# CSE 478 Robot Autonomy
# Motion Planning

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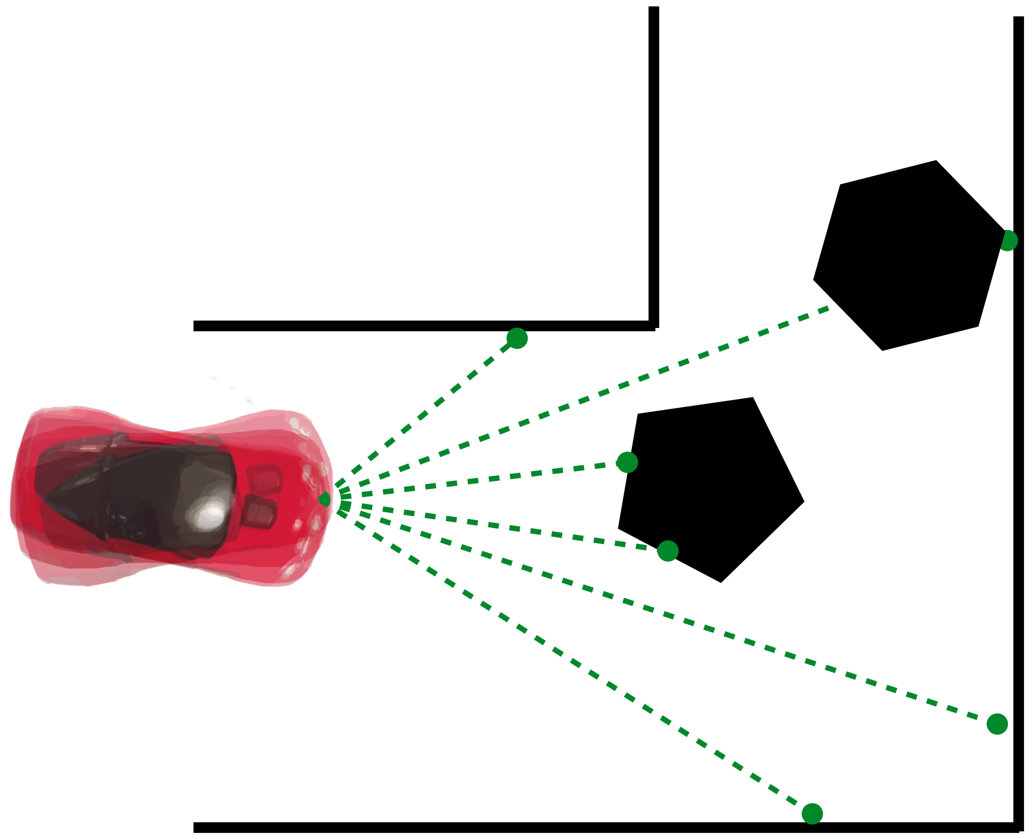
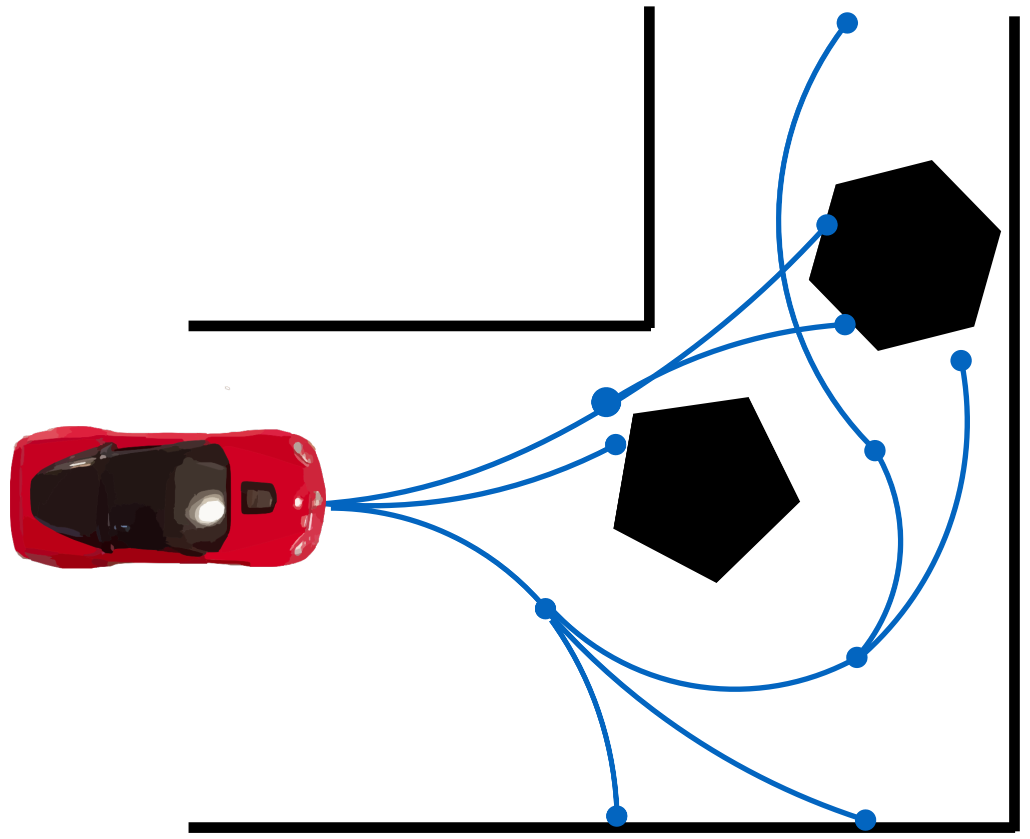
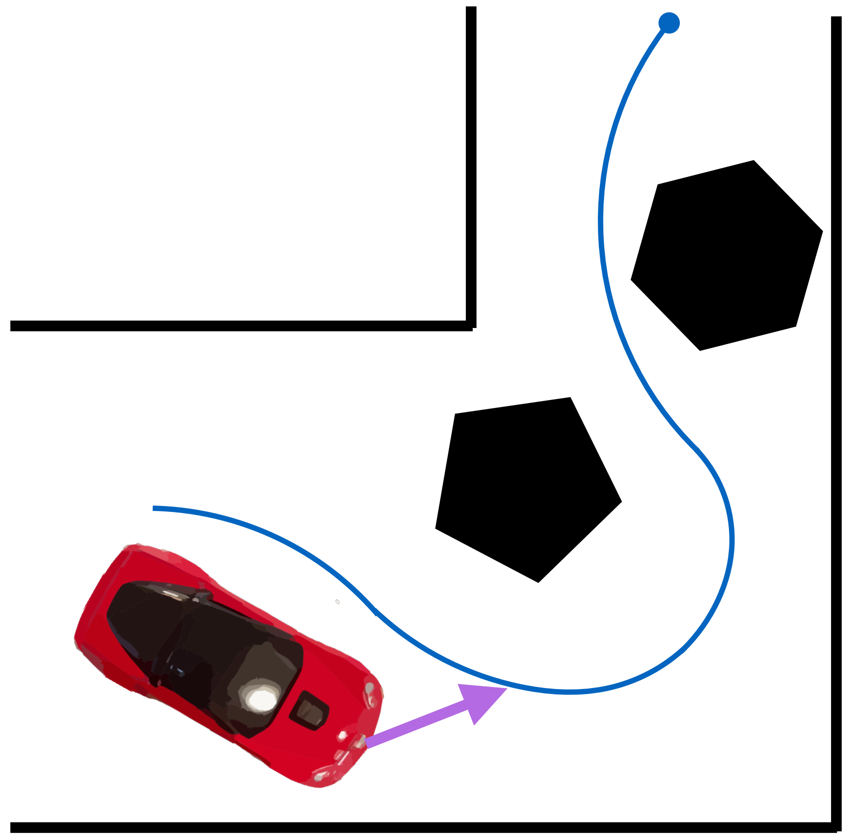
**Estimate state**

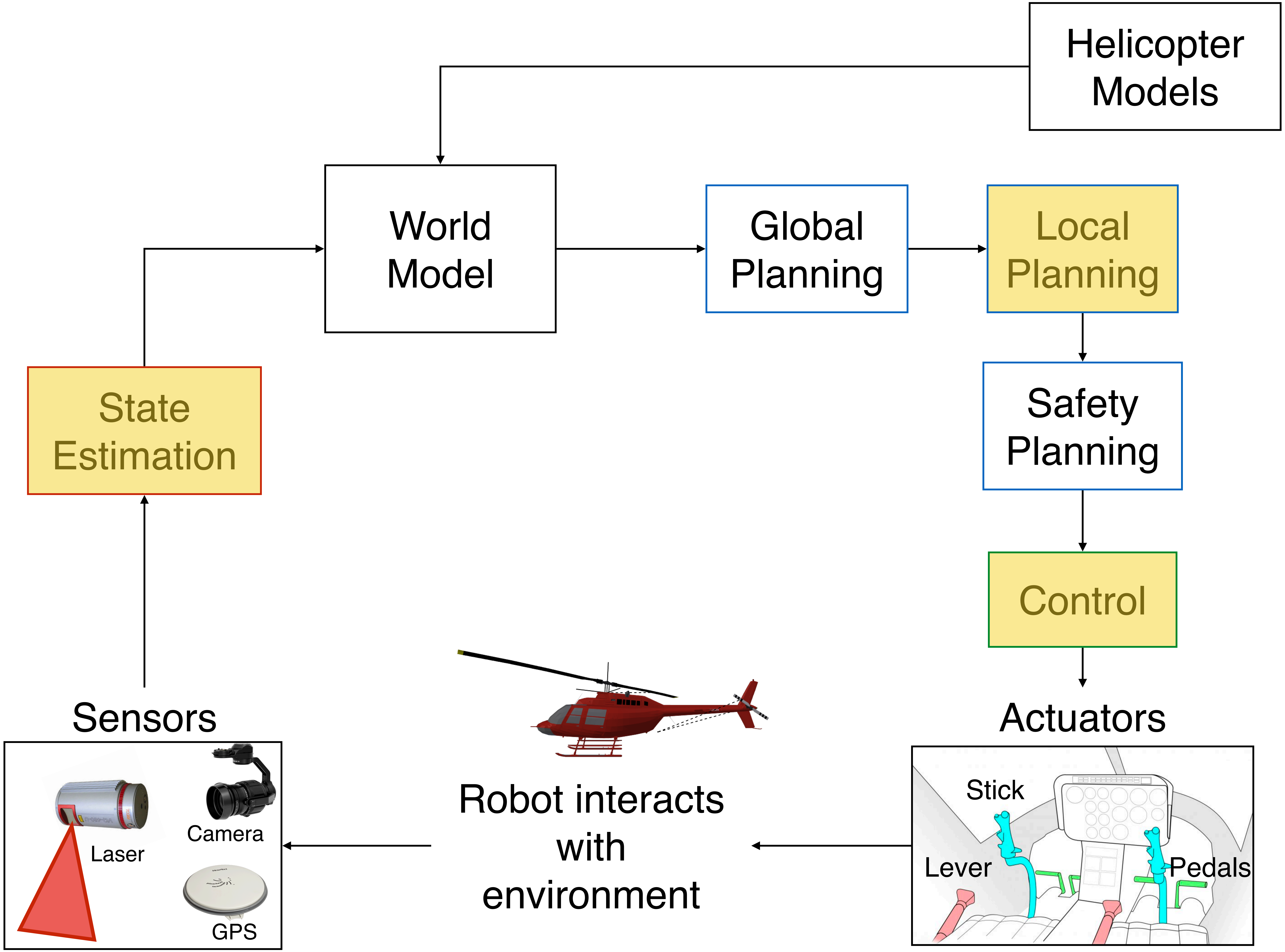


**Plan a sequence of motions**



**Control robot to follow plan**

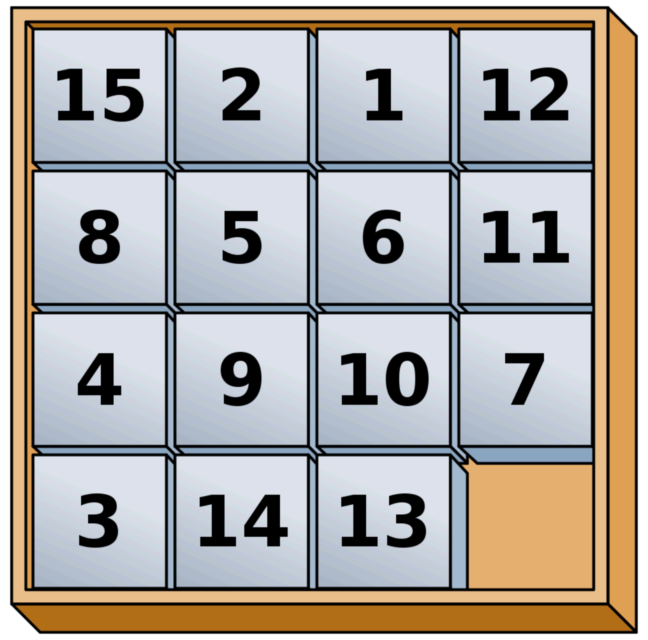


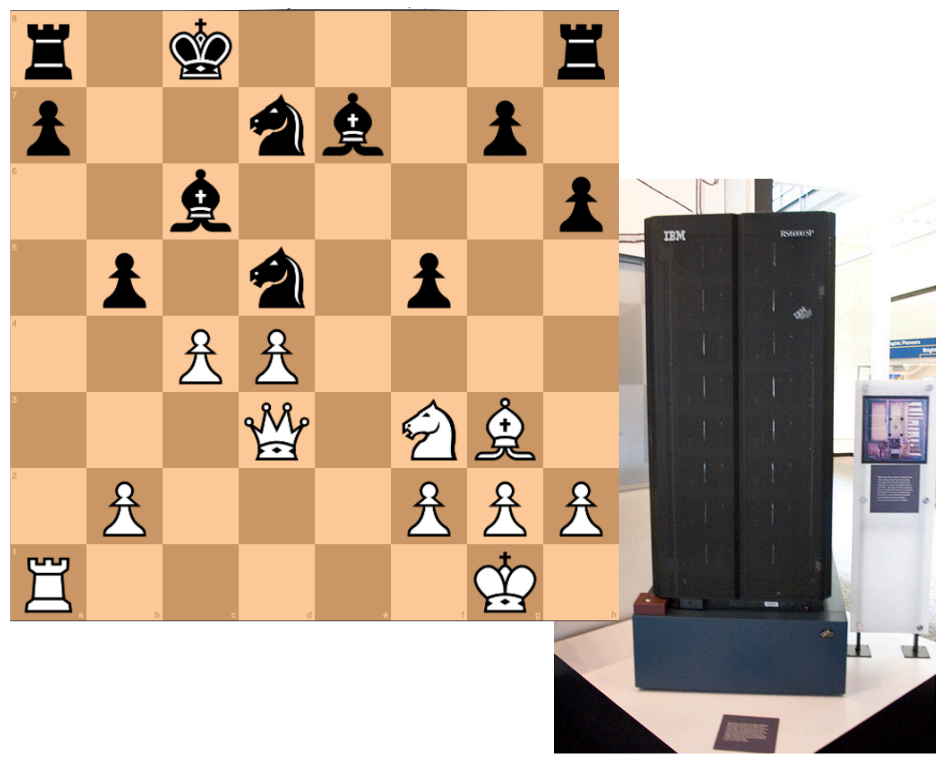


Helicopter Models
World Model
Global Planning
Local Planning
State Estimation
Safety Planning
Control
Sensors
Laser
Camera
GPS
Robot interacts with environment
Actuators
Stick
Lever
Pedals

# What Makes (Motion) Planning Difficult?

- Classic AI planning problems: Rubik's cube, sliding-tile puzzle, chess
  - Discrete state space, strictly-defined rules, humans have great intuition
  - Developed many of the tools that are still used today!
- (Some) challenges in motion planning: continuous state space, expensive action simulation, robot model uncertainty, nonholonomic constraints



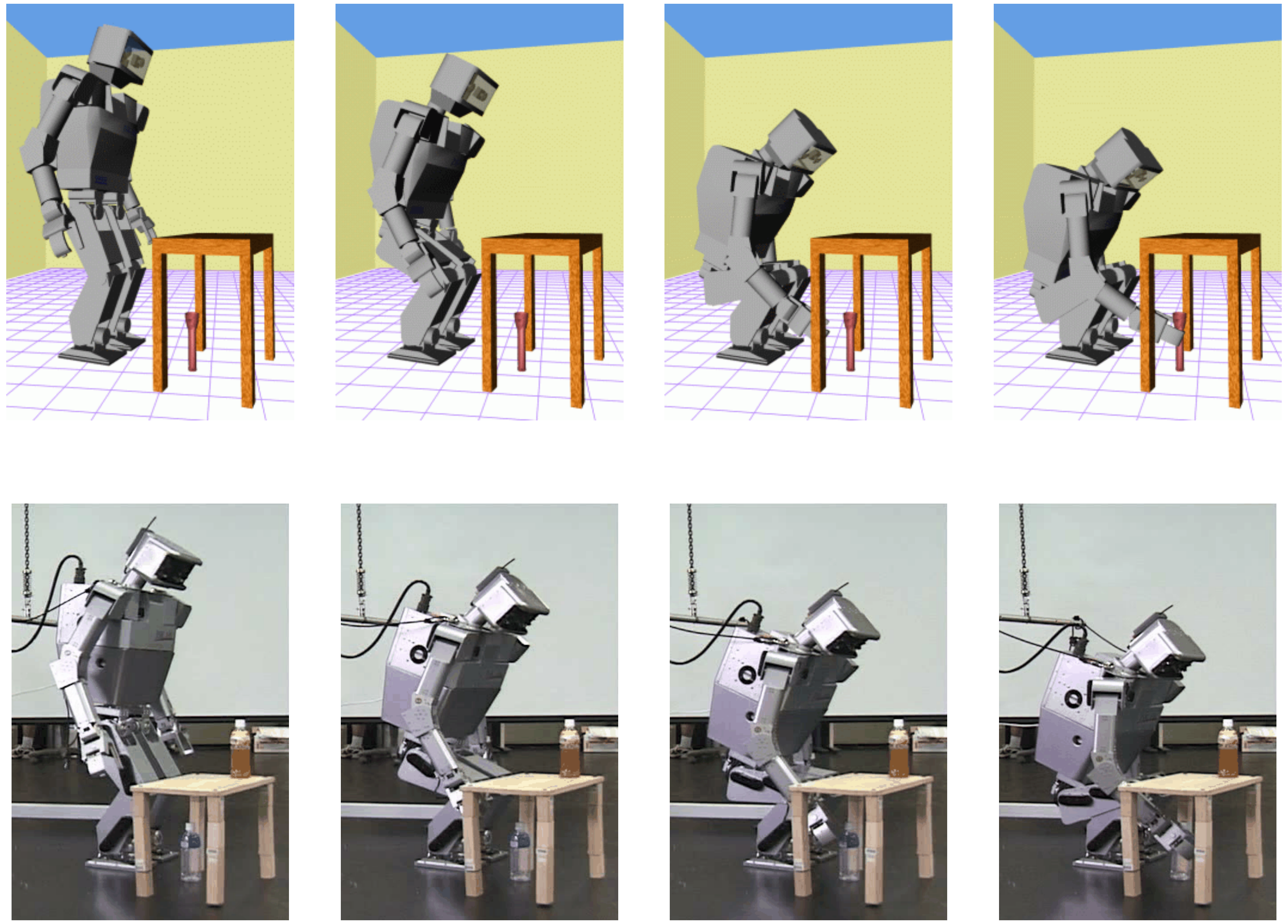
(WIKIPEDIA)

# The Piano Mover's Problem

SCHWARTZ AND SHARIR, 1983

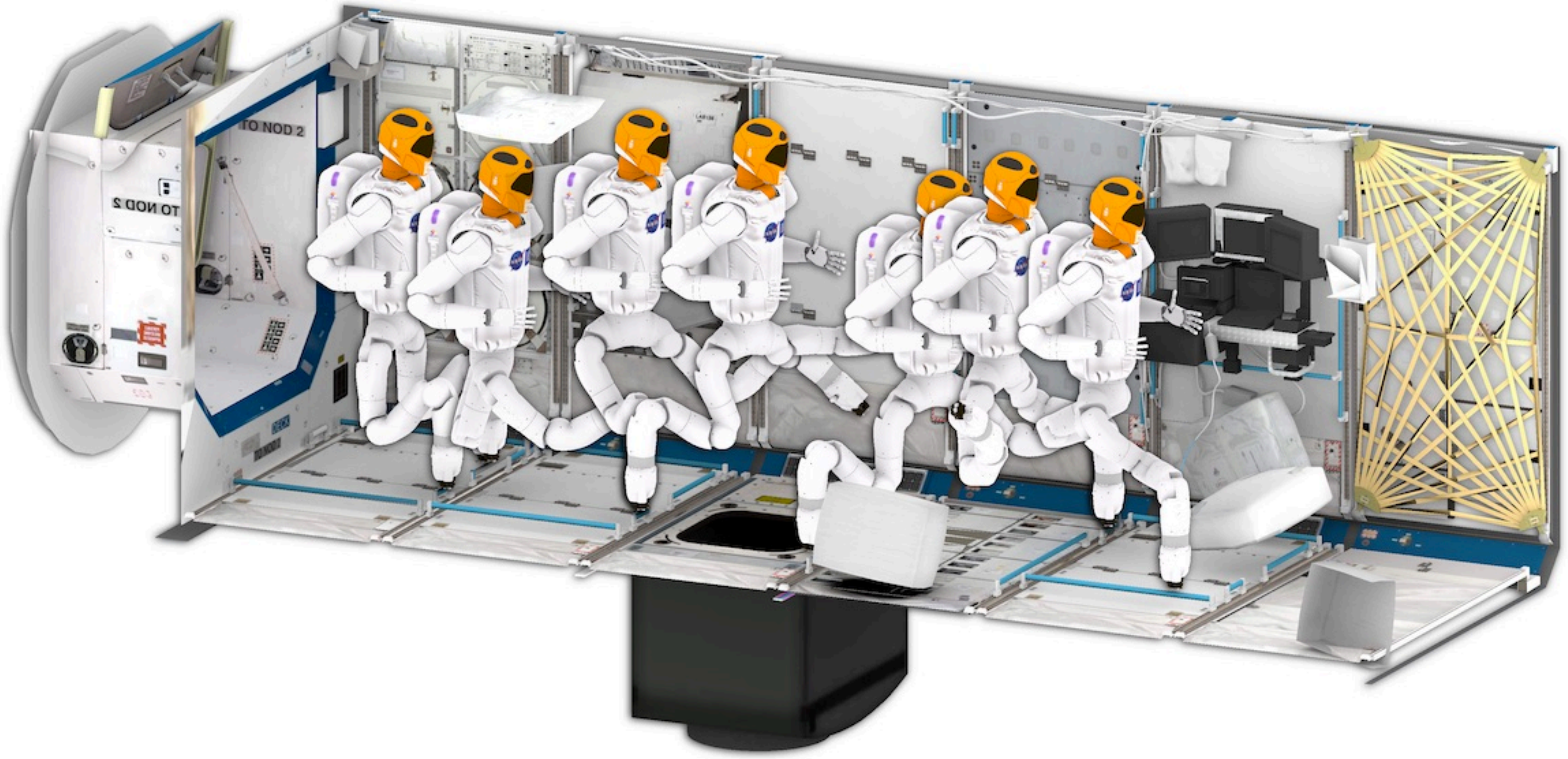
HTTPS://YOUTU.BE/UBAGTSNZABK

# High-Dimensional Planning



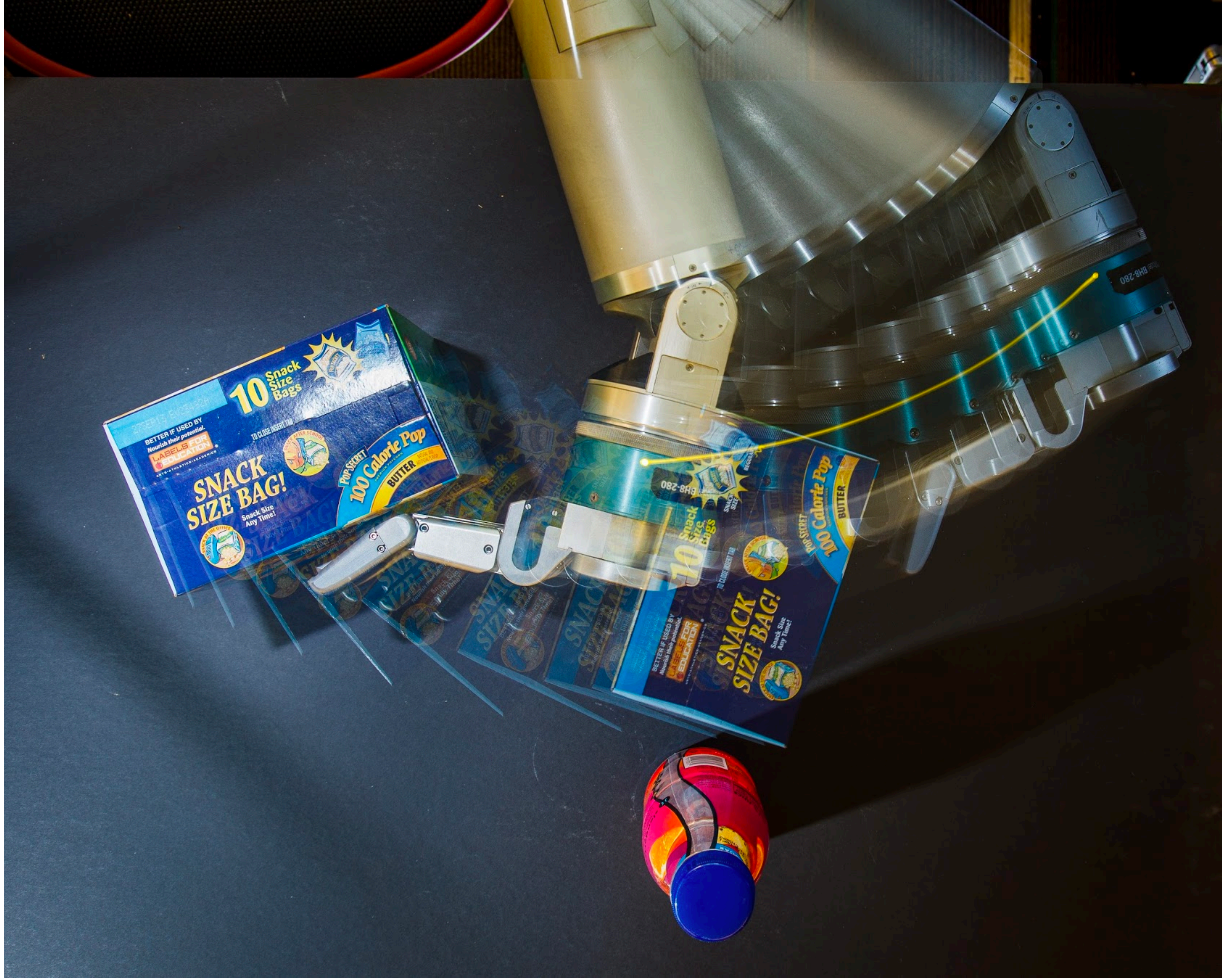
HONDA H7 HUMANOID ROBOT

KUFFNER ET AL., 2003



NASA R2 HUMANOID ROBOT

KINGSTON ET AL., 2019



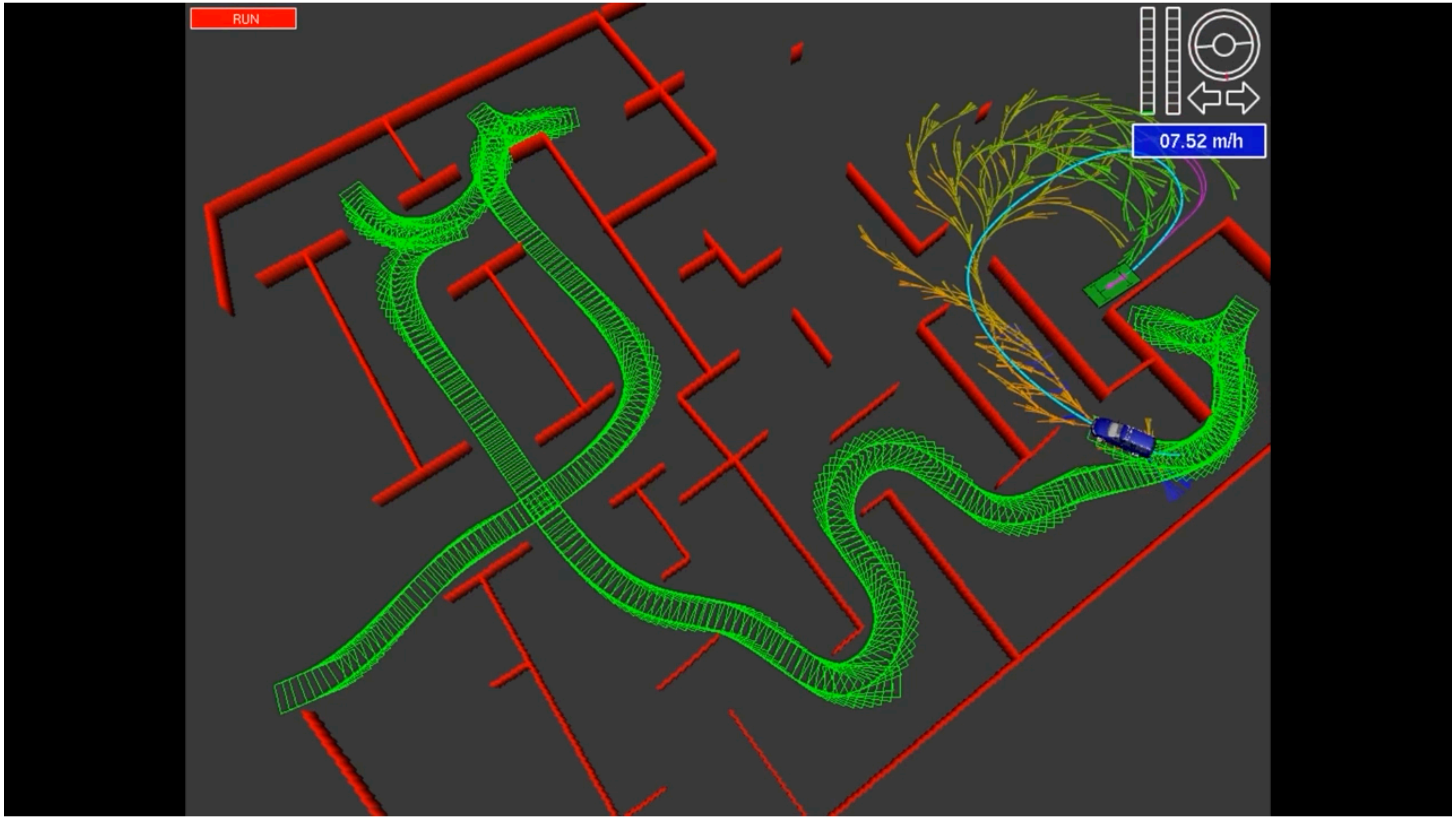

MANIPULATION PLANNING, HERB

# Real-time planning

Willow garage, 2009

https://www.youtube.com/watch?v=qbQDJ1c_Nxk&feature=youtu.be

# Real-Time Planning

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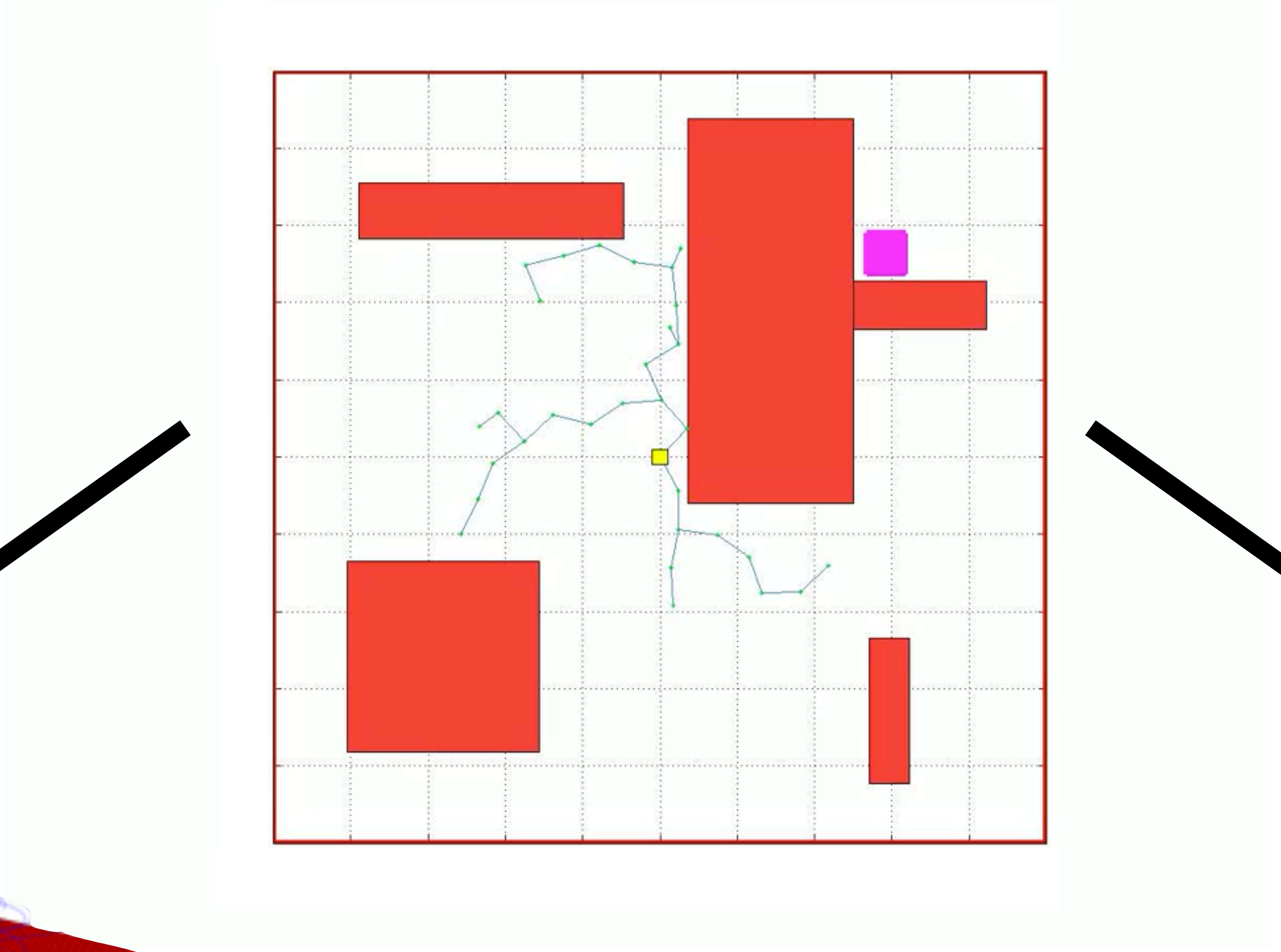
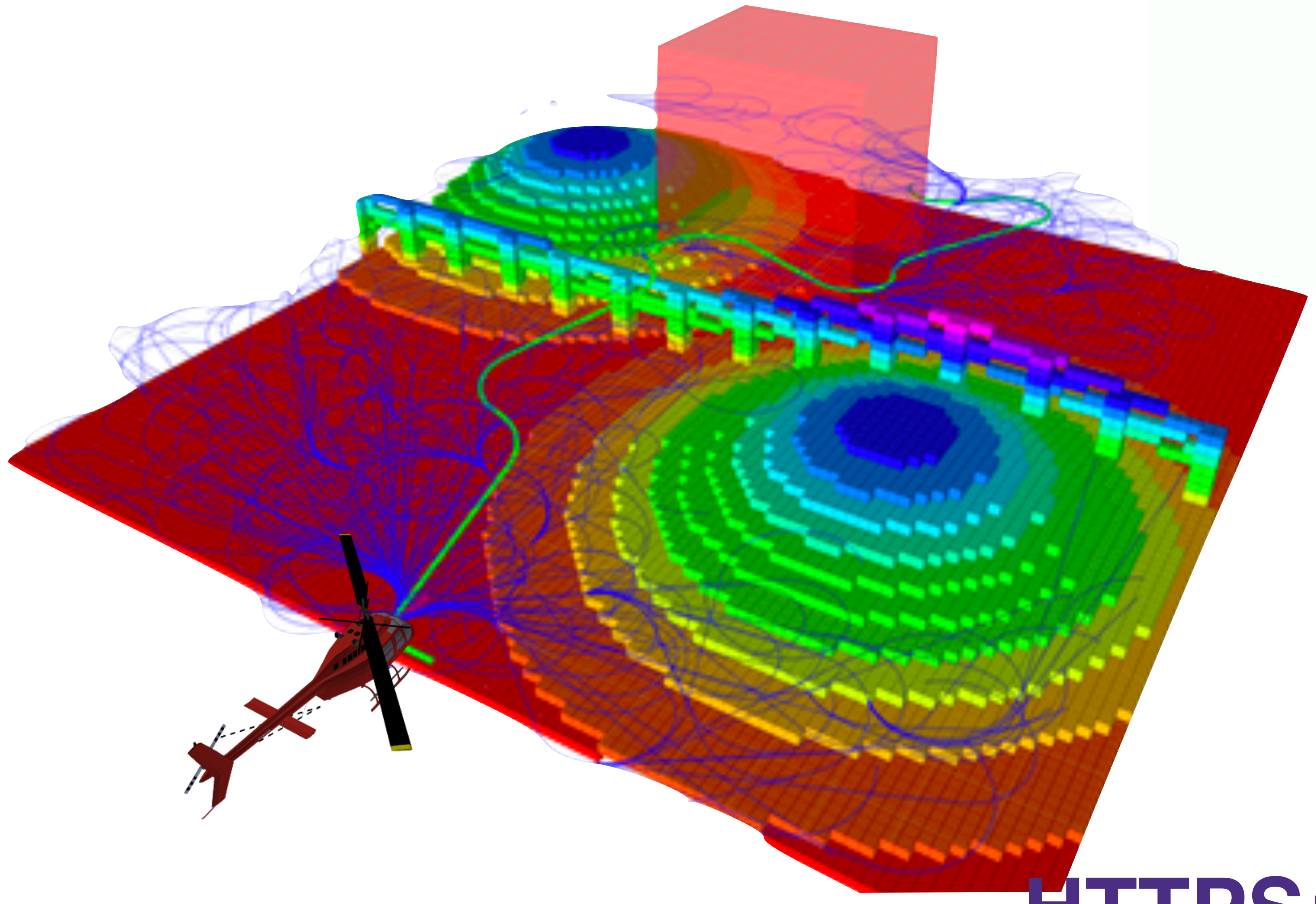
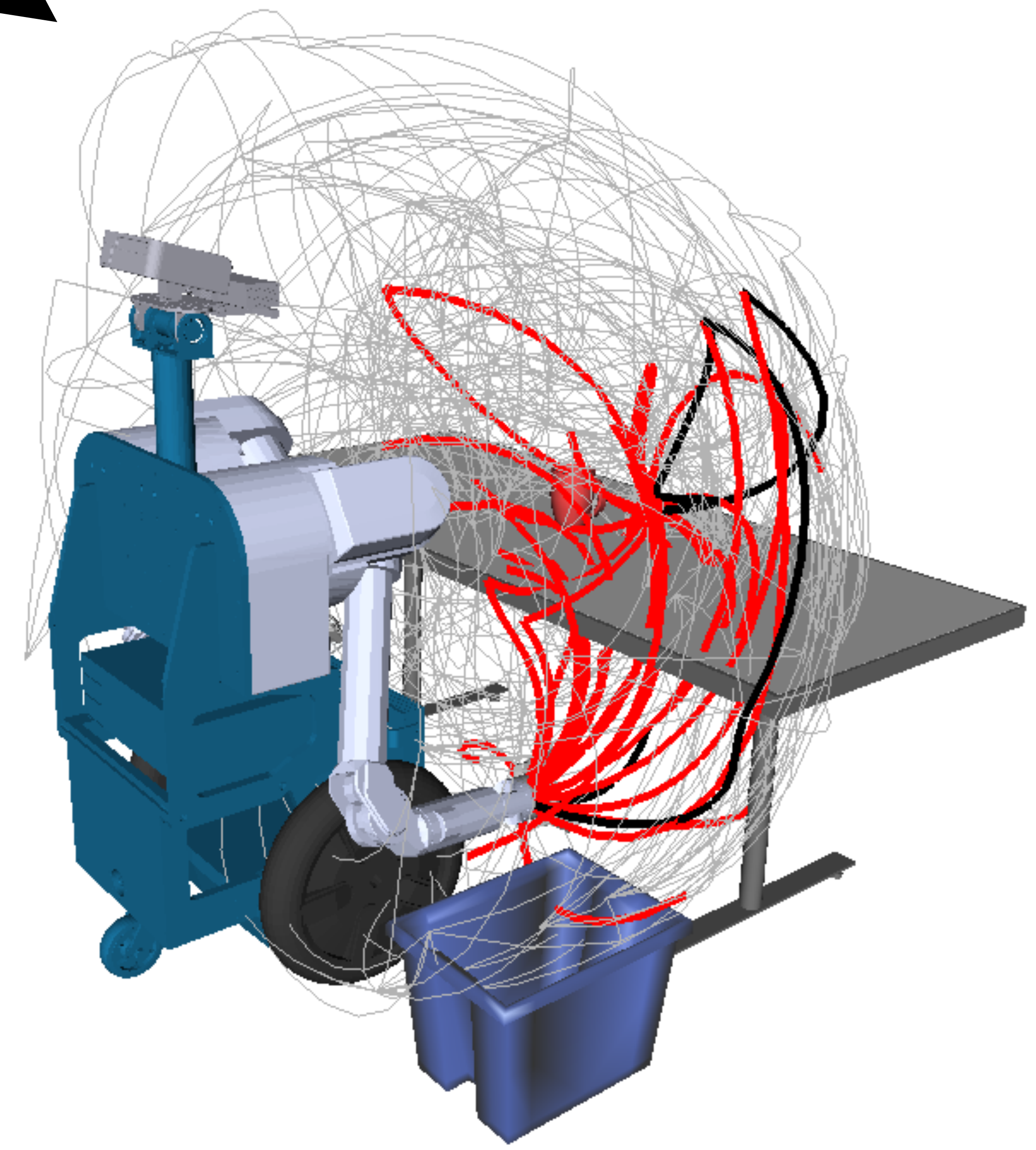
# Algorithms that Generalize

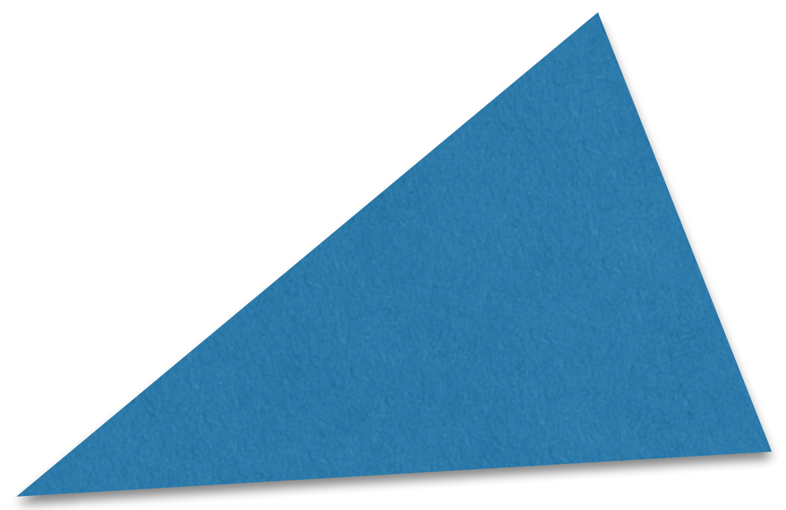




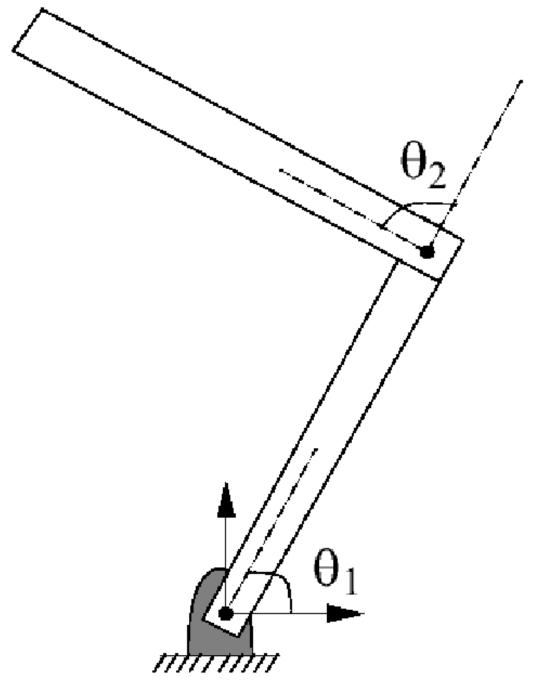


HTTPS://YOUTU.BE/FAFW8DOKVIK

# Representations that Generalize



(a) Translating Triangle




(b) 2-joint planar arm

(c) Racecar

(d) Manipulator

# The Configuration Space

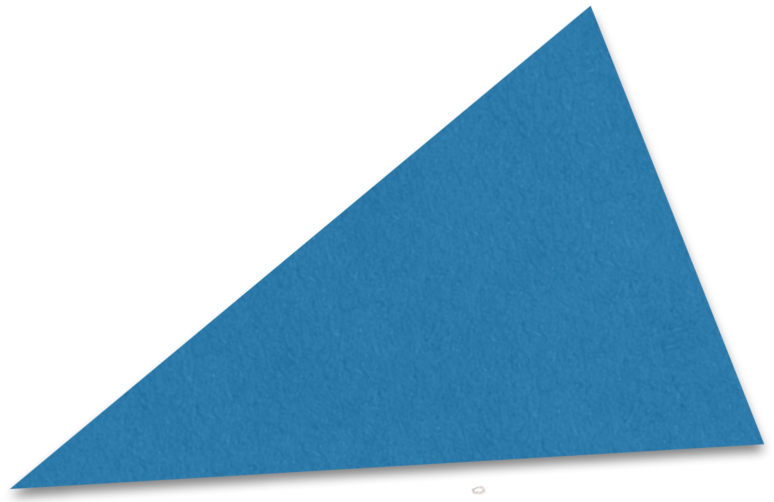
The configuration space or C-space is the manifold that contains the set of transformations achievable by the robot.

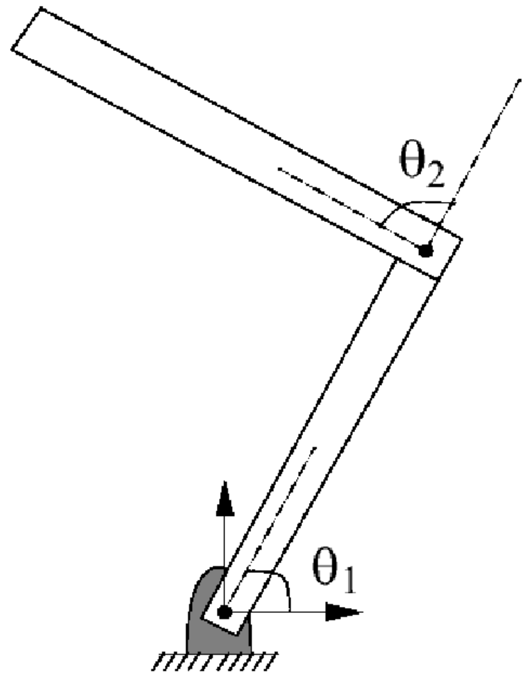
$$\mathcal{C}$$

Complete specification of the location of every point on robot geometry

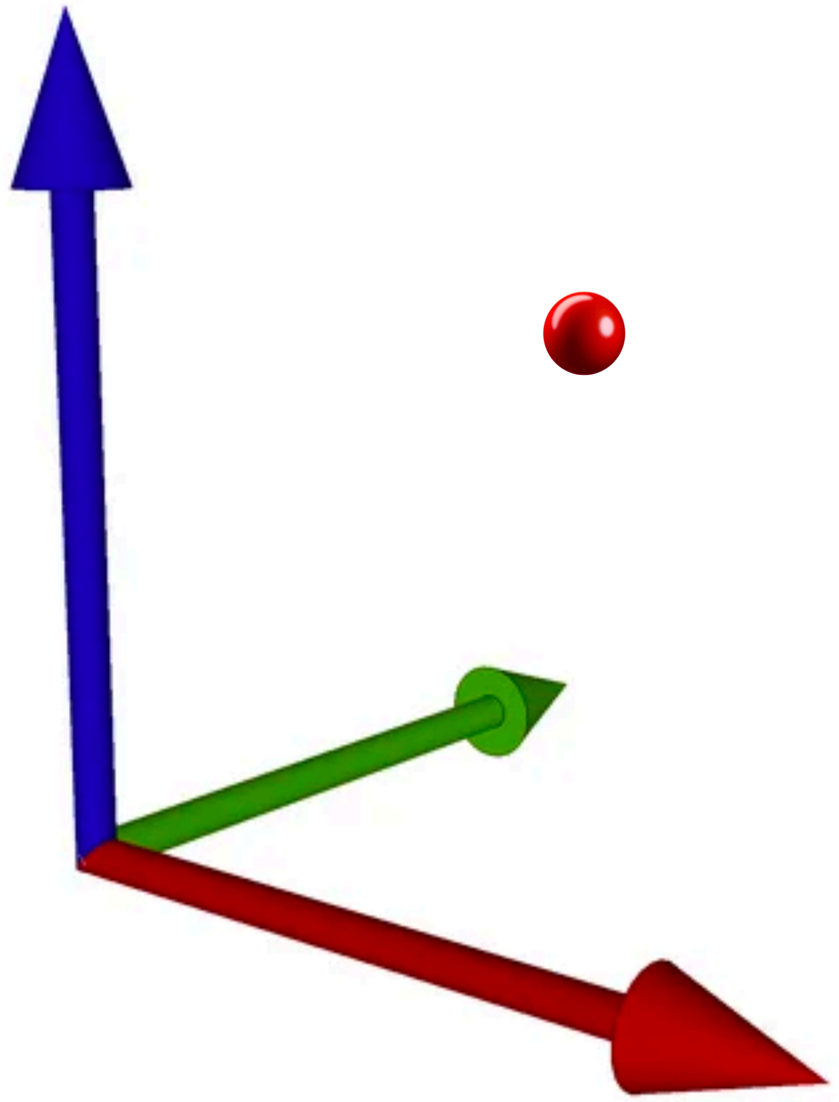
# Key Insight

Represent the robot as a point in some high-dimensional space

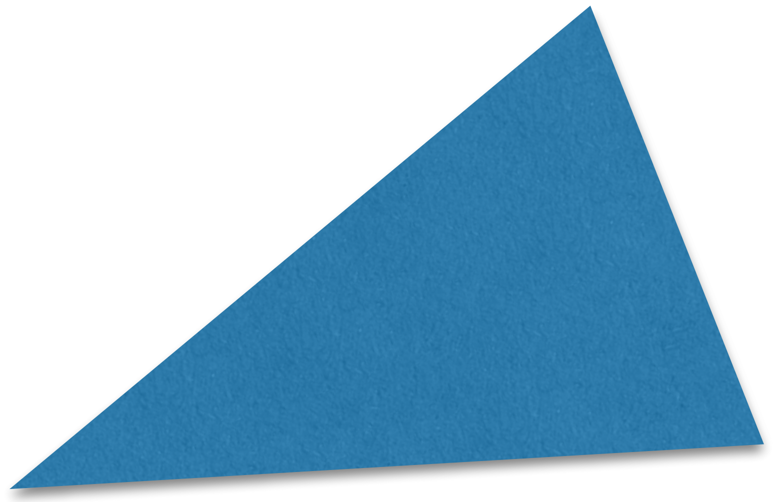





$$\mathcal{C}$$



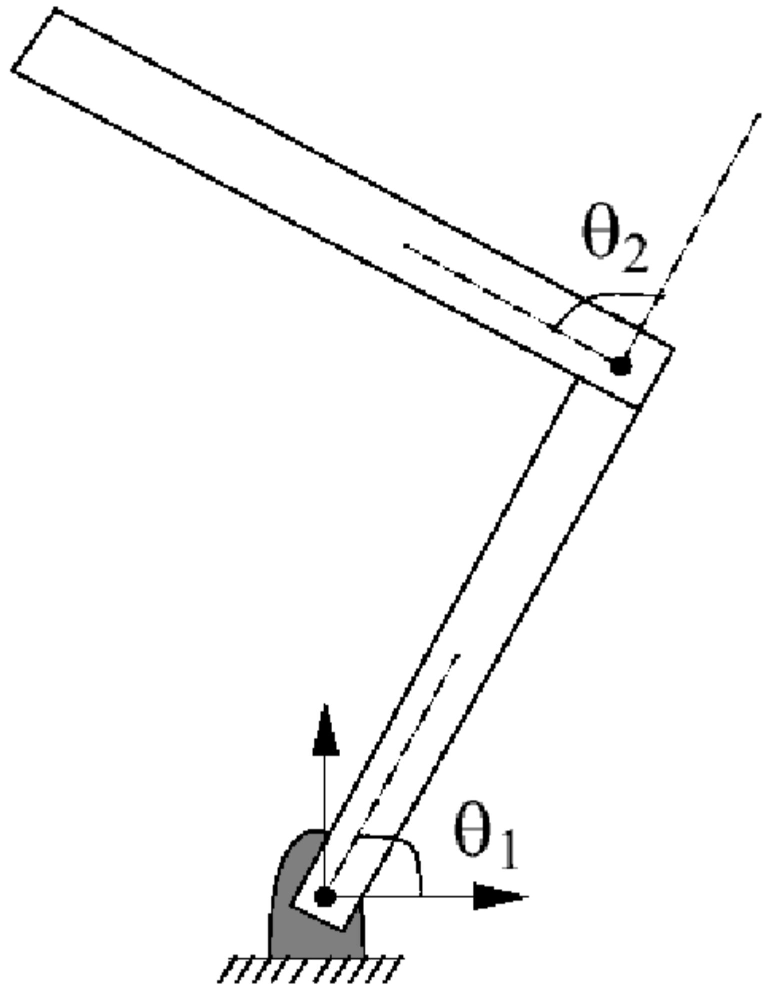
# Example 1: Translating triangle

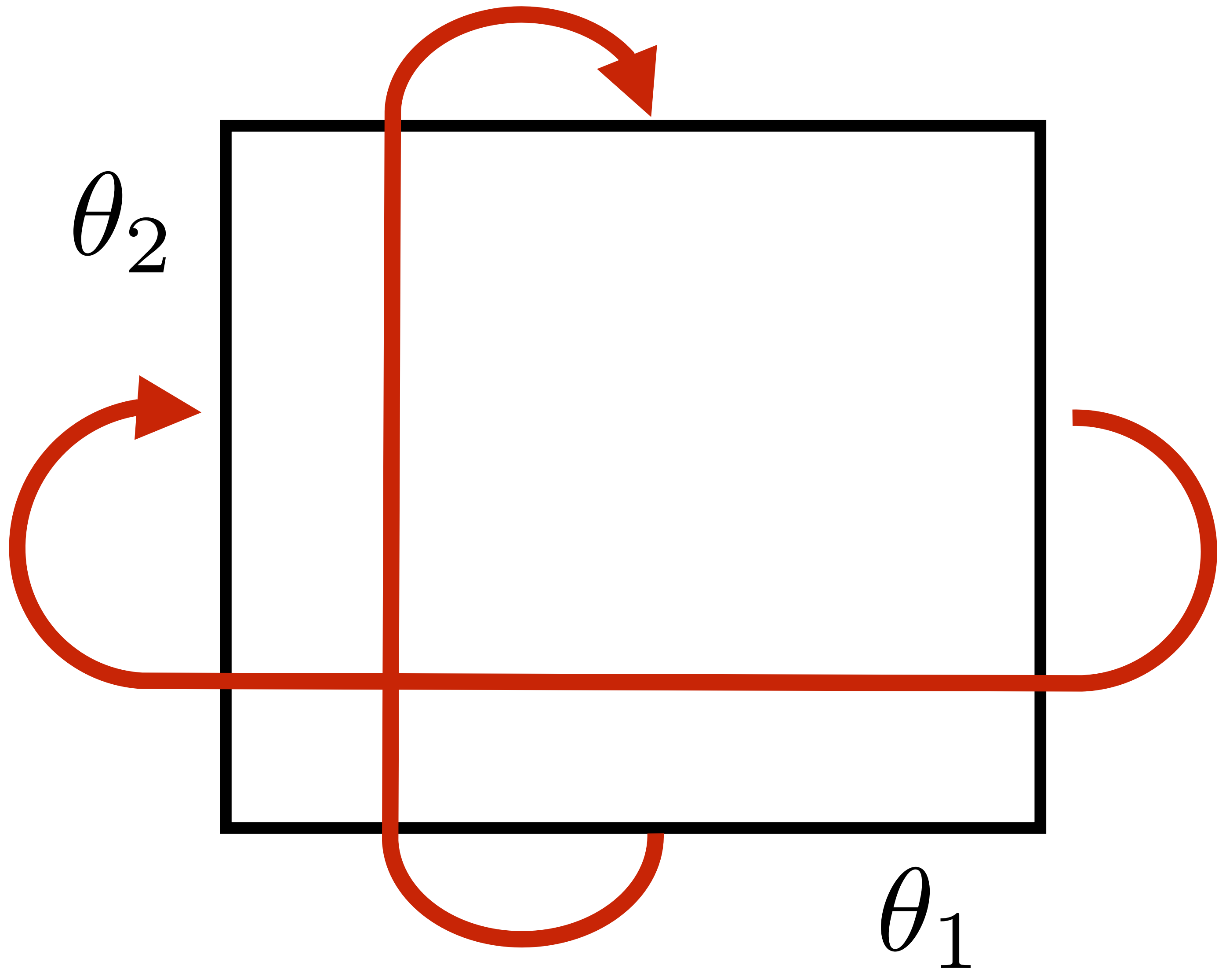


$$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$$

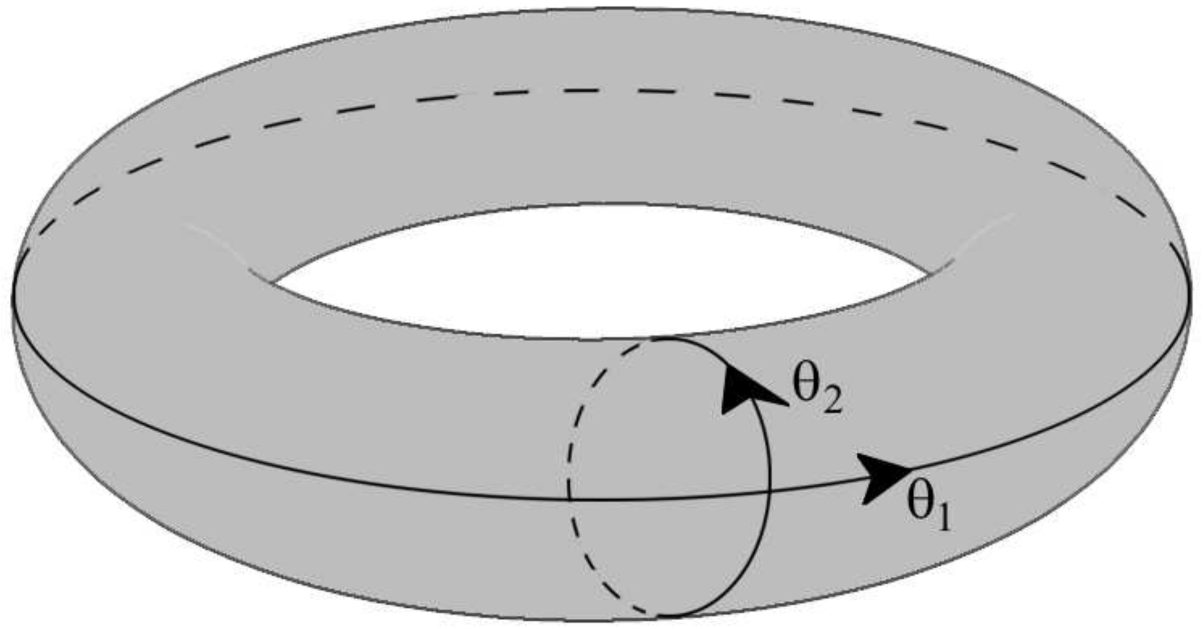
(cartesian product)

# Example 2: 2-joint planar arm




$$\mathbb{S}^1 \times \mathbb{S}^1 = \mathbb{T}^2$$




Circle

$$\mathbb{S}^1 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}.$$

# Example 3: Racecar

$$\mathbb{R}^2 \times \mathbb{S}^1$$

special euclidean group $SE(2)$

# Common C-spaces

| **Type of Robot** | **$\mathcal{C}$-space Representation** |
|---|---|
| Mobile robot translating in the plane | $\mathbb{R}^2$ |
| Mobile robot translating and rotating in the plane | $SE(2)$ or $\mathbb{R}^2 \times S^1$ |
| Rigid body translating in the three-space | $\mathbb{R}^3$ |
| A spacecraft | $SE(3)$ or $\mathbb{R}^3 \times SO(3)$ |
| An $n$-joint revolute arm | $T^n$ |
| A planar mobile robot with an attached $n$-joint arm | $SE(2) \times T^n$ |

(Kavraki and LaValle)

# Obstacles

# Obstacle specification

Robot operates in a 2D / 3D workspace $\mathcal{W} = \mathbb{R}^2 \text{ or } \mathbb{R}^3$

Subset of this space is obstacles $\mathcal{O} \subset \mathcal{W}$

semi-algebraic models (polygons, polyhedra)

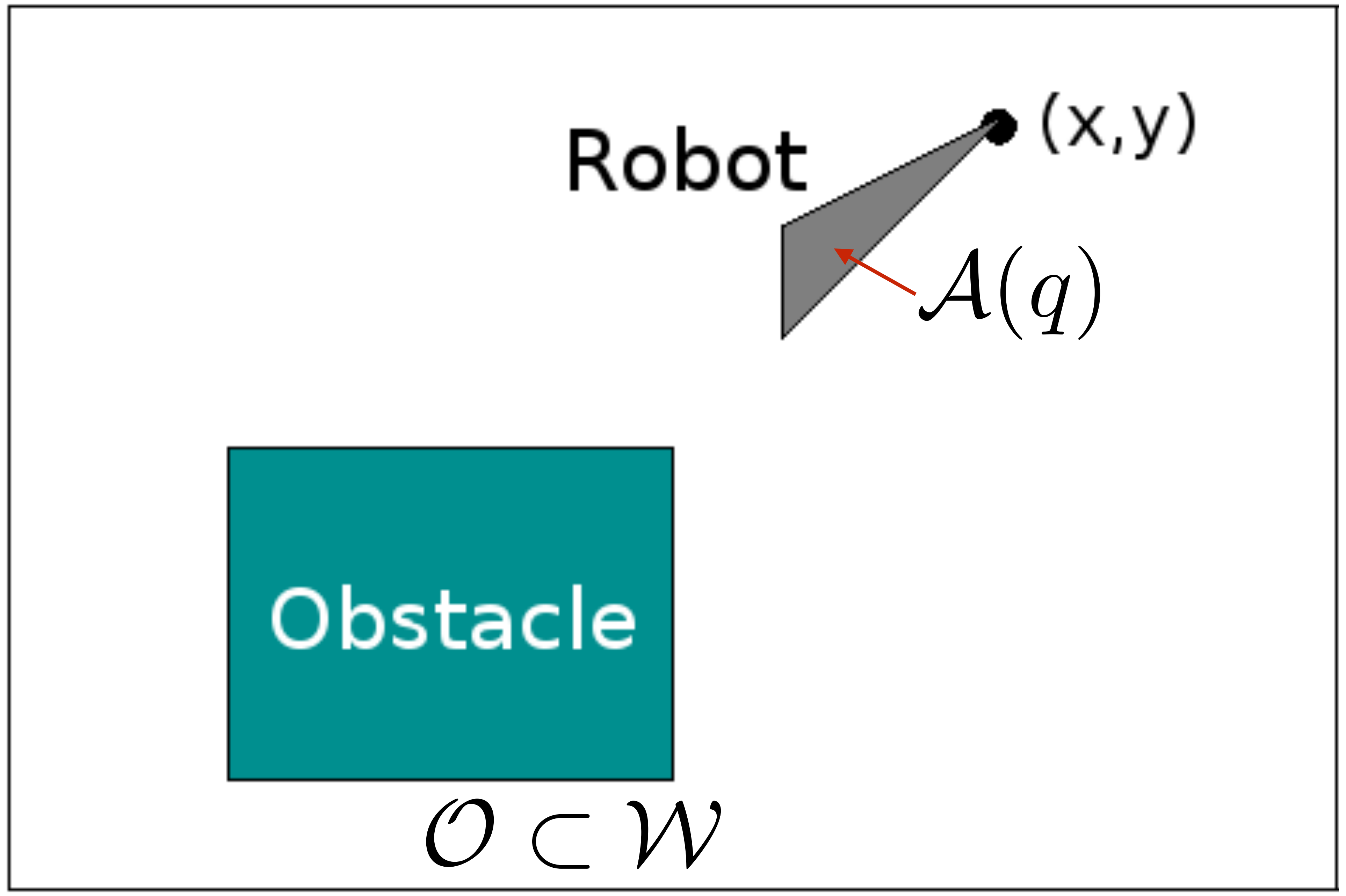
Geometric shape of the robot
(set of points occupied by robot at a config) $\mathcal{A}(q) \subset \mathcal{W}$

C-space obstacle region

$$\mathcal{C}_{obs} = \{\boldsymbol{q} \in \mathcal{C} \mid \mathcal{A}(\boldsymbol{q}) \cap \mathcal{O} \neq \emptyset\}$$

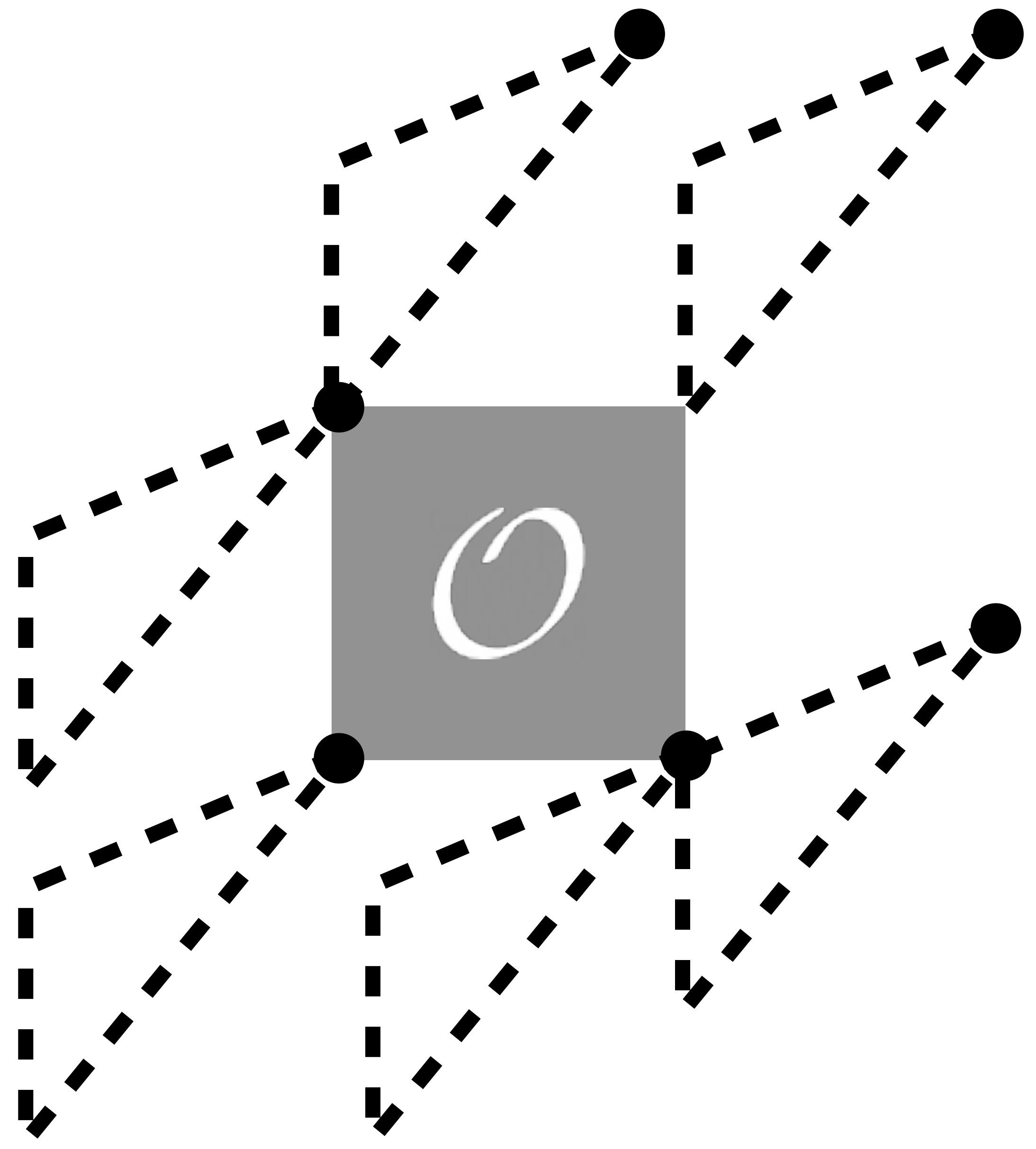
$$\mathcal{C}_{free} = \mathcal{C} \setminus \mathcal{C}_{obs}$$

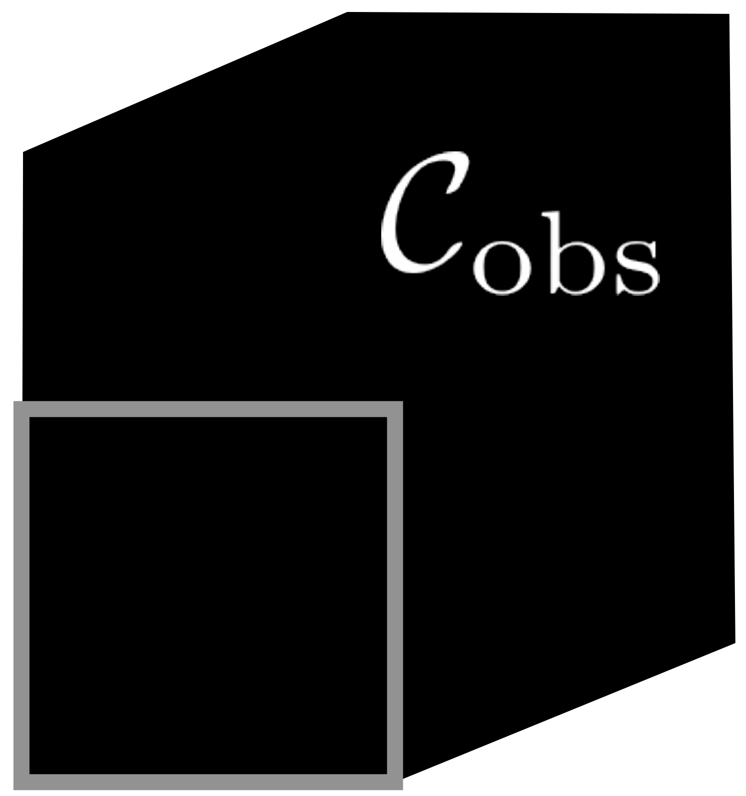
# Example 1: Translating triangle




Can be efficiently computed using Minkowski sum

# Example: Translating Triangle in Plane

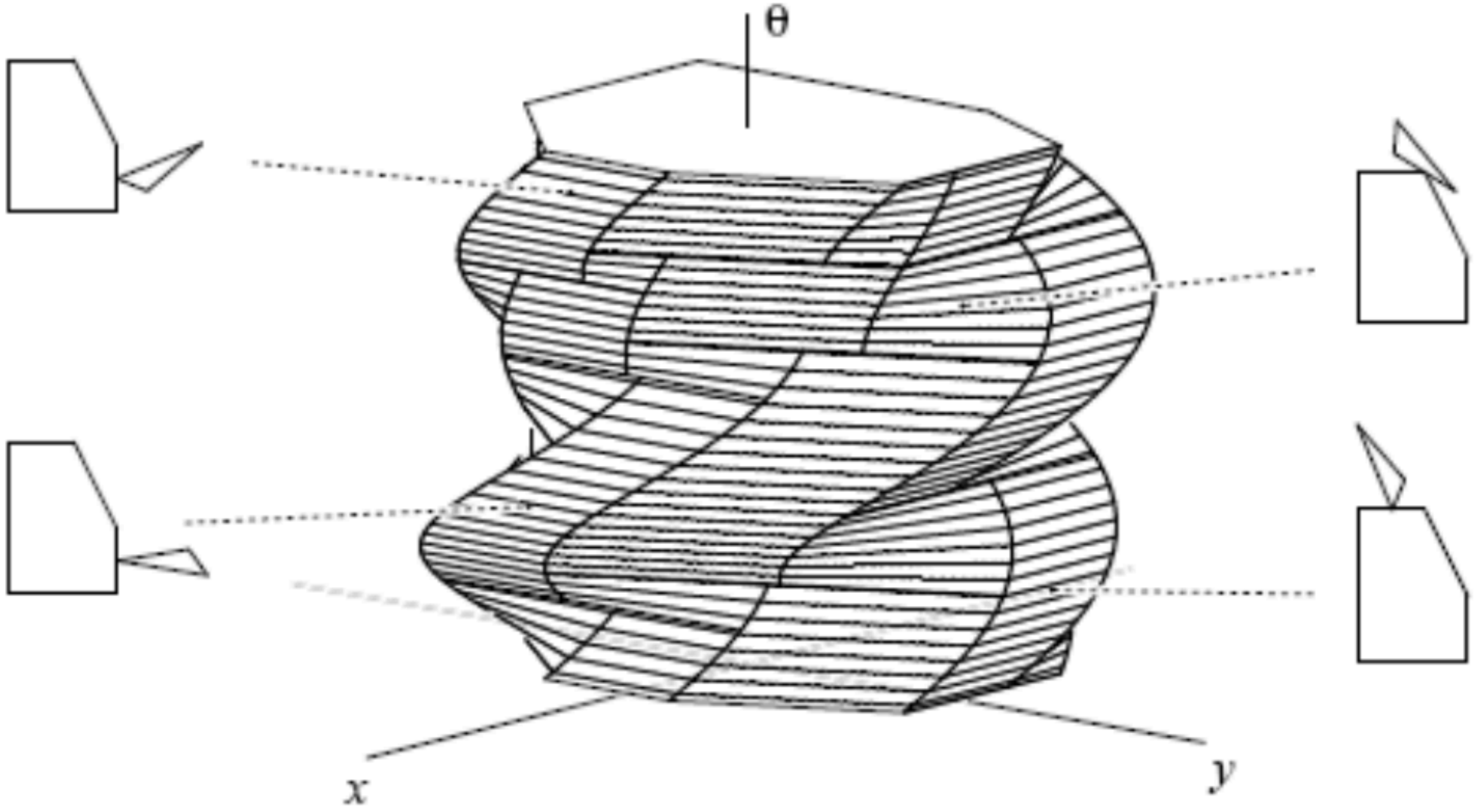



Can be computed for convex polygons (Minkowski sum)

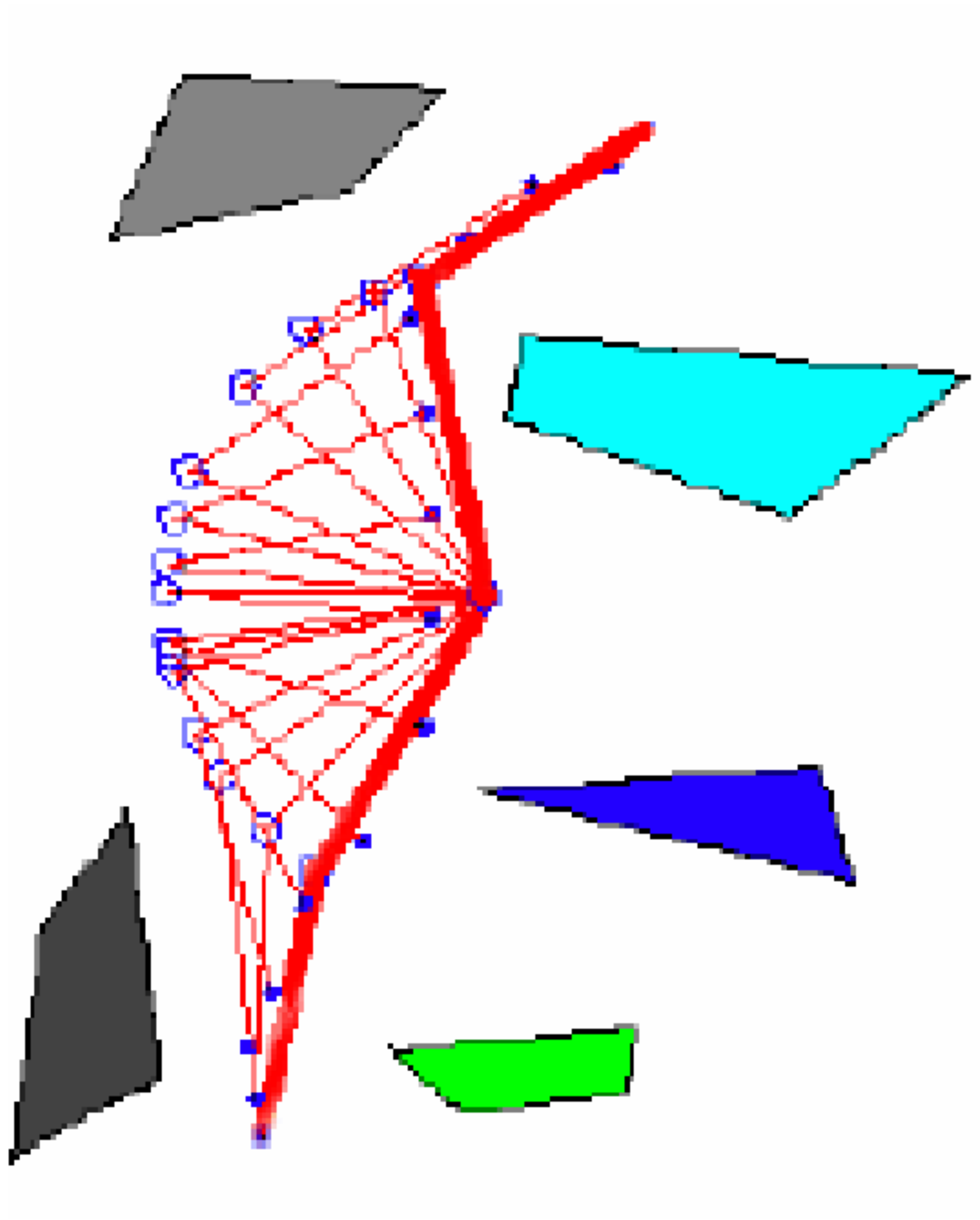
(EXAMPLE FROM LYDIA KAVRAKI AND STEVEN LAVALLE)

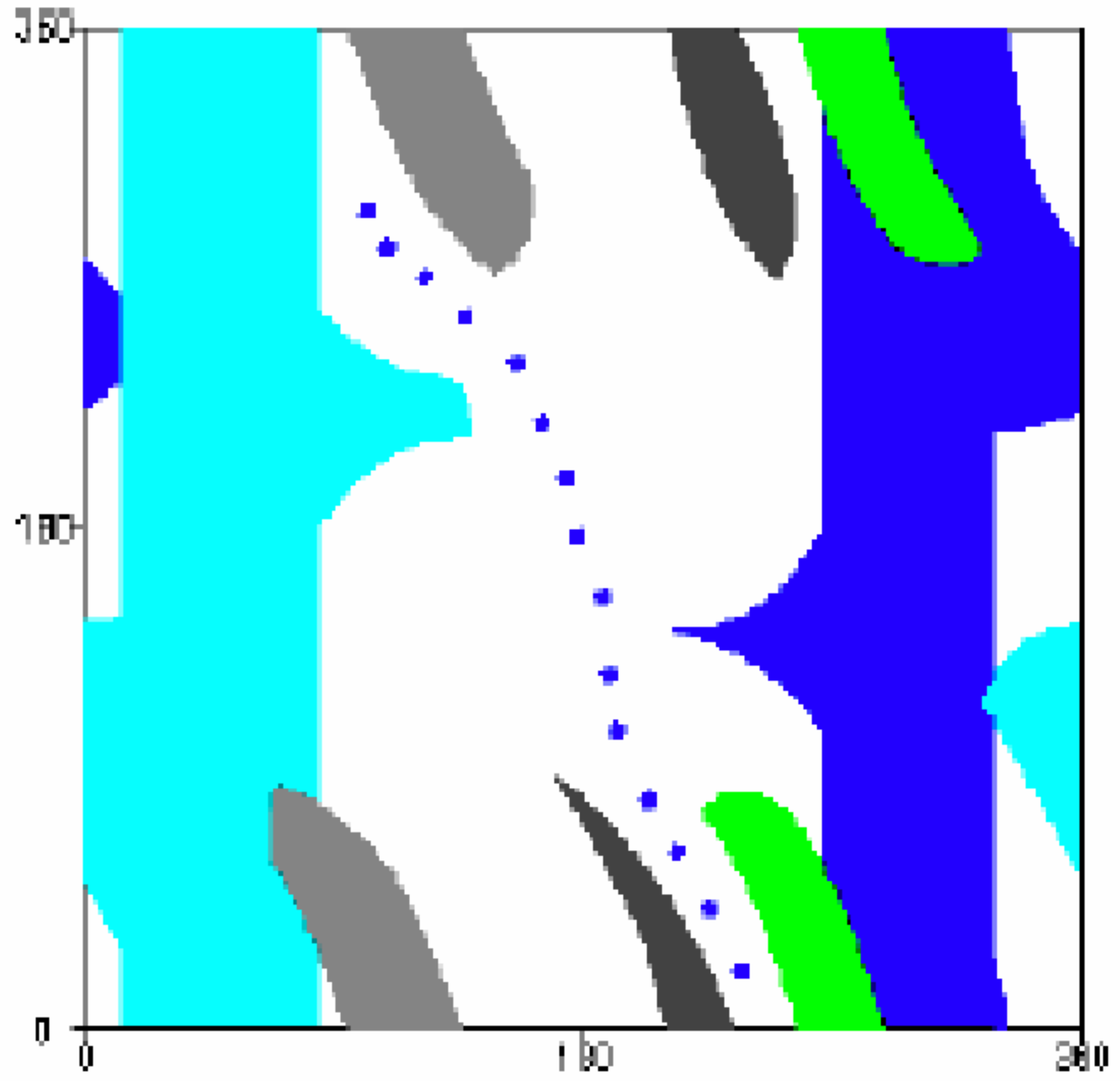
# Example: Translating and Rotating Triangle




(EXAMPLE FROM HOWIE CHOSET)

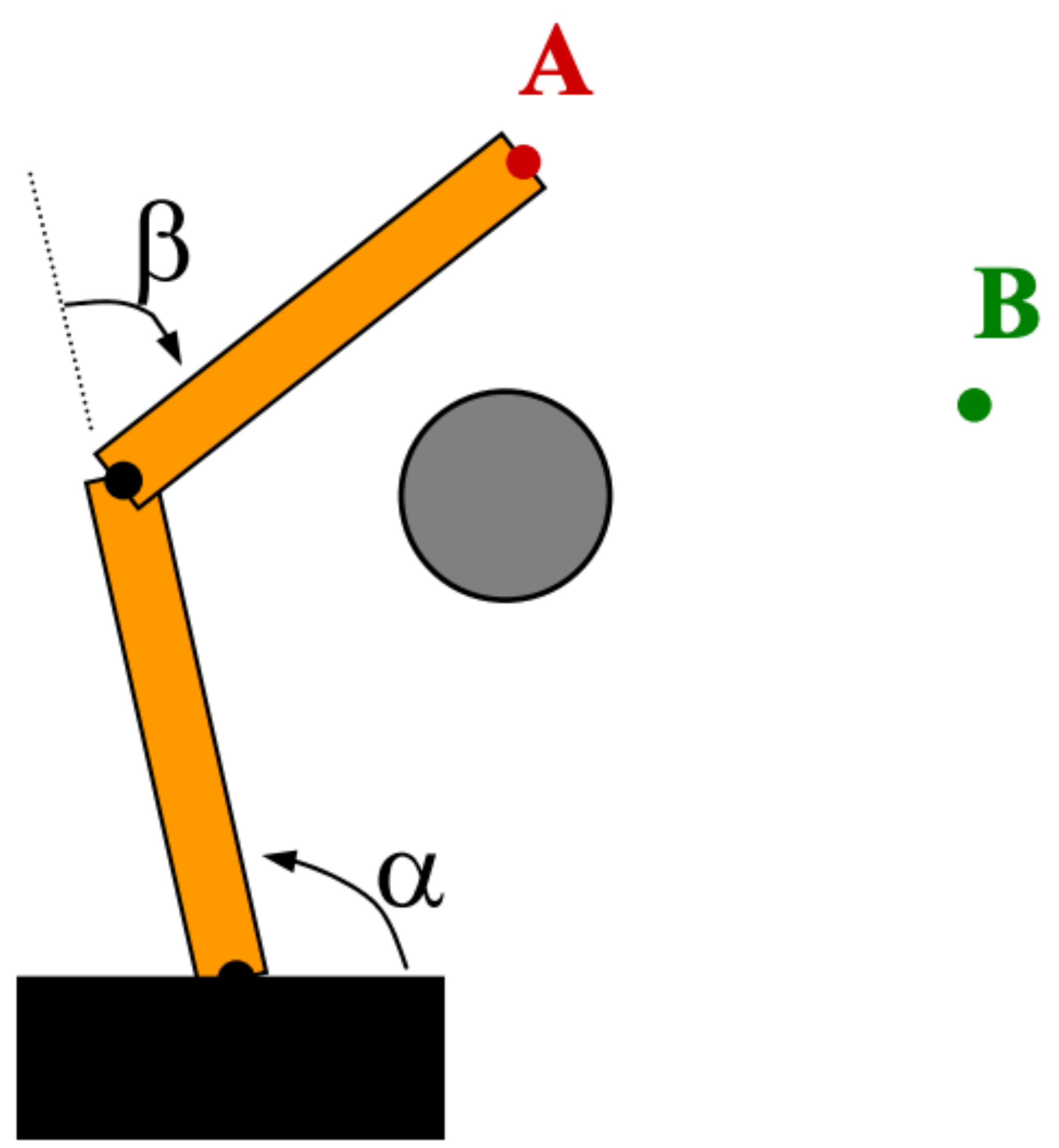
# Example 3: 2-link planar arm

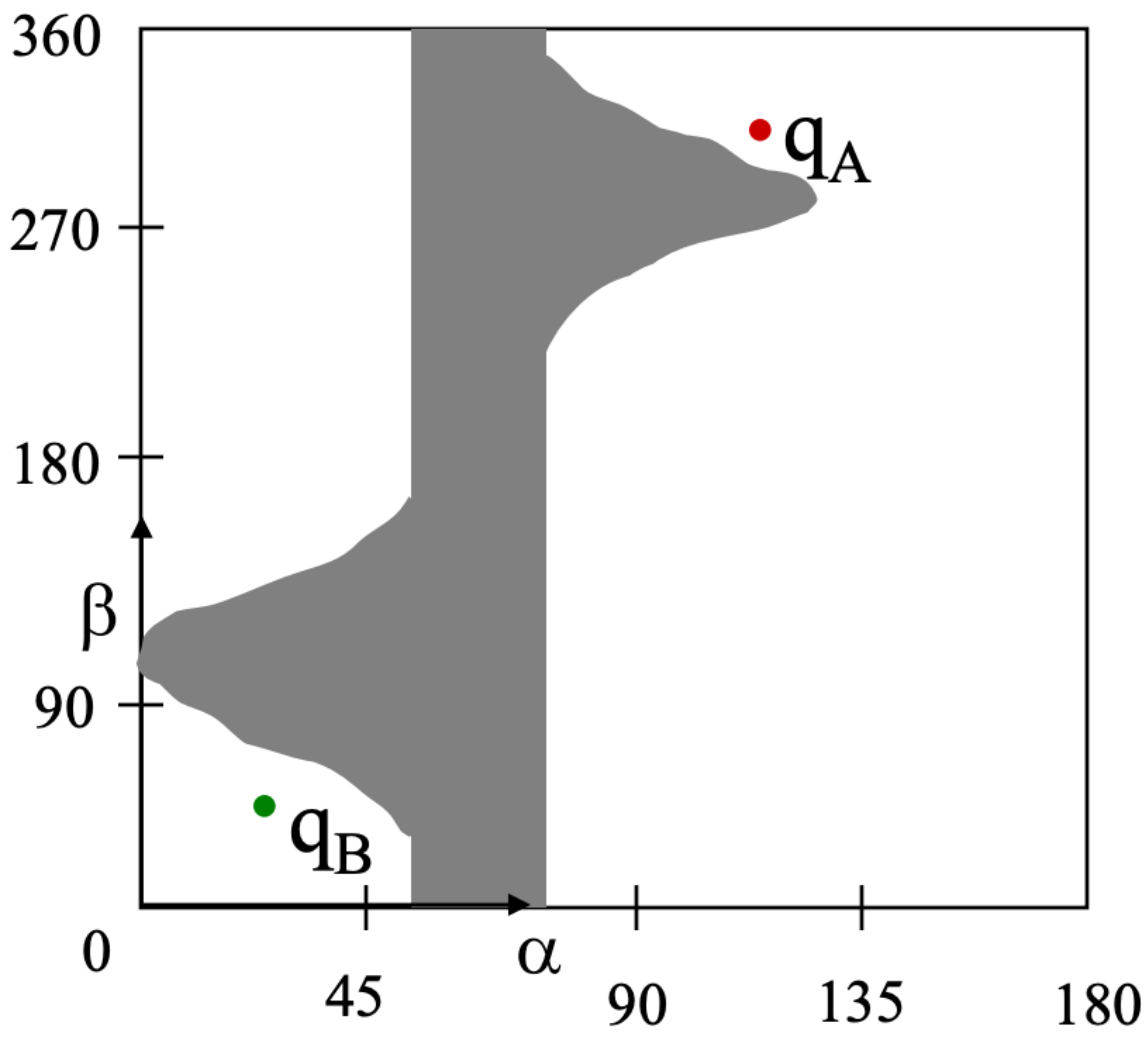





Courtesy Tapomayukh Bhattacharya

# Example 3: 2-link planar arm

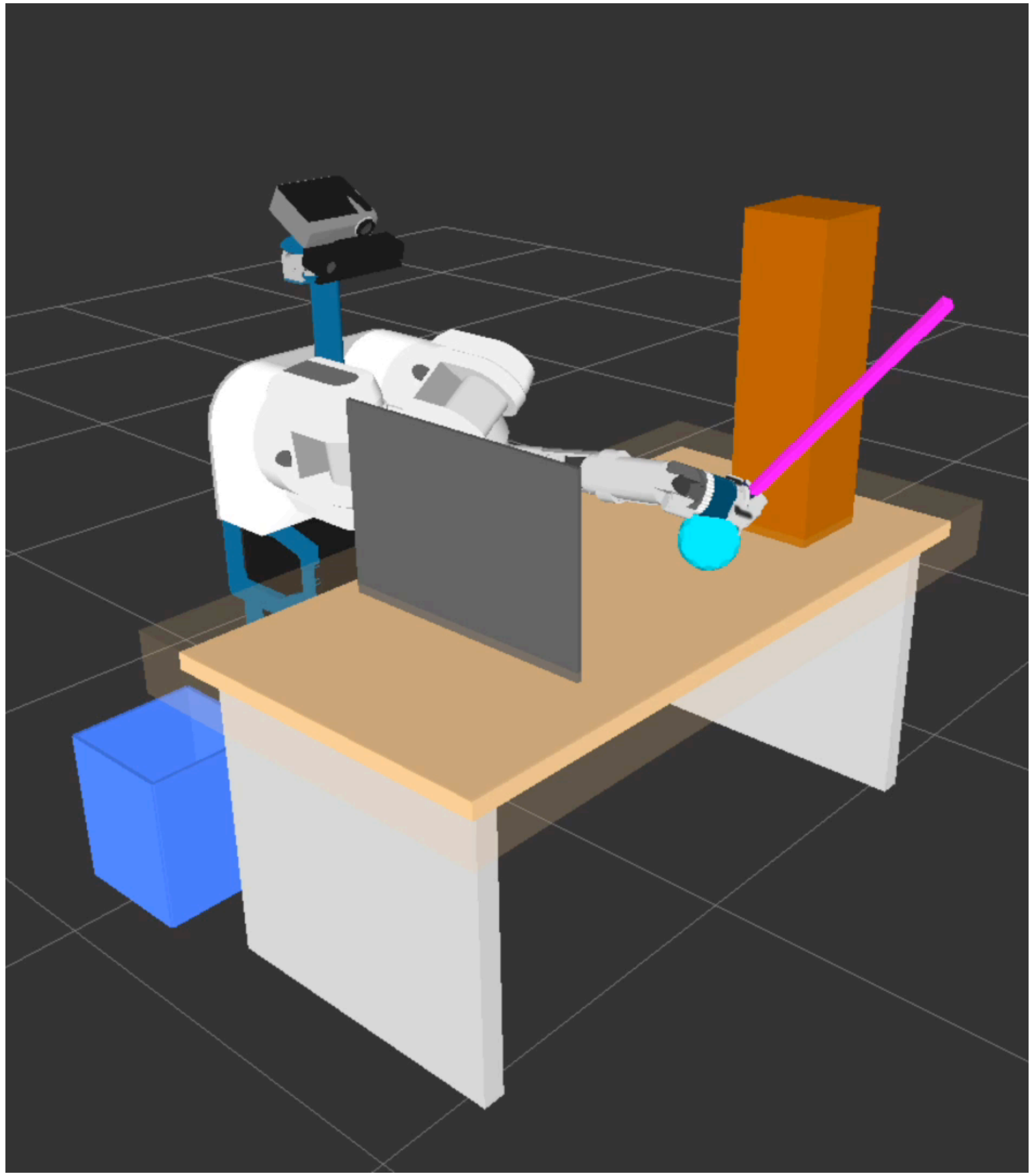



Courtesy Tapomayukh Bhattacharya

# Geometric Path Planning Problem

# Geometric Path Planning Problem



Also known as
Piano Mover's Problem (Reif 79)

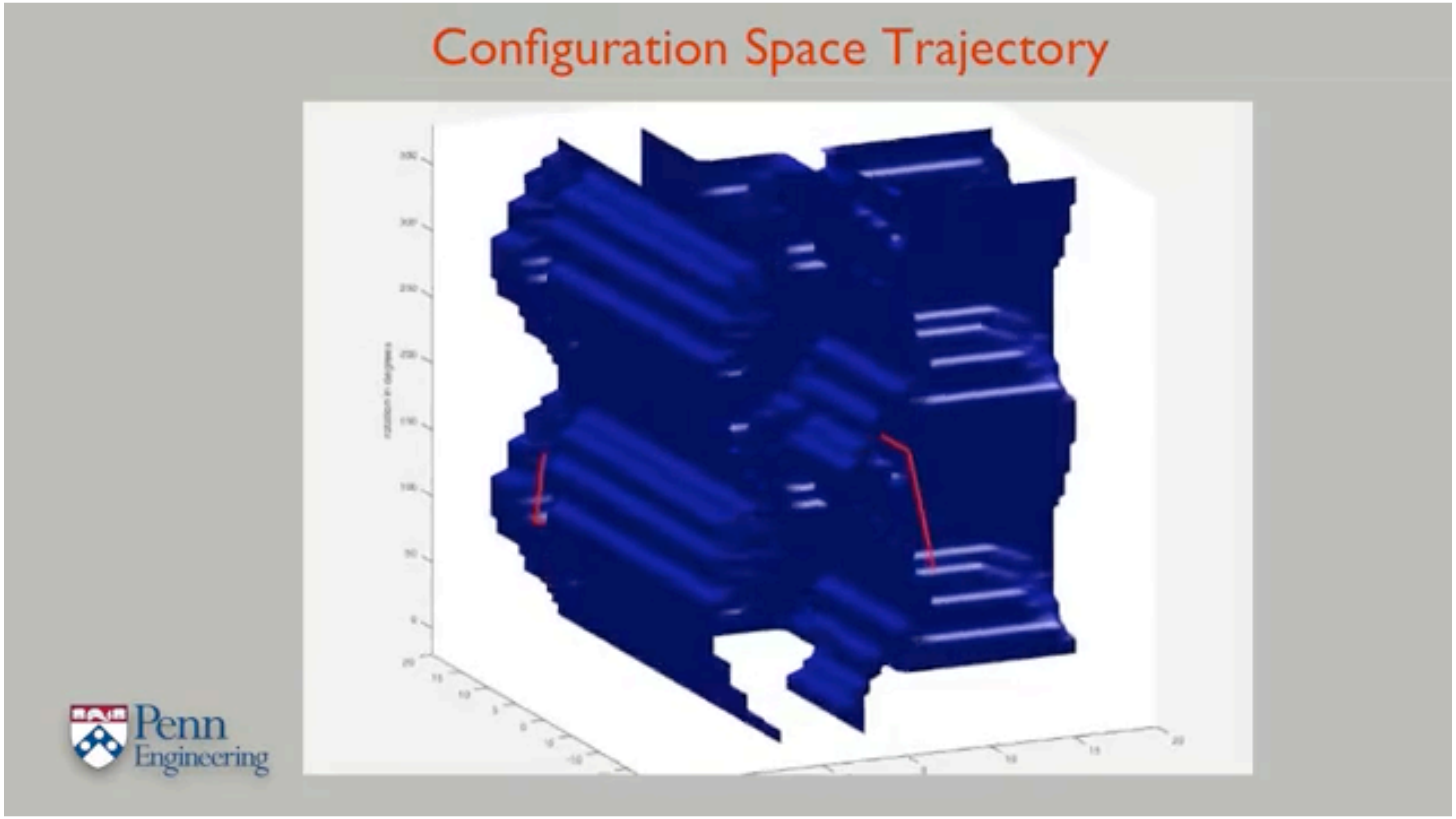
Given:

1. A *workspace* $\mathcal{W}$, where either $\mathcal{W} = \mathbb{R}^2$ or $\mathcal{W} = \mathbb{R}^3$.
2. An *obstacle region* $\mathcal{O} \subset \mathcal{W}$.
3. A *robot* defined in $\mathcal{W}$. Either a rigid body $\mathcal{A}$ or a collection of $m$ links: $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$.
4. The *configuration space* $\mathcal{C}$ ($C_{obs}$ and $C_{free}$ are then defined).
5. An *initial configuration* $\boldsymbol{q_I} \in \mathcal{C}_{free}$.
6. A *goal configuration* $\boldsymbol{q_G} \in \mathcal{C}_{free}$. The initial and goal configuration are often called a *query* $(\boldsymbol{q_I}, \boldsymbol{q_G})$.

*Compute a (continuous)* path, $\tau : [0,1] \rightarrow \mathcal{C}_{free}$, *such that* $\tau(0) = \boldsymbol{q_I}$ *and* $\tau(1) = \boldsymbol{q_G}$.

Also may want to minimize cost $c(\tau)$

# Planning in Configuration Space

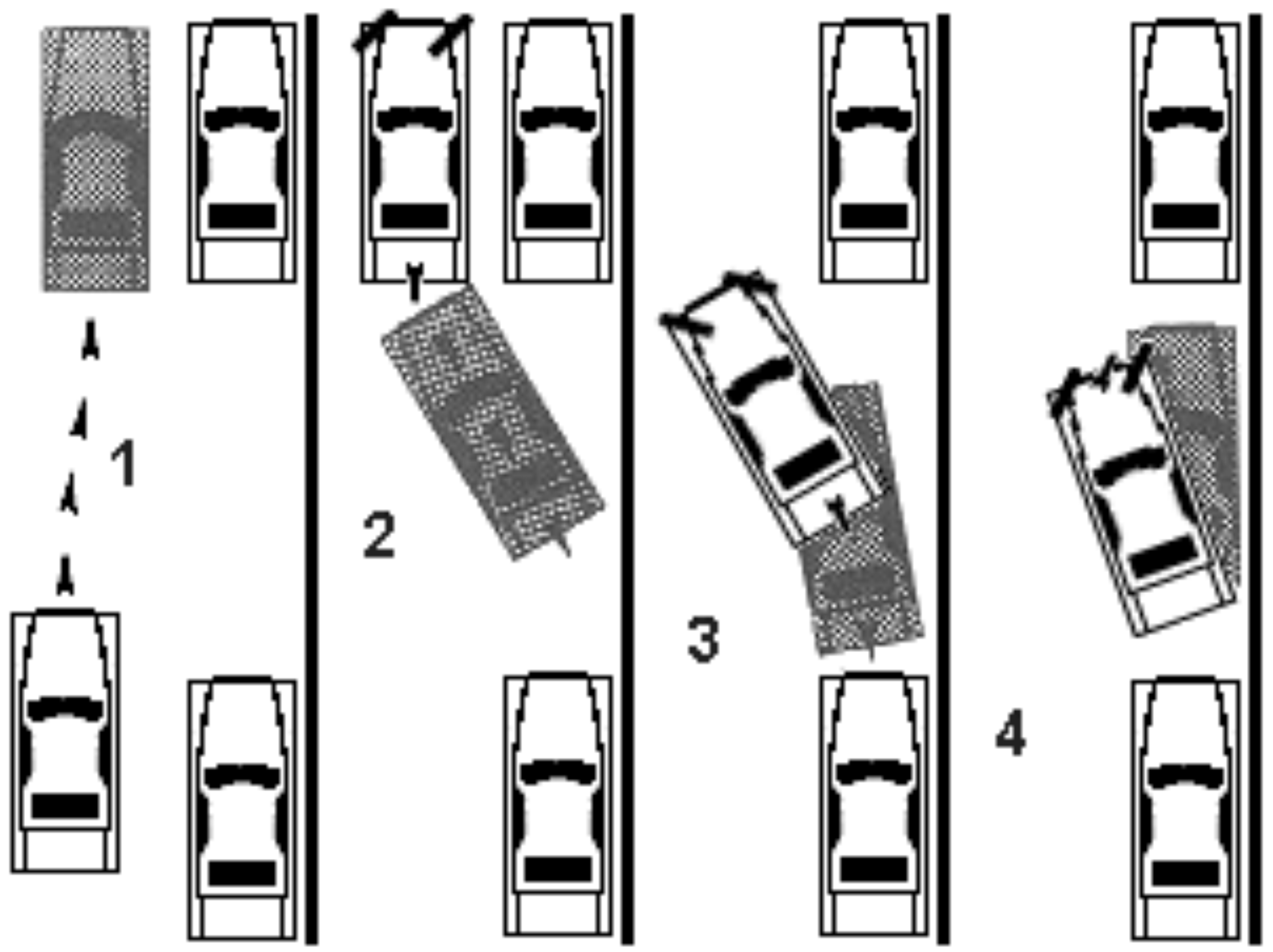


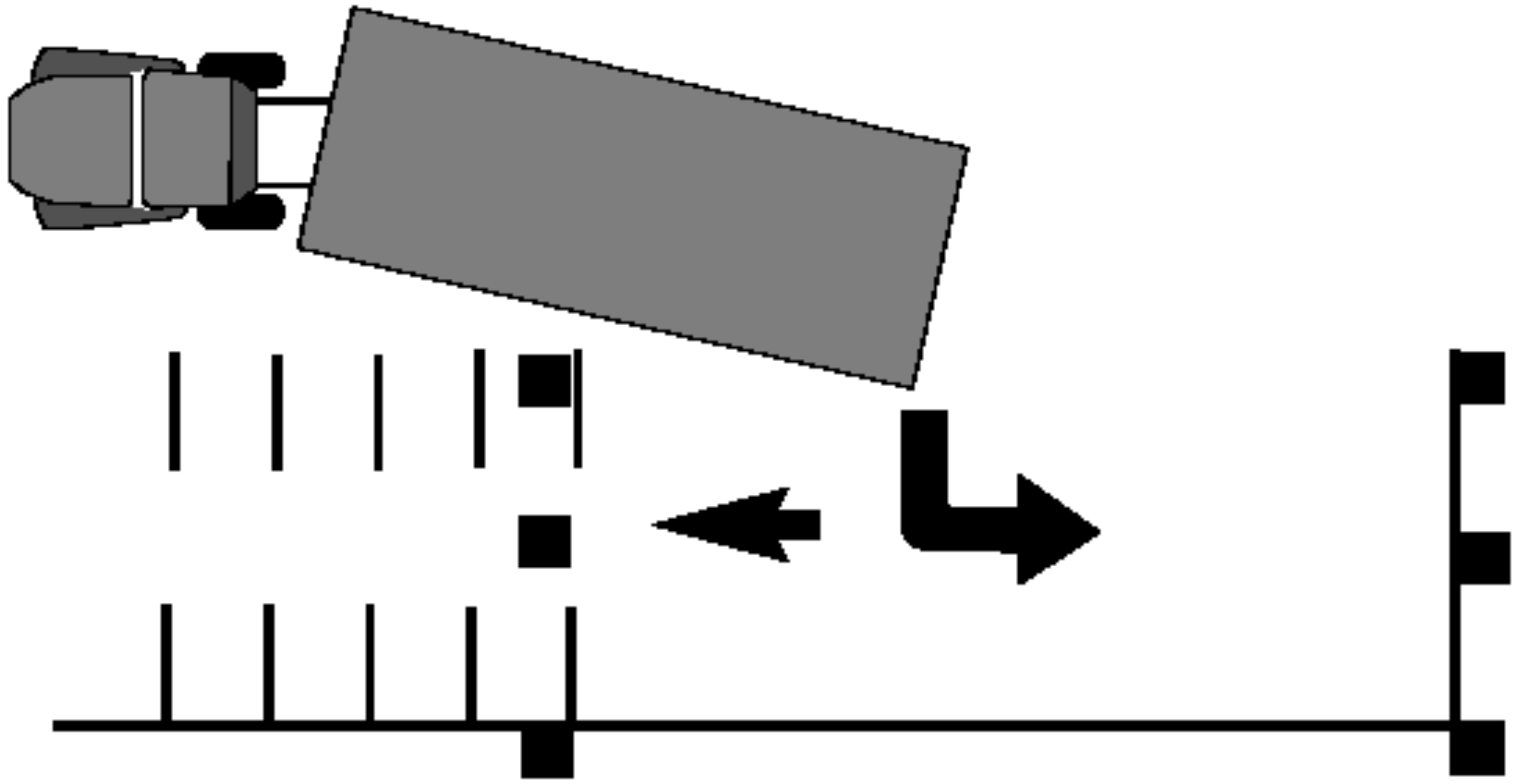

(EXAMPLE FROM CJ TAYLOR)

So, are we done?

No! Planning is hard

# Differential constraints make things even harder



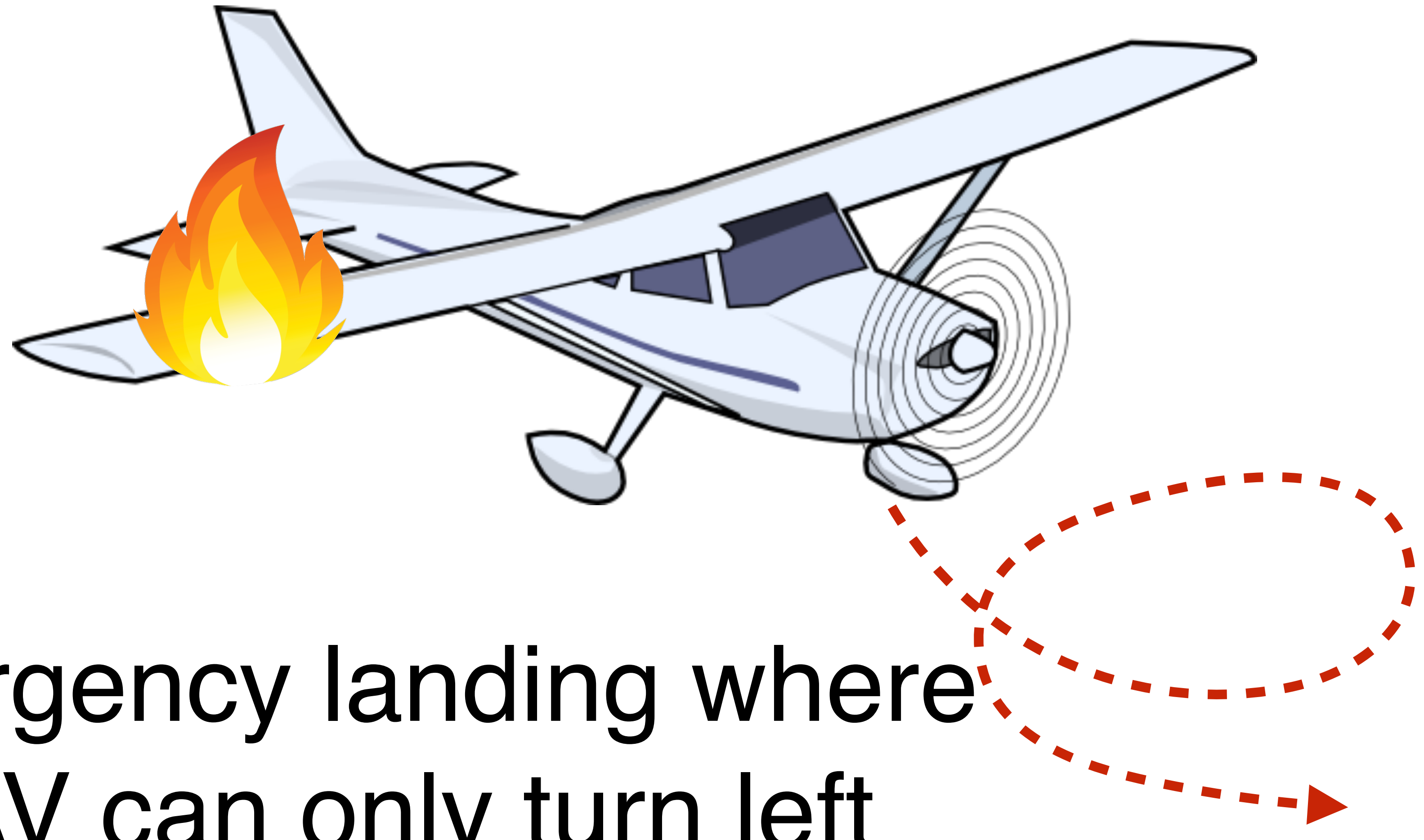



These are examples of non-holonomic system

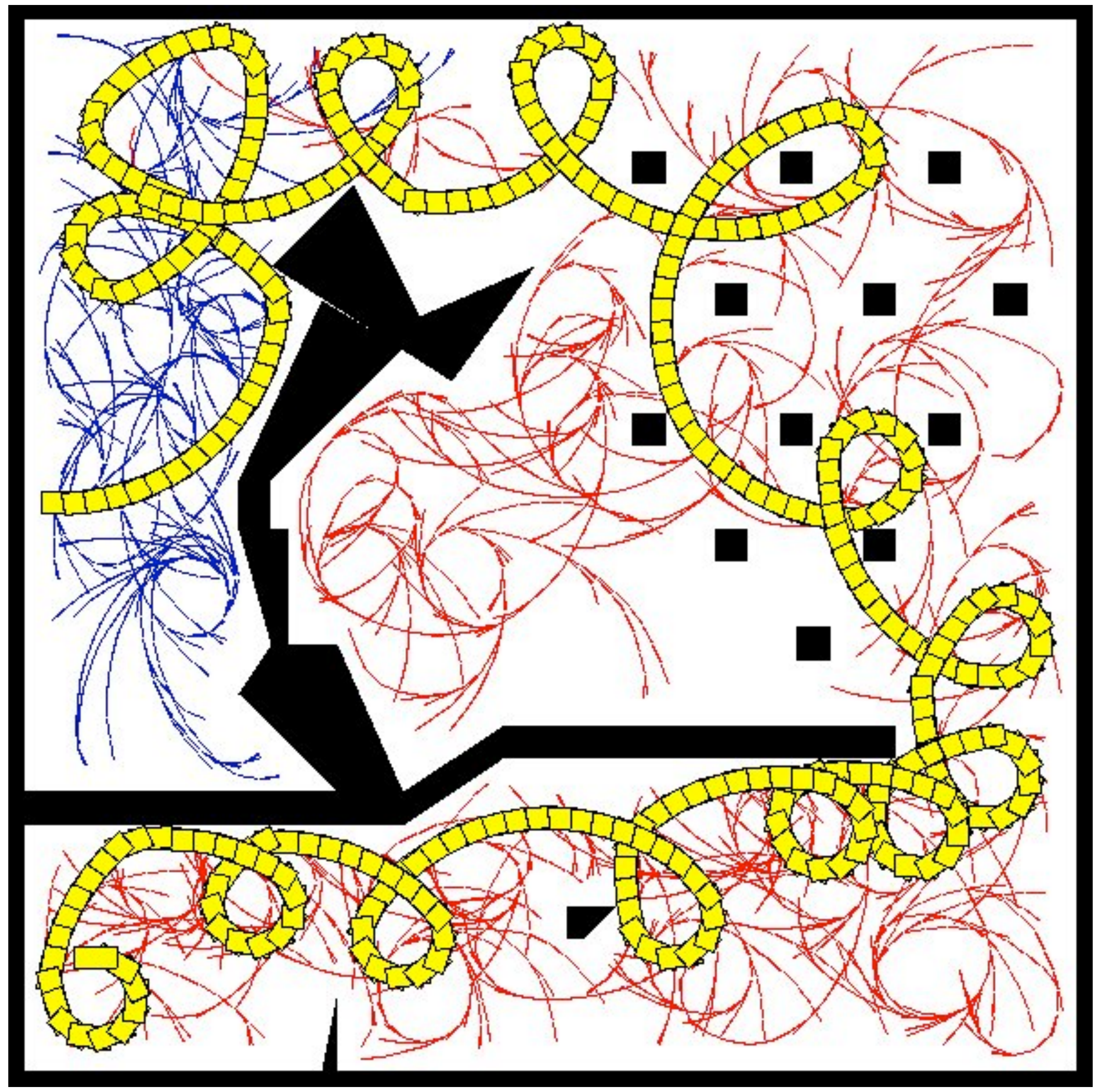
non-holonomic differential constraints are not completely integrable

i.e. the system is trapped in some sub-manifold of the config space

# Differential constraints make things even harder



Emergency landing where
UAV can only turn left



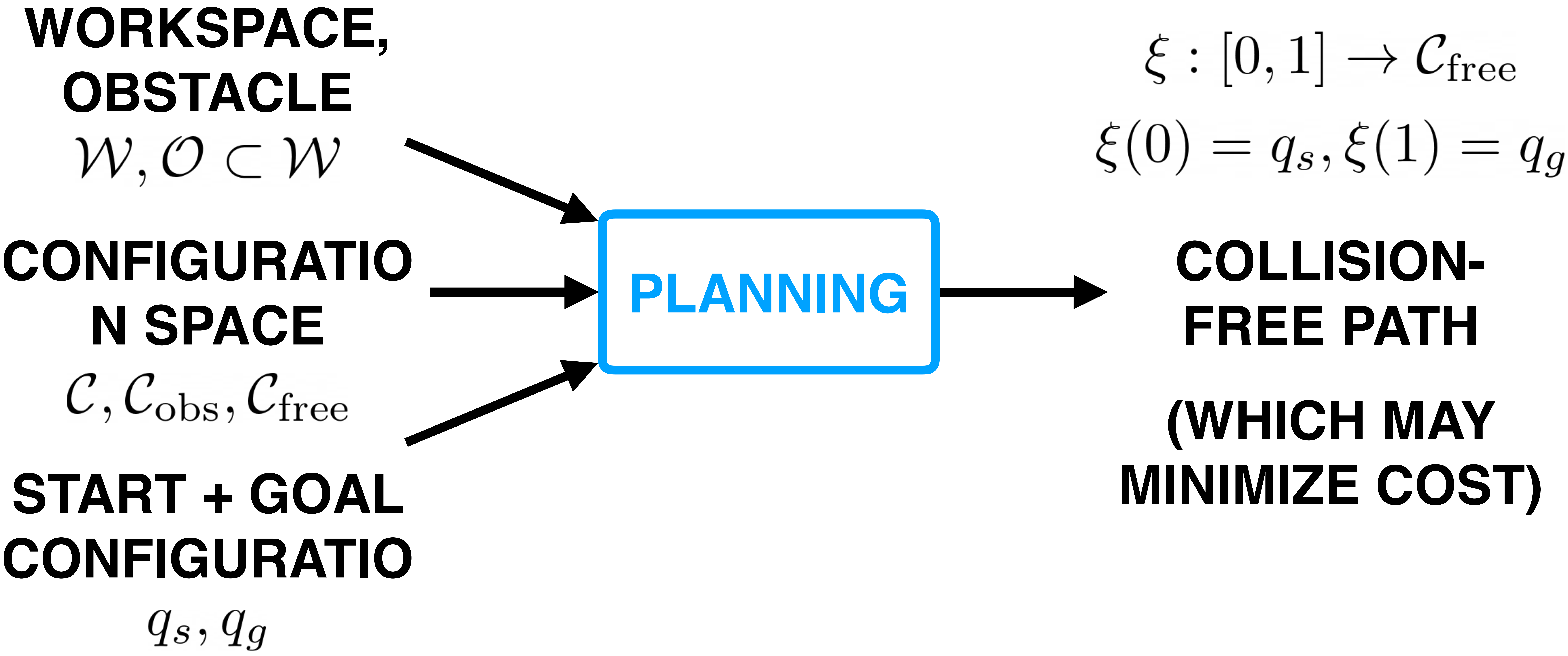
"Left-turning-car"

These are examples of non-holonomic system

non-holonomic differential constraints are not completely integrable

i.e. the system is trapped in some sub-manifold of the config space

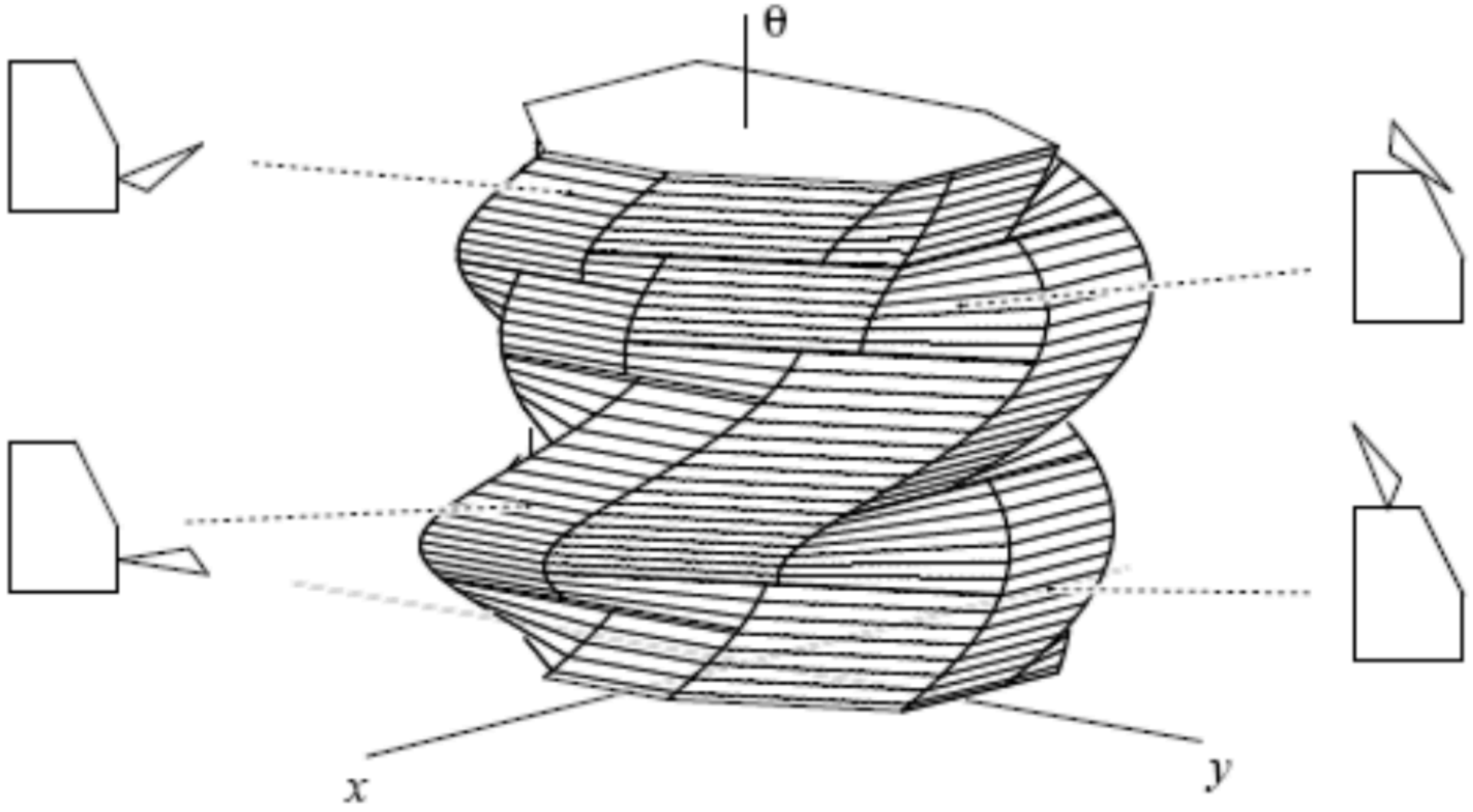
# Geometric Path Planning



# Challenges in Motion Planning

- Computing configuration-space obstacles **HARD!**
- Planning in continuous high-dimensional space **HARD!**

Goal: tractable approximations with provable guarantees!




**(EXAMPLE FROM HOWIE CHOSET)**

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