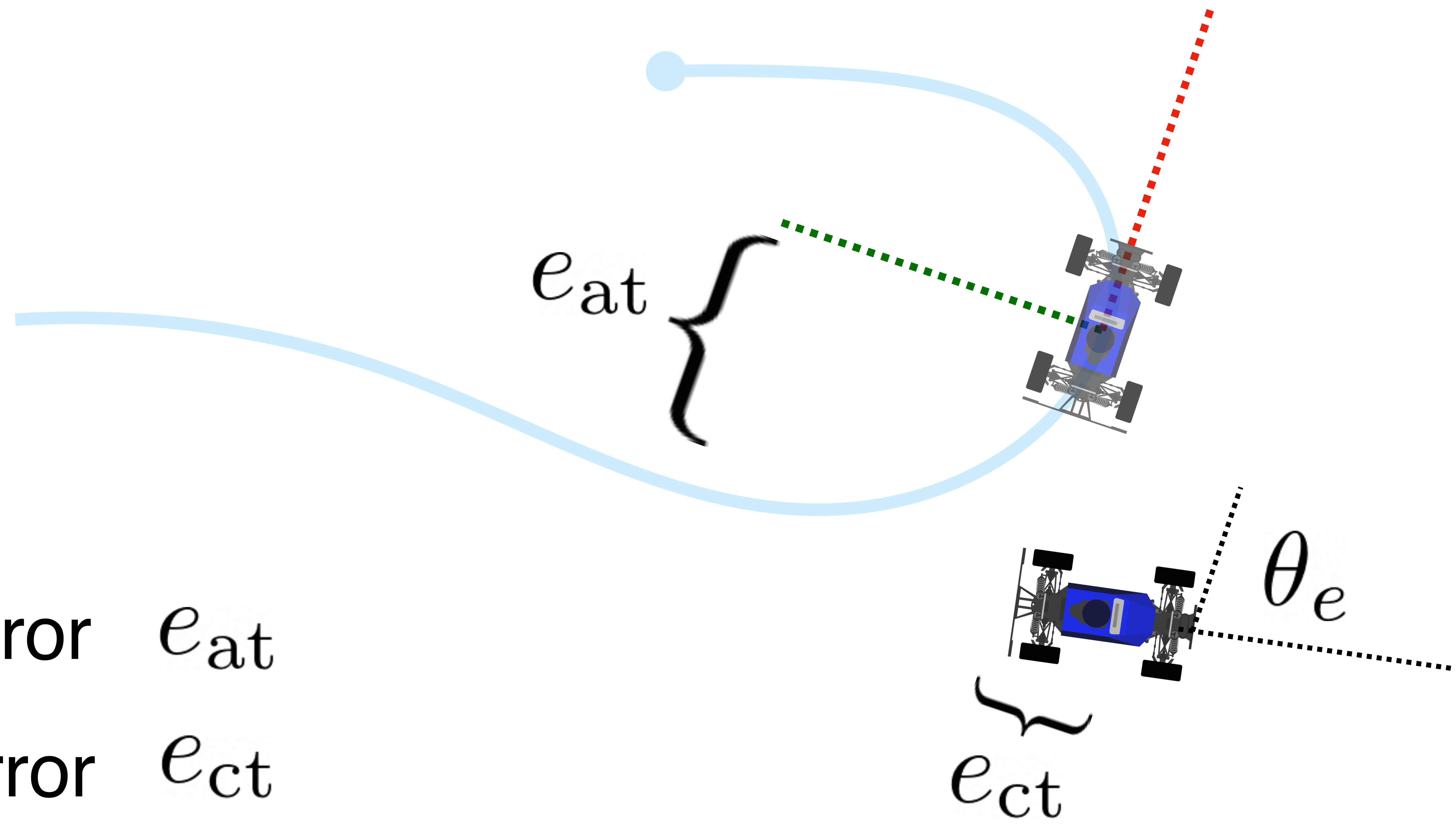
How do we choose a reference state?



For an **index**-parameterized path, there are multiple options.

Lookahead $$\tau_{\text{ref}} = \arg\min_{\tau} \left(\| \begin{bmatrix} x & y \end{bmatrix}^\top - \begin{bmatrix} x(\tau) & y(\tau) \end{bmatrix}^\top \| - \ell \right)^2$$

Step 3: Compute error to reference state

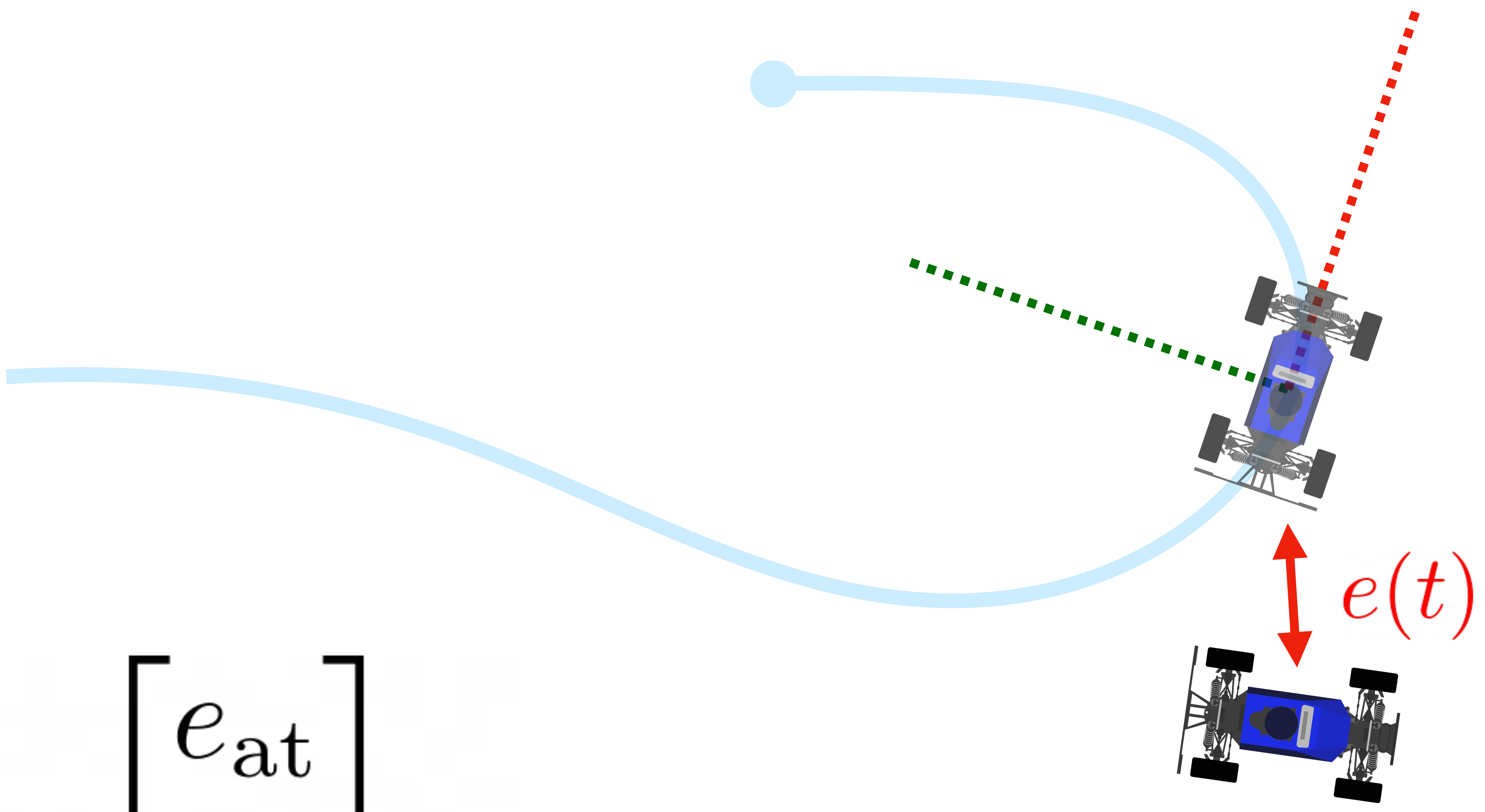


Along-track error e_{at}

Cross-track error e_{ct}

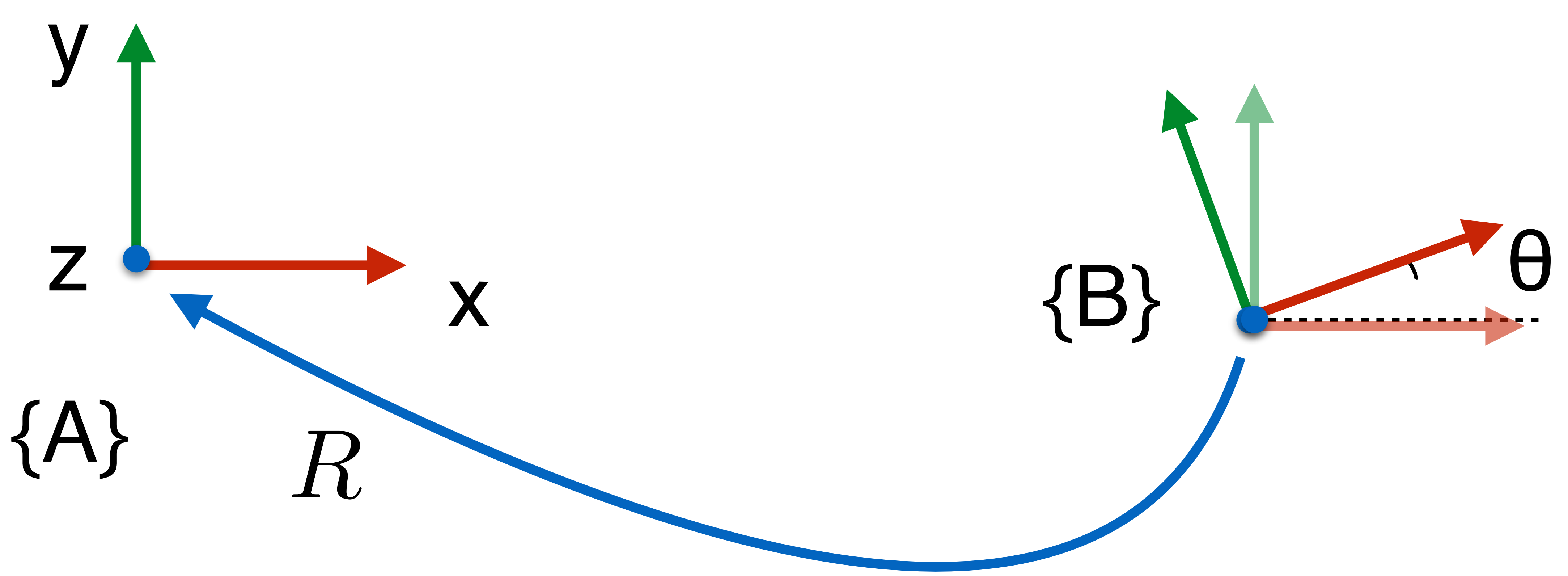
Heading error θ_e

Step 3: Compute error to reference state



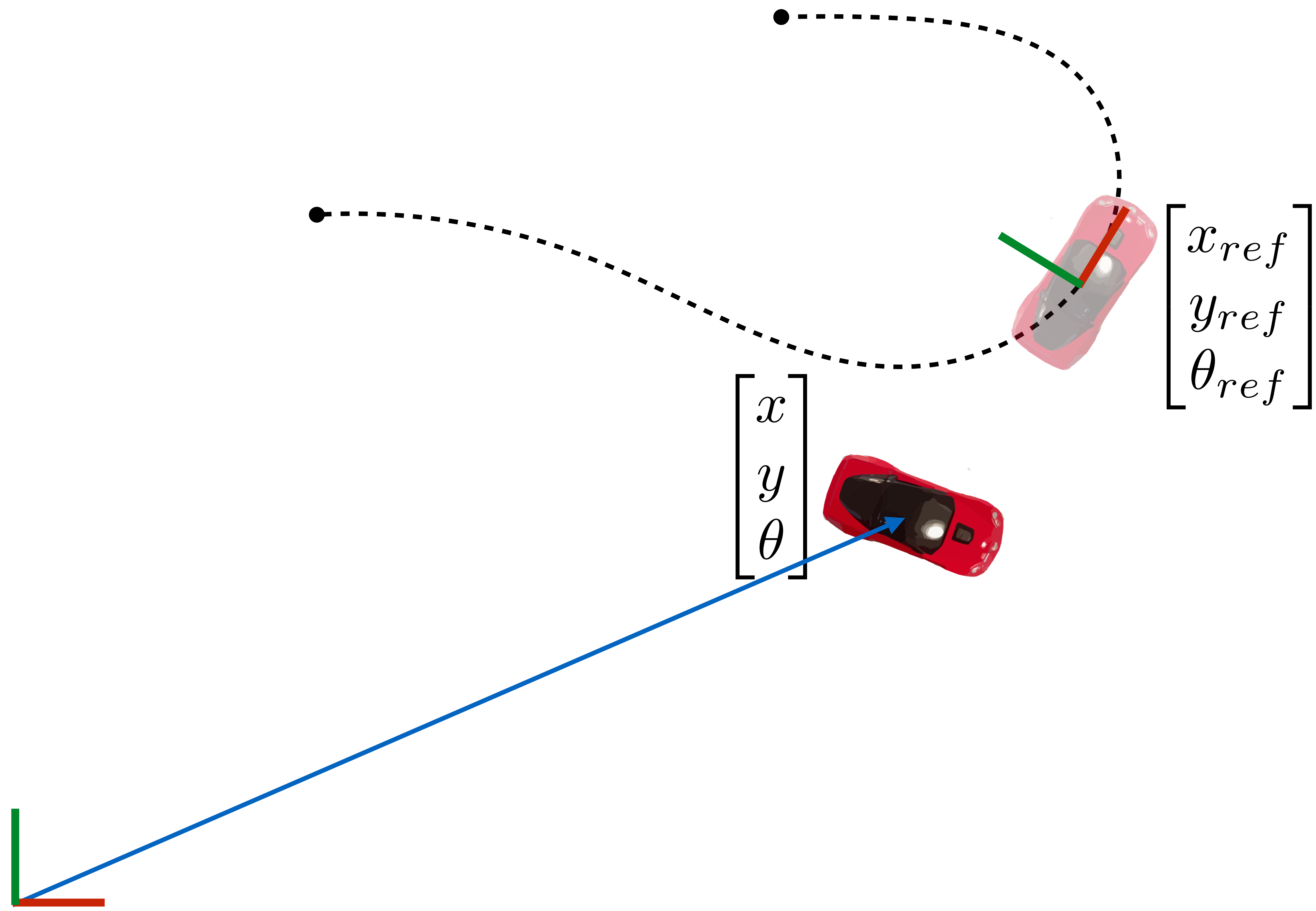
$$\begin{bmatrix} x \\ y \\ \theta \end{bmatrix}_G \rightarrow \begin{bmatrix} e_{\mathrm{at}} \\ e_{\mathrm{ct}} \\ \theta_e \end{bmatrix}_{\mathrm{ref}}$$

Aside: Rotation Matrices (Plane)

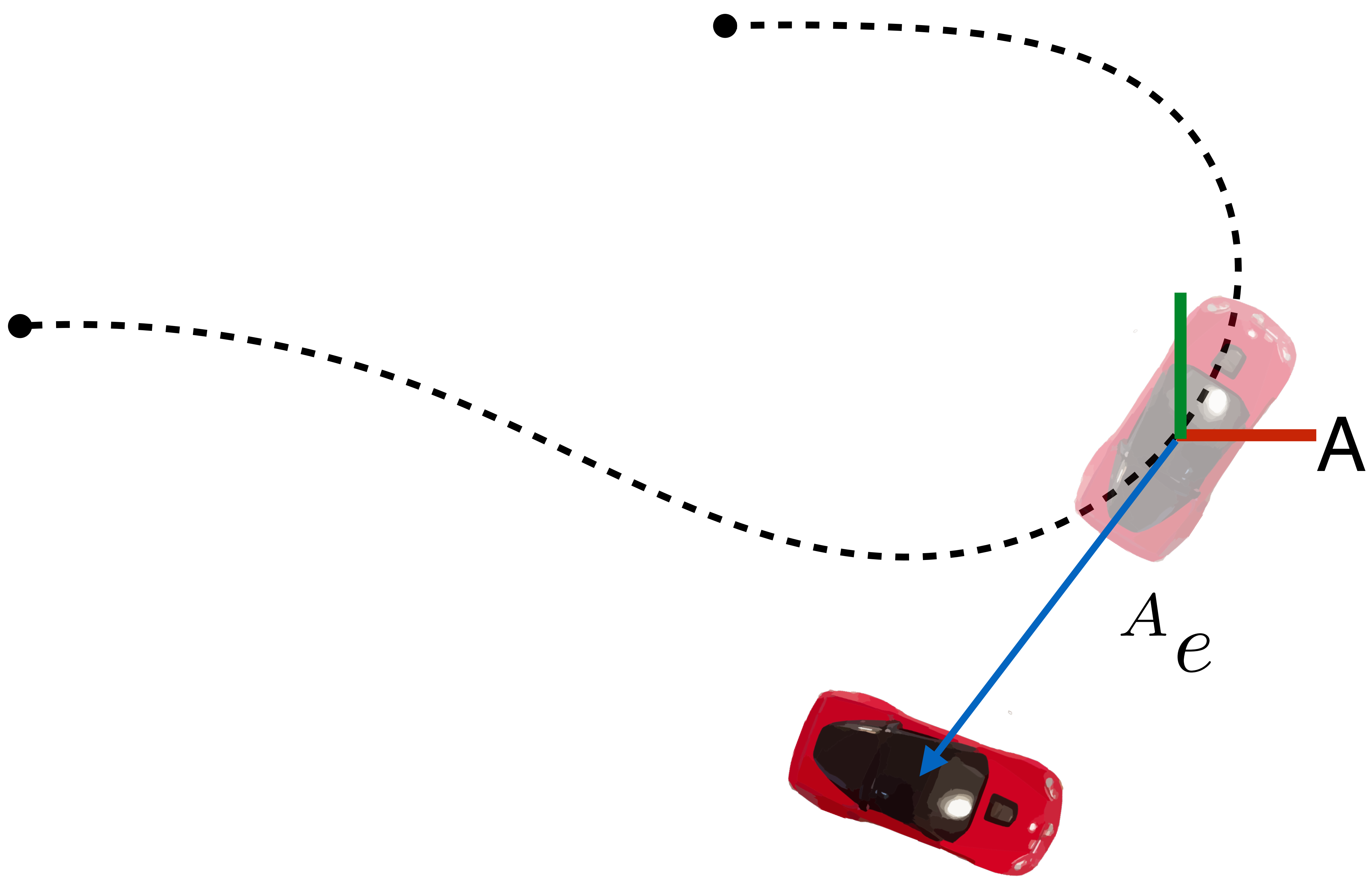


$$R = R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Step 3: Compute error to reference state



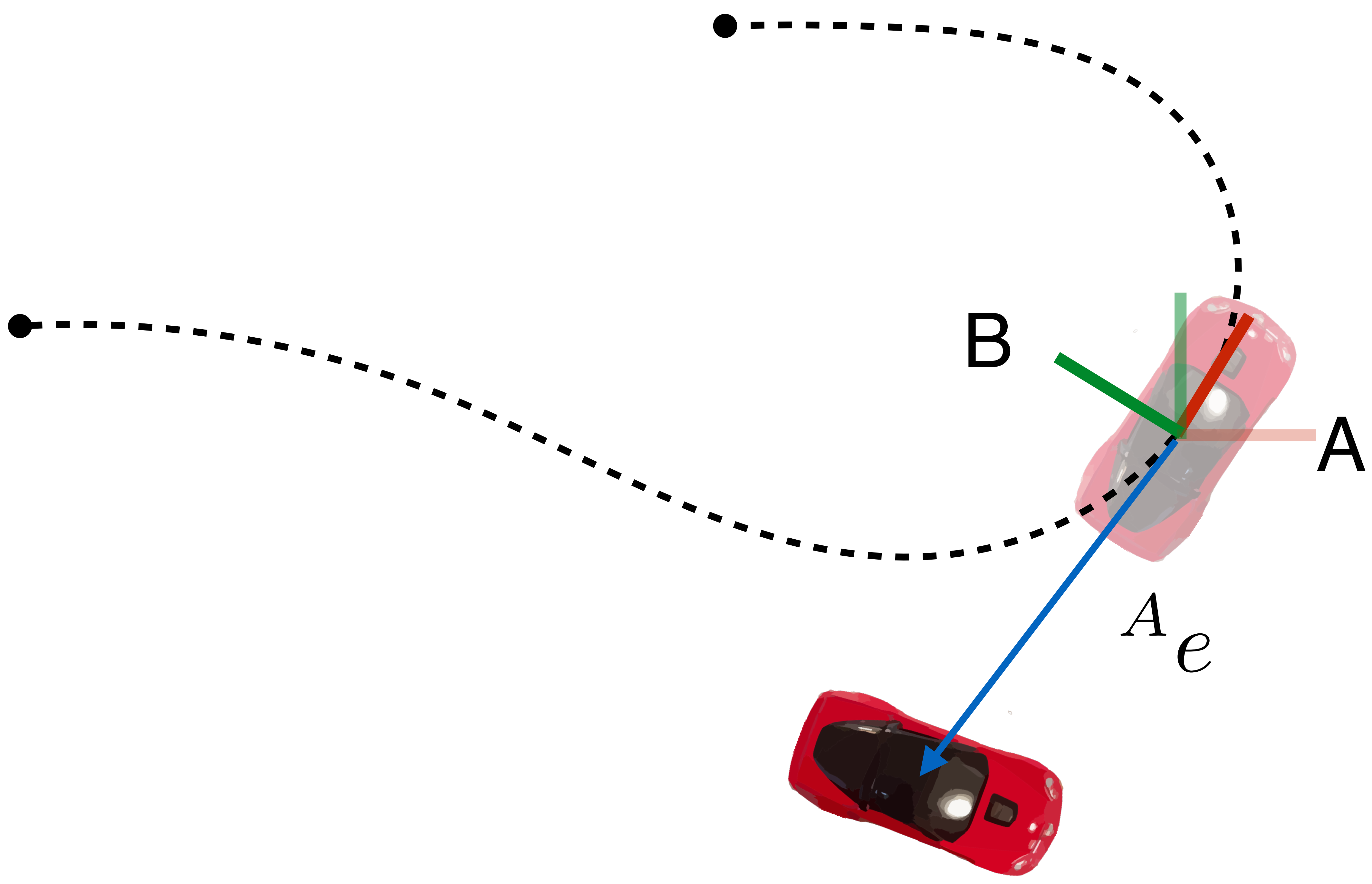
Step 3: Compute error to reference state



Position in frame A

$$^A e = \begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix}$$

Step 3: Compute error to reference state



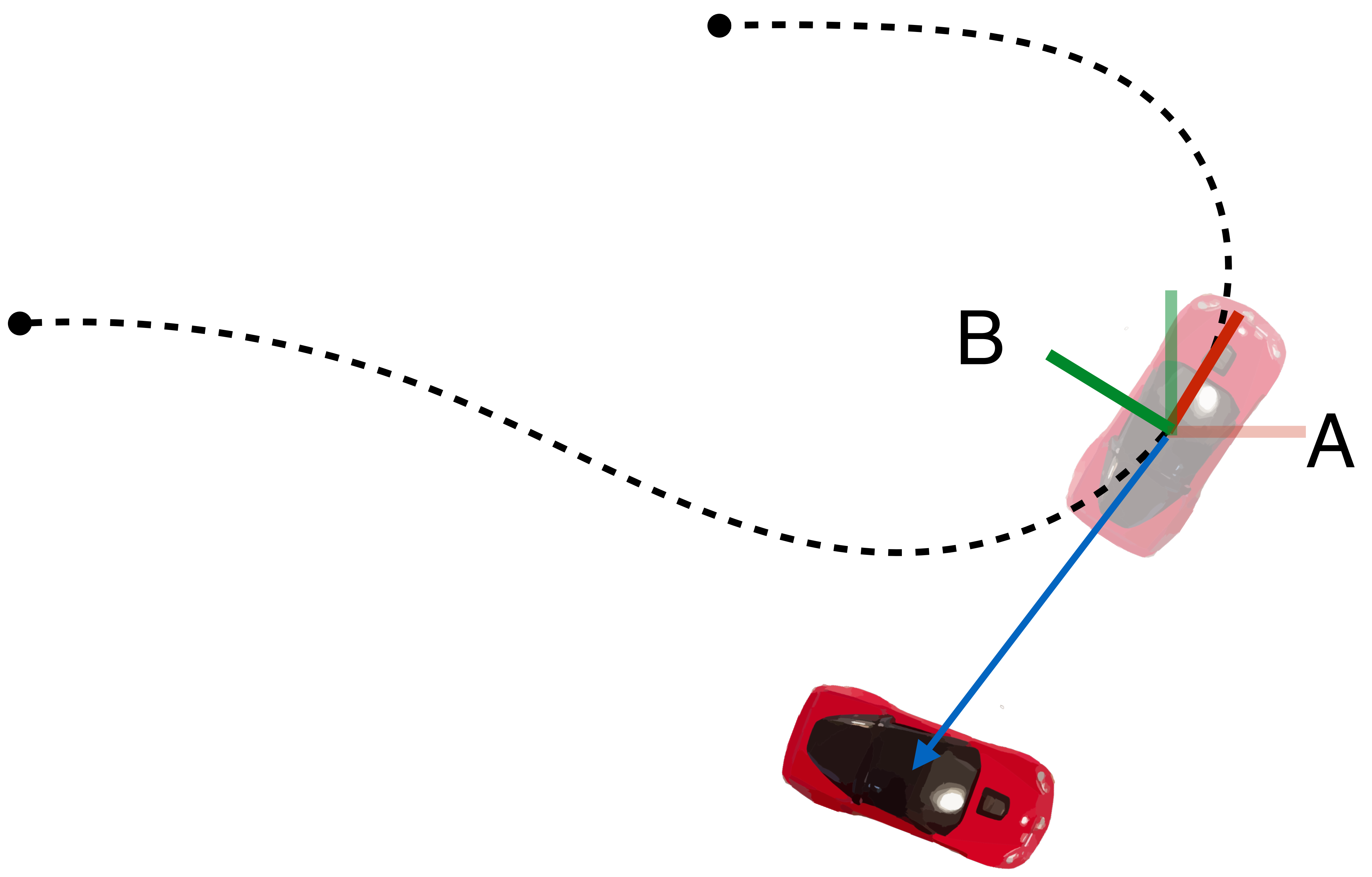
We want position in frame B

$$^{B}e = {}^{B}_{A}R \;\; ^{A}e = R(-\theta_{ref}) \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix} \right)$$

(rotation of A w.r.t B)

(rotation of A w.r.t B)

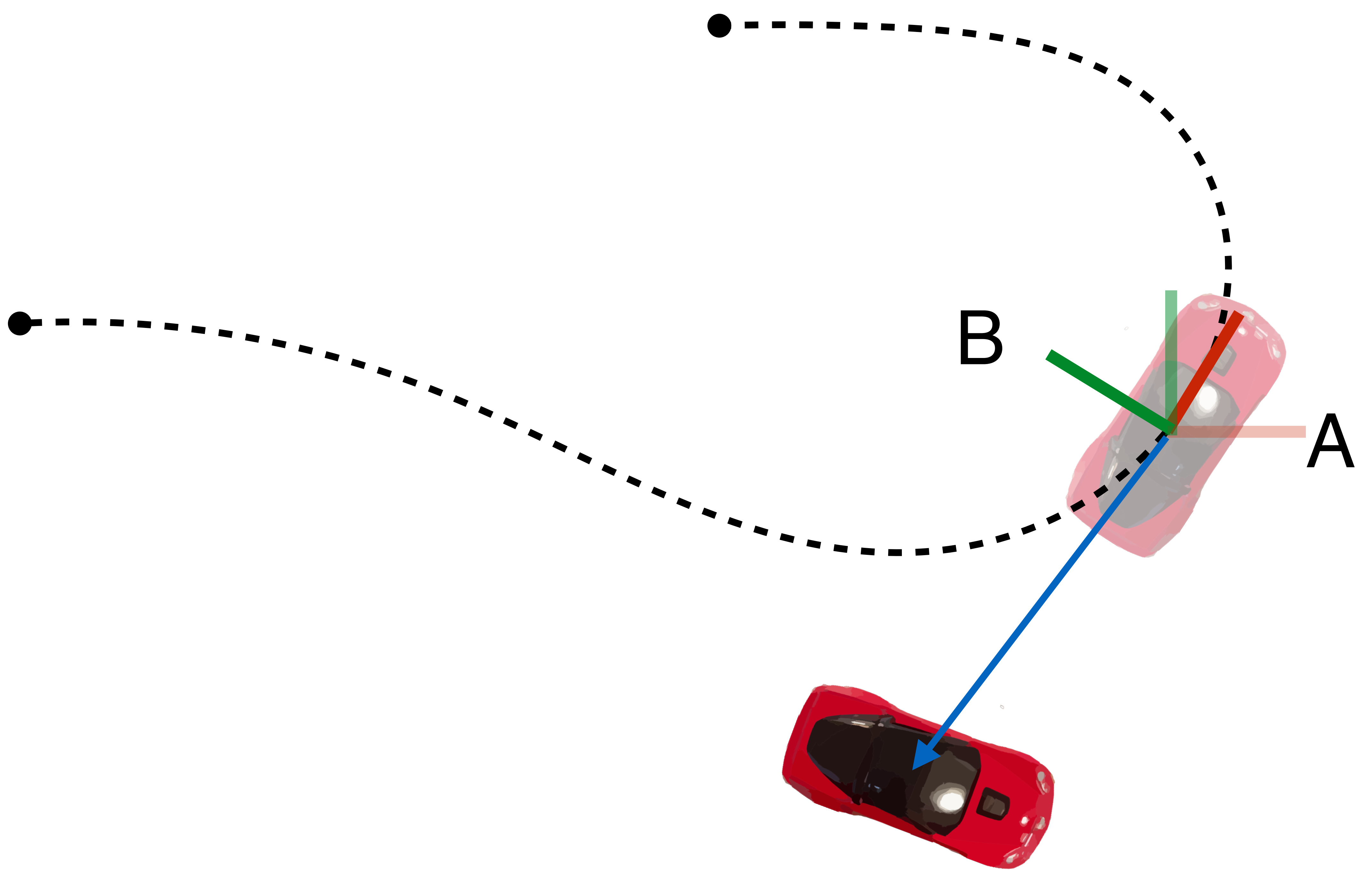
Step 3: Compute error to reference state



We want position in frame B

$$^{B}e = \begin{bmatrix} e_{at} \\ e_{ct} \end{bmatrix} = \begin{bmatrix} \cos(\theta_{ref}) & \sin(\theta_{ref}) \\ -\sin(\theta_{ref}) & \cos(\theta_{ref}) \end{bmatrix} \left(\begin{bmatrix} x \\ y \end{bmatrix} - \begin{bmatrix} x_{ref} \\ y_{ref} \end{bmatrix} \right)$$

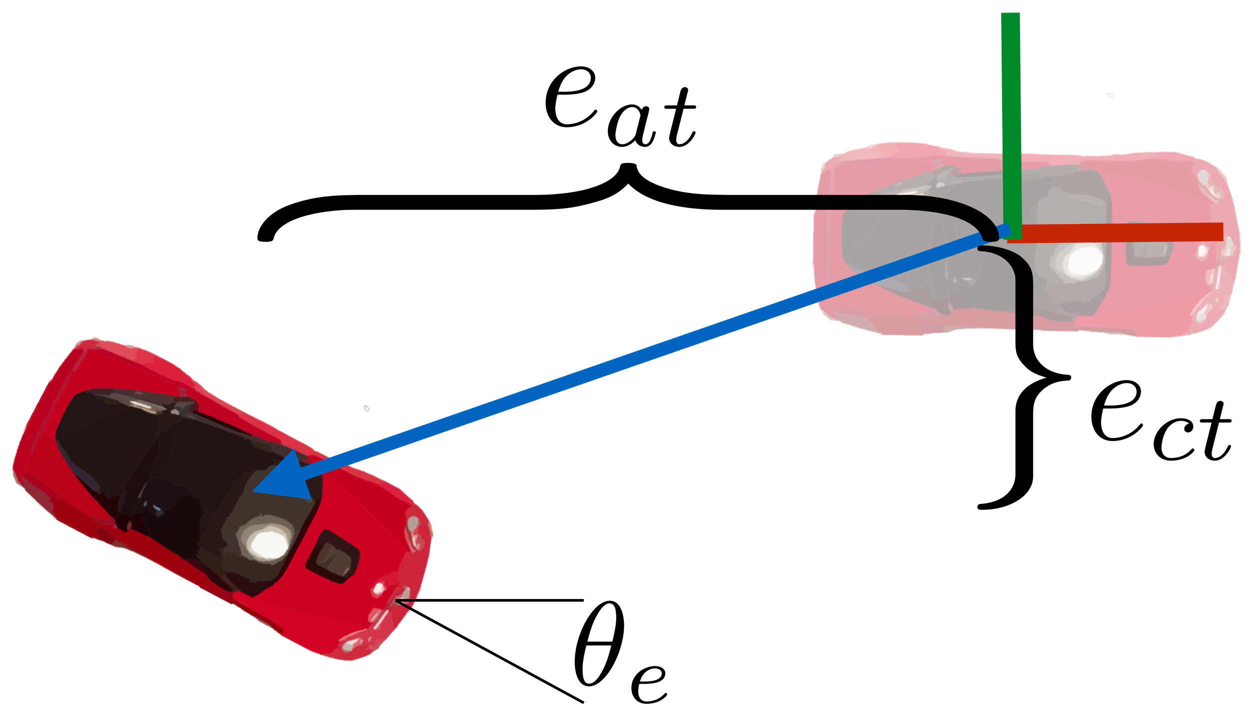
Step 3: Compute error to reference state



Heading error

$$\theta_e = \theta - \theta_{ref}$$

Step 3: Compute error to reference state

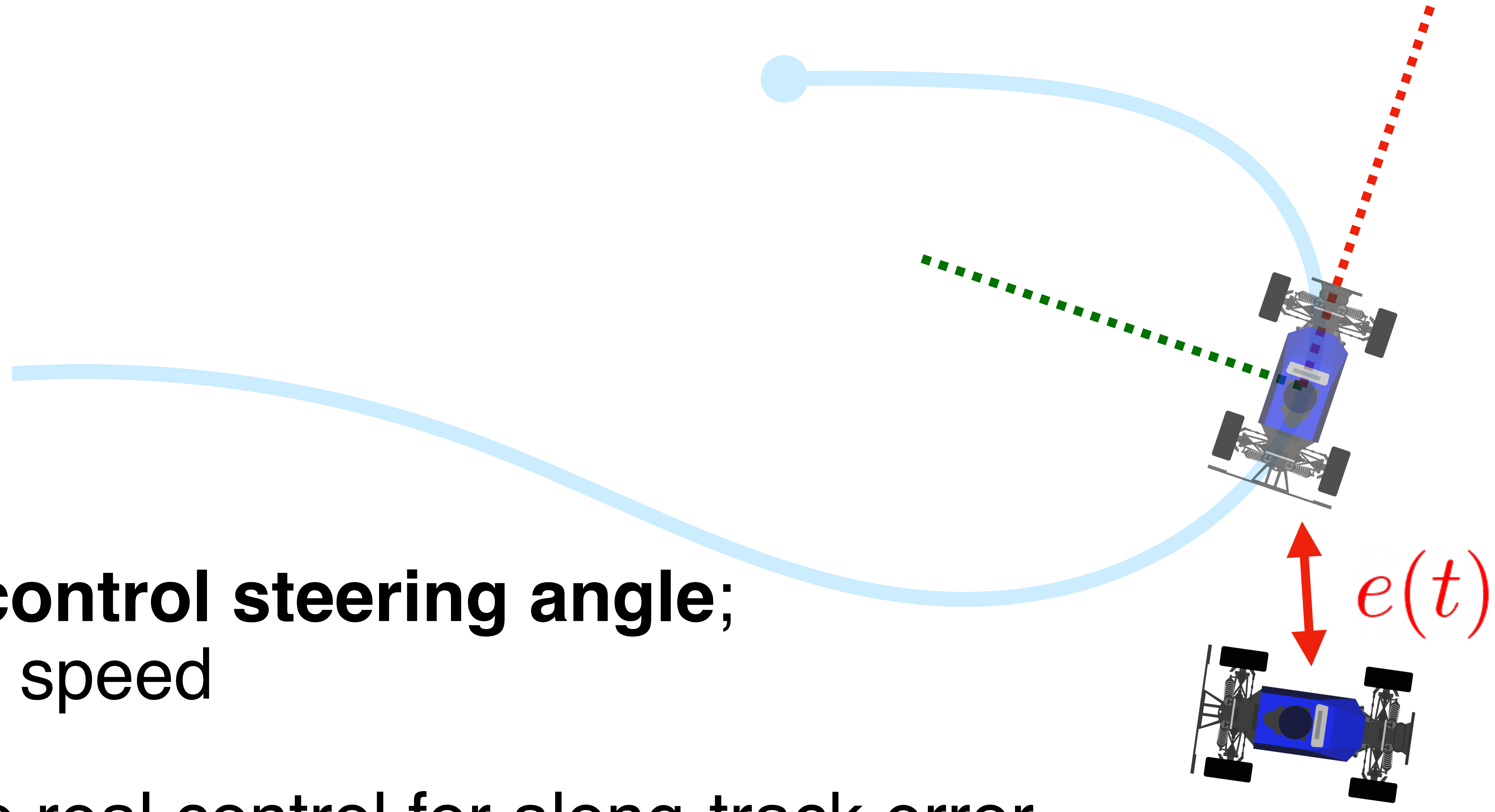


(Along-track) $e_{at} = \cos(\theta_{ref})(x - x_{ref}) + \sin(\theta_{ref})(y - y_{ref})$

(Cross-track) $e_{ct} = -\sin(\theta_{ref})(x - x_{ref}) + \cos(\theta_{ref})(y - y_{ref})$

(Heading) $\theta_e = \theta - \theta_{ref}$

Step 4: Compute control law



- We will **only control steering angle**; fixed constant speed
- As a result, no real control for along-track error
- Some control laws will only minimize cross-track error, others will also minimize heading

$$u = K(e)$$

Different Control Laws

- Proportional-integral-derivative (PID) control
- Pure-pursuit control
- Model-predictive control (MPC)
- Linear-quadratic regulator (LQR)
- And many many more!

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