Autonomous Robotics
Winter 2025

Abhishek Gupta

TAs: Carolina Higuera, Entong Su, Bernie Zhu

Slides borrowed from many sources – Sidd Srinivasa, Sanjiban Choudhury, M Likhachev

Class Outline

State Estimation

- Robotic System Design
- Filtering
- Localization
- SLAM

Control

- Feedback Control
- PID Control
- MPC
- LQR

Planning

- Search
- Heuristic Search
- Motion Planning
- Lazy Search

Learning

- Imitation Learning
- Policy Gradient
- Actor-Critic
- Model-Based RL

Logistics

- 2 papers in seeded discussion:
  - Paper 1: RRT-Connect, Kuffner et al
  - Paper 2: LazySP, Dellin et al

- Post questions, discuss any issues you are having on Ed.
- Students with **no** access to 002, e-mail us with your student ID.
- Students that have not been added to the class, email abhgupta@cs.washington.edu with the subject-line "Waitlisted for CSE478"

Lecture Outline

Best-First Search

↓

Heuristics and A*

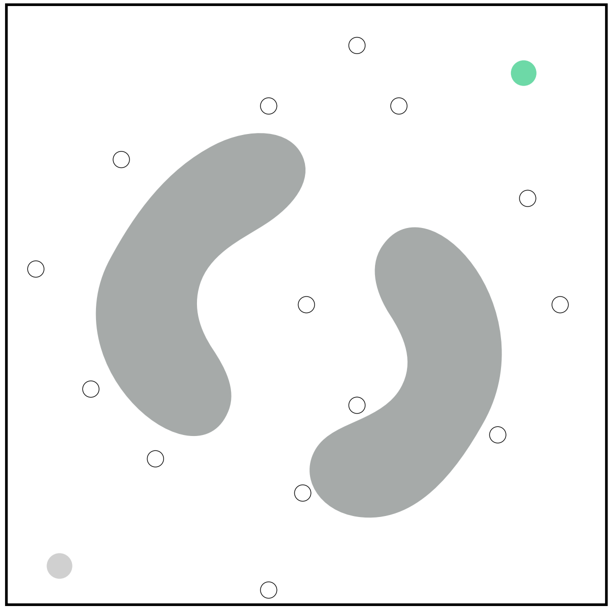
↓

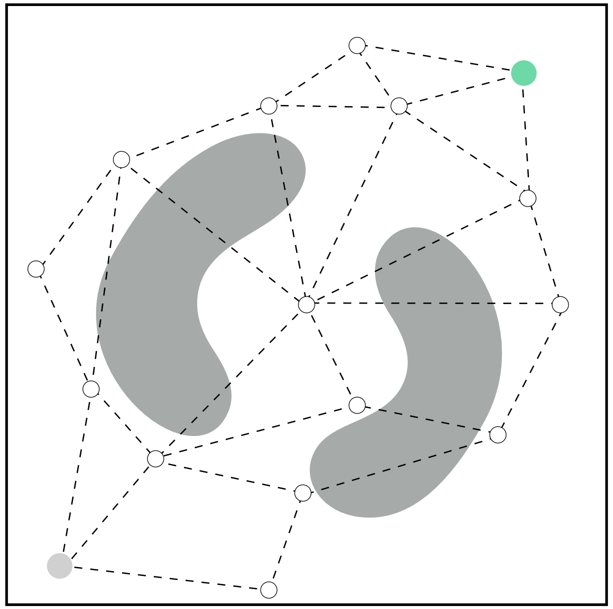
Lazy A*

Creating a Graph

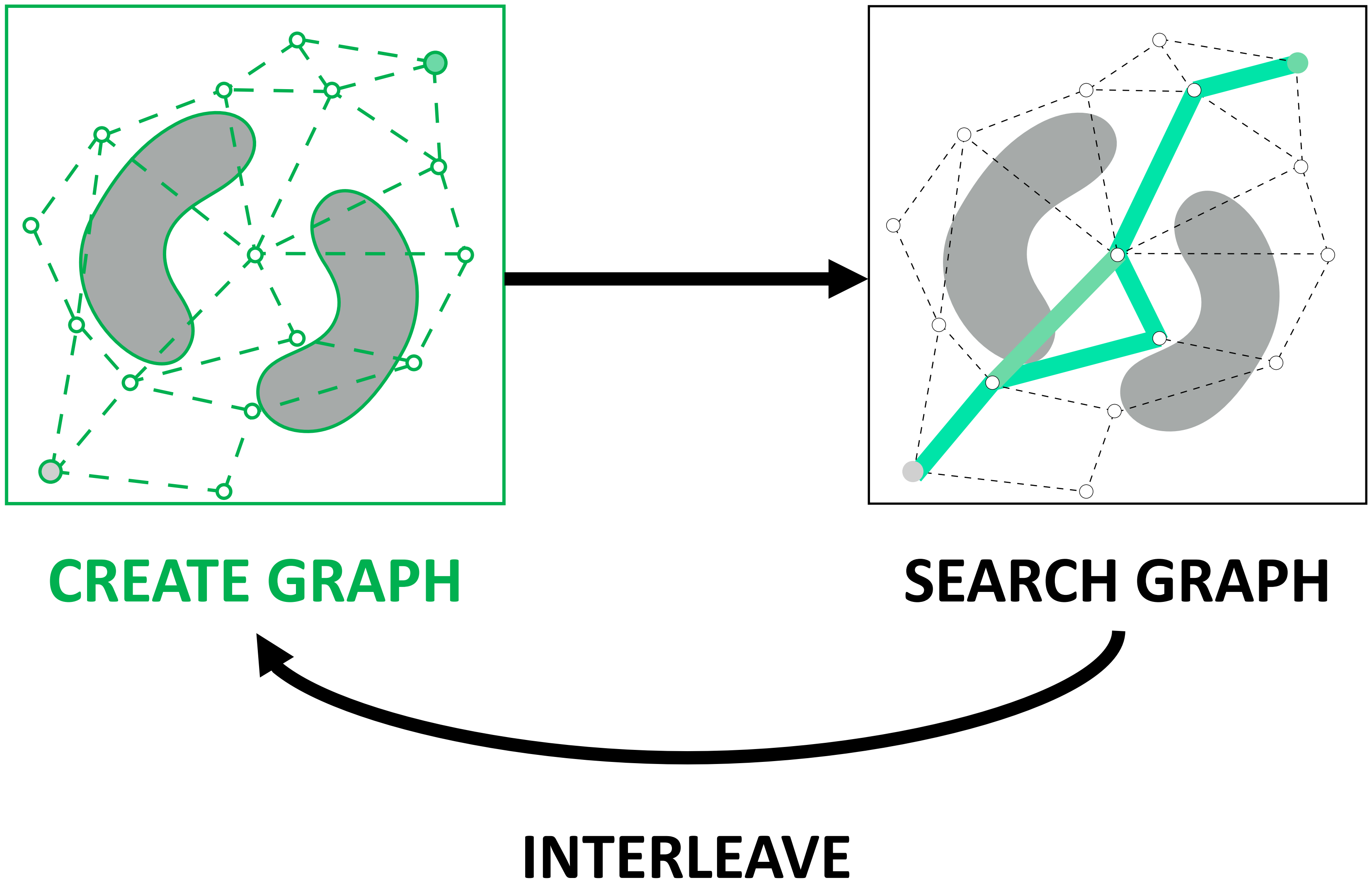
$$G = (V, E)$$

1. **Sample collision-free configurations as vertices (including start and goal)**
2. **Connect neighboring vertices with simple movements as edges**

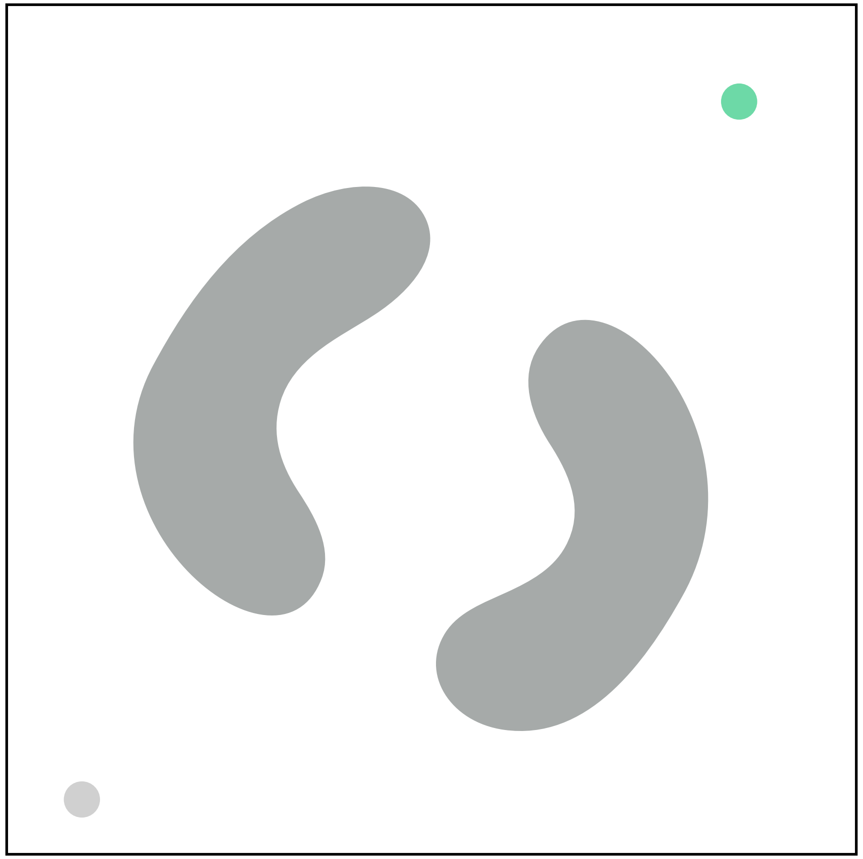




Sampling-Based Motion Planning



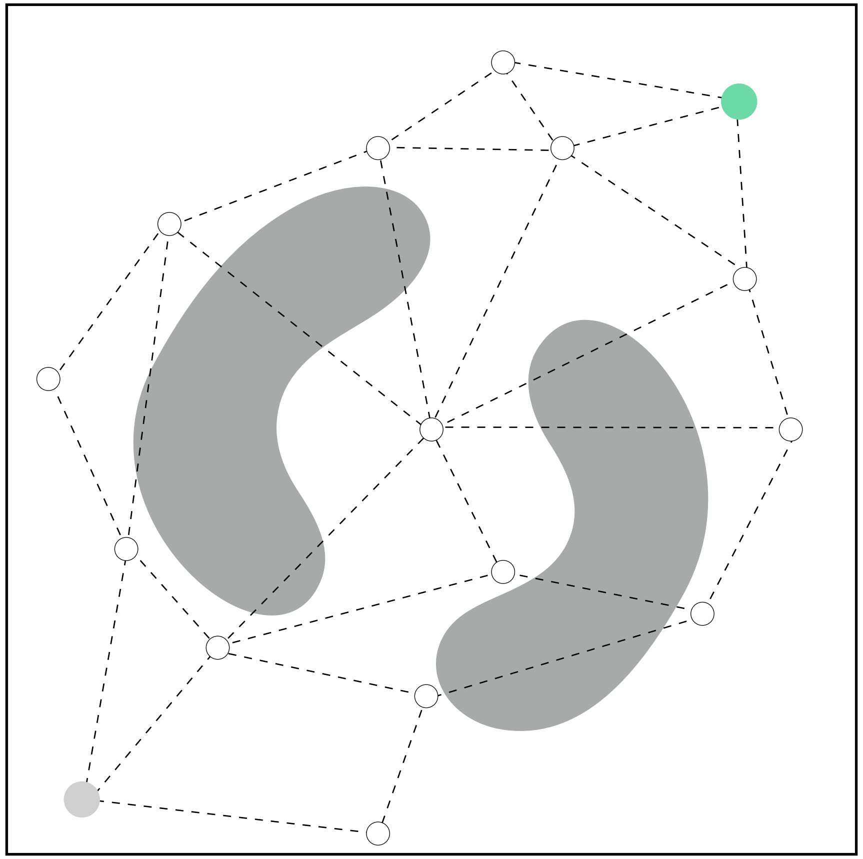
Minimal Cost Path on a Graph



START, GOAL

COST (E.G. LENGTH)

Minimal Cost Path on a Graph

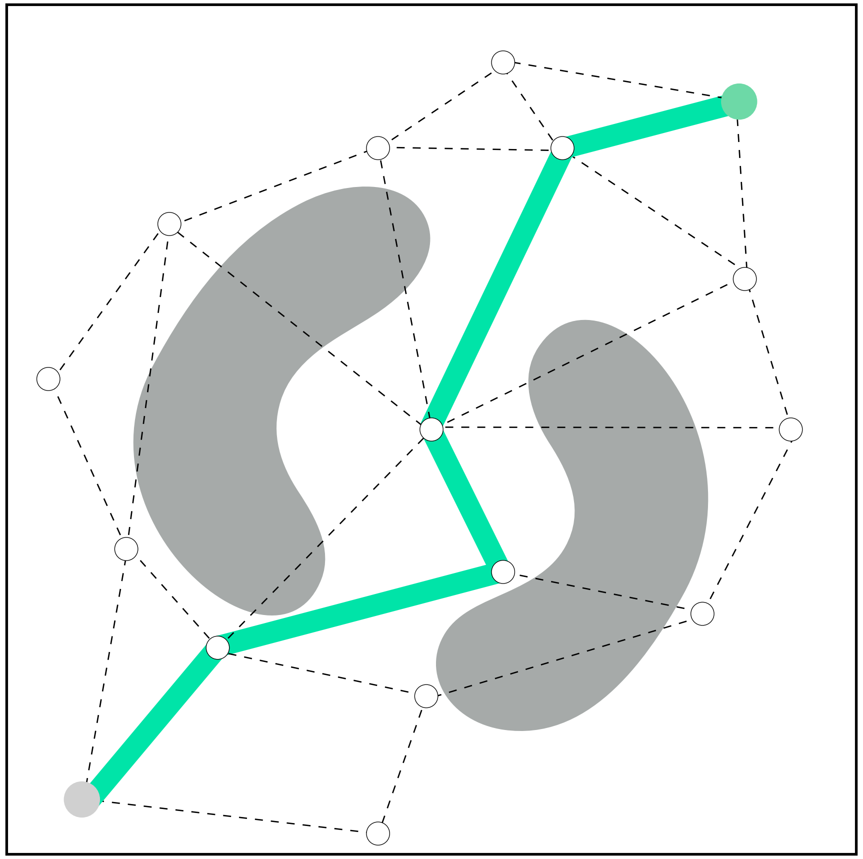


START, GOAL

COST (E.G. LENGTH)

GRAPH (VERTICES, EDGES)

Minimal Cost Path on a Graph



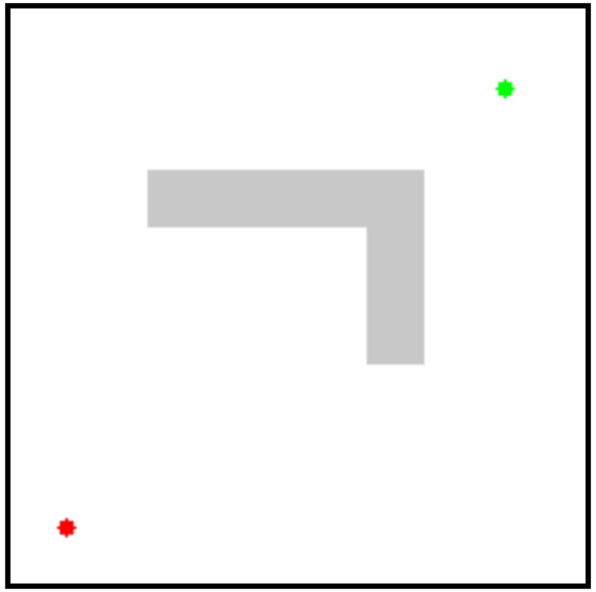
START, GOAL

COST (E.G. LENGTH)

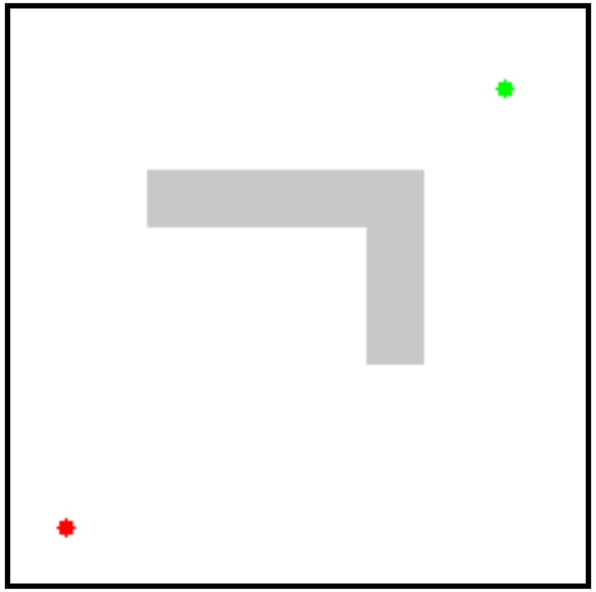
GRAPH (VERTICES, EDGES)

High-order bit

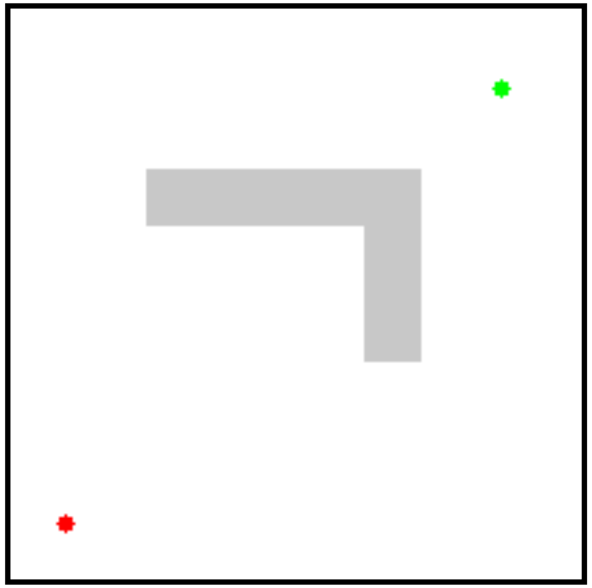
Expansion of a search wavefront from start to goal



Djikstra



A*



Weighted A*

Courtesy wikipedia

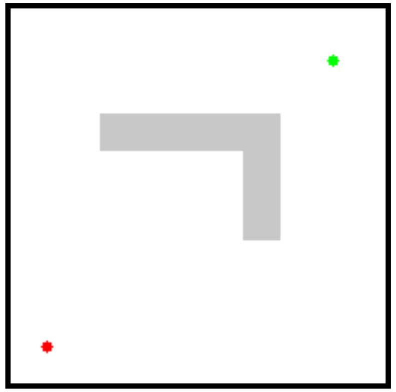
What do we want?

1. Search to systematically reason over the space of paths

2. Find a (near)-optimal path quickly

(minimize planning effort)

Best first search



This is a meta-algorithm

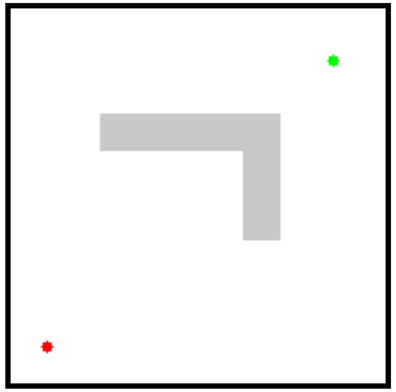
BFS maintains a priority queue of promising nodes

Each node is ranked by a function f(s)

Populate queue initially with start node

Element (Node)	**Priority Value (f-value)**
Node A	f(A)
Node B	f(B)
…..	……

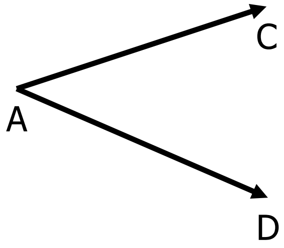
Best first search



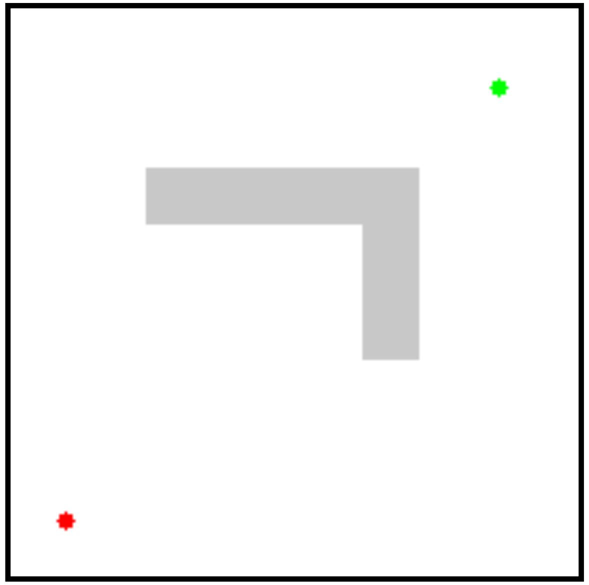
Search explores graph by expanding most promising node min *f(s)*

Terminate when you find the goal

Element (Node)	**Priority Value (f-value)**
~~Node A~~	~~f(A)~~
Node B	f(B)
…..	……



Best first search



Key Idea: Choose *f(s)* wisely!

- when goal found, it has (near) optimal path

- minimize the number of expansions

Notations

Given:

Start s_{start} Goal s_{goal}

Cost $c(s, s')$

Objects created:

OPEN: priority queue of nodes to be processed

CLOSED: list of nodes already processed

$g(s)$: estimate of the least cost from start to a given node

Pseudocode

Push *start* into OPEN

While *goal* not expanded

Pop *best* from OPEN

Add *best* to CLOSED

For every successor s'

If $g(s') > g(s) + c(s,s')$

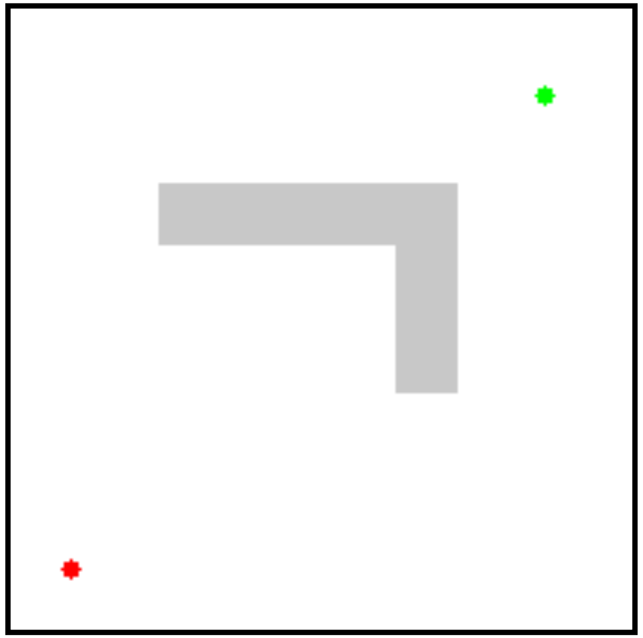
$g(s') = g(s) + c(s,s')$

Add (or update) s' to OPEN

Dijkstra’s Algorithm

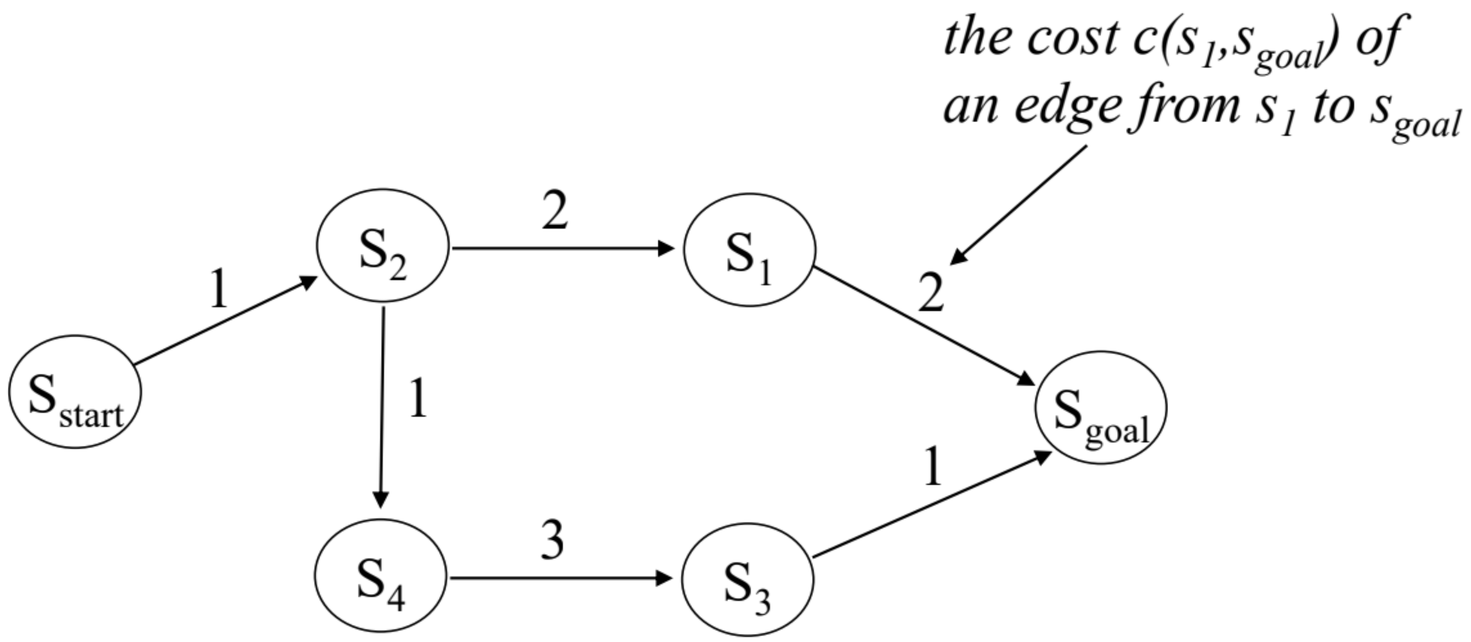
Set
$f(s) = g(s)$

Sort nodes by their cost to come



Dijkstra's Algorithm

– optimal values satisfy: $g(s) = \min_{s'' \in pred(s)} g(s'') + c(s'',s)$



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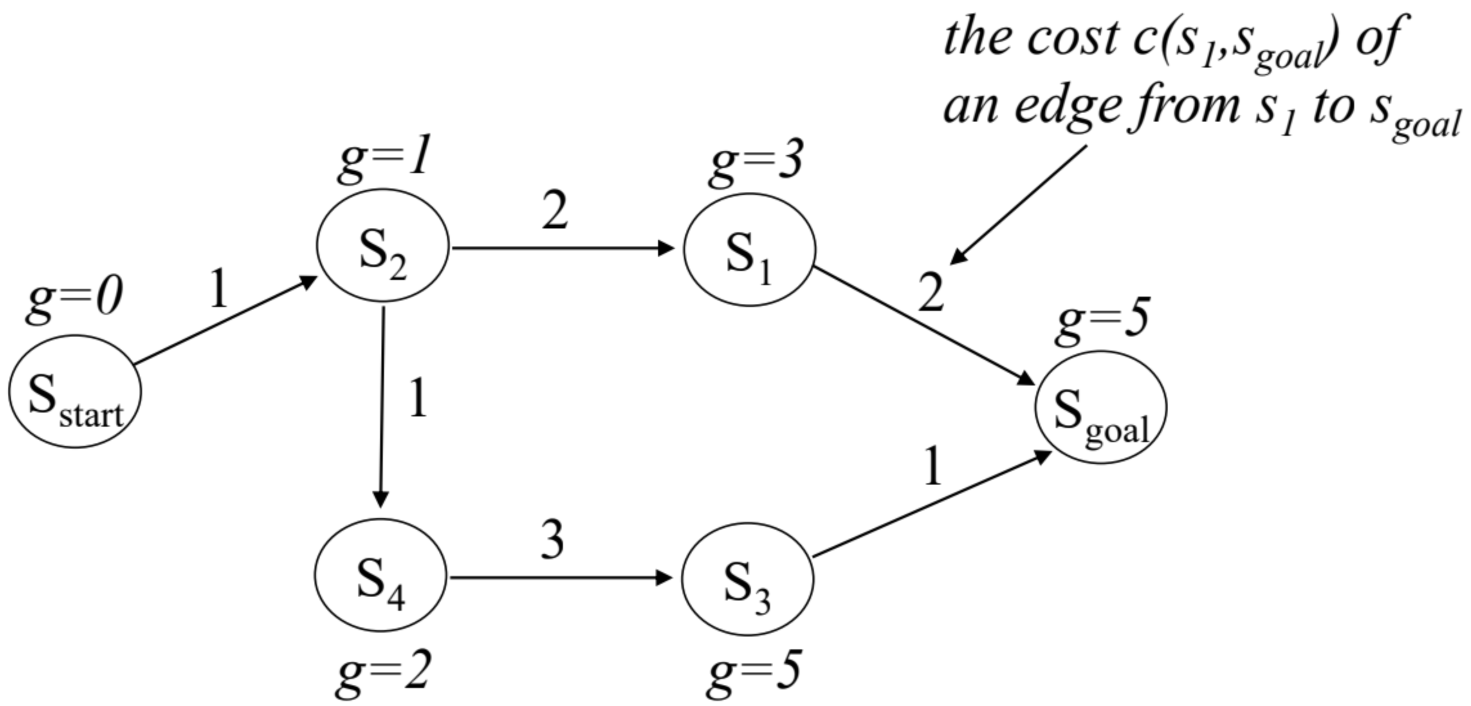


Nice property:
Only process nodes ONCE. Only process cheaper nodes than goal.

Lecture Outline

Best-First Search

↓

Heuristics and A*

↓

Lazy A*

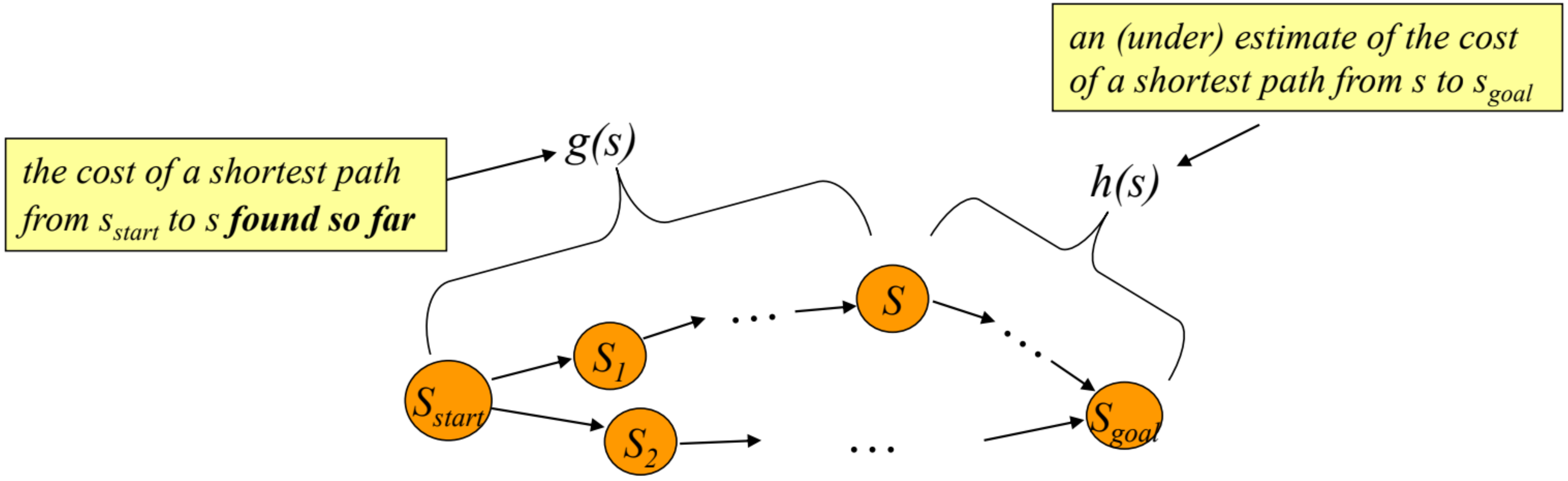
Can we have a better f(s)?

Yes!

f(s) should estimate the
cost of the path to goal

Heuristics

What if we had a heuristic *h(s)* that estimated the cost to goal?

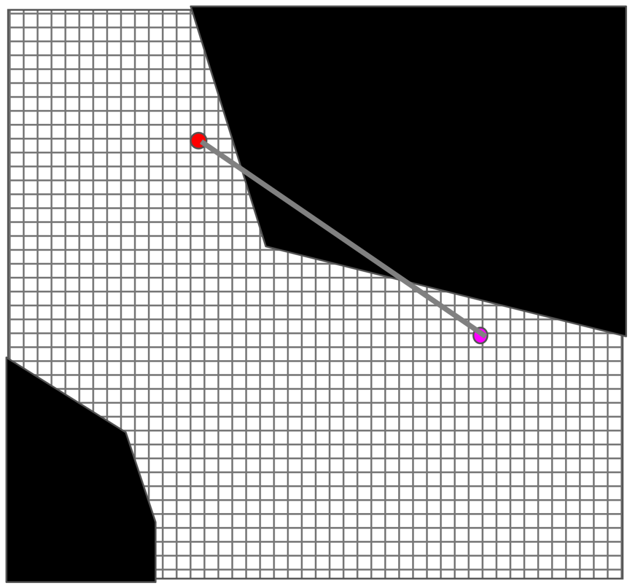


Set the evaluation function f(s) = g(s) + h(s)

Example of heuristics?

1. Minimum number of nodes to go to goal

2. Euclidean distance to goal (if you know your cost is measuring length)



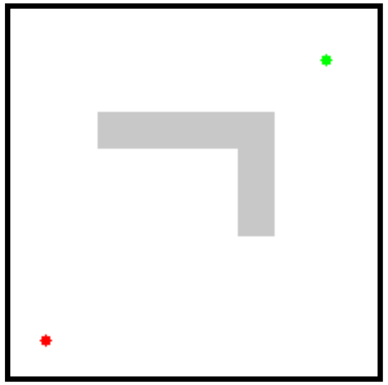
3. Solution to a relaxed problem

4. Domain knowledge / Learning ….

A* [Hart, Nillson, Raphael, '68]

Let L be the length of the shortest path

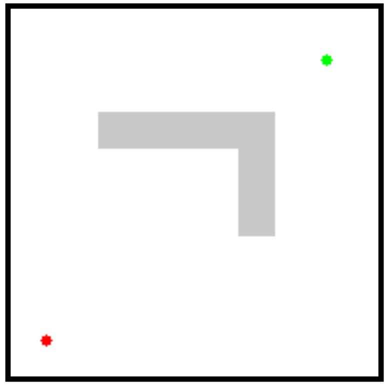
Djikstra



Expand every state

g(s) < L

A*



Expand every state

f(s) = g(s) + h(s) < L

Both find the optimal path …

but A* only expands relevant states, i.e., does much less work!

A* Search

- Computes optimal g-values for relevant states

while(s_{goal} is not expanded)
  remove s with the smallest $[f(s) = g(s)+h(s)]$ from *OPEN*;
  insert s into *CLOSED*;
  for every successor s' of s such that s' not in *CLOSED*
    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s')$;
      insert s' into *OPEN*;

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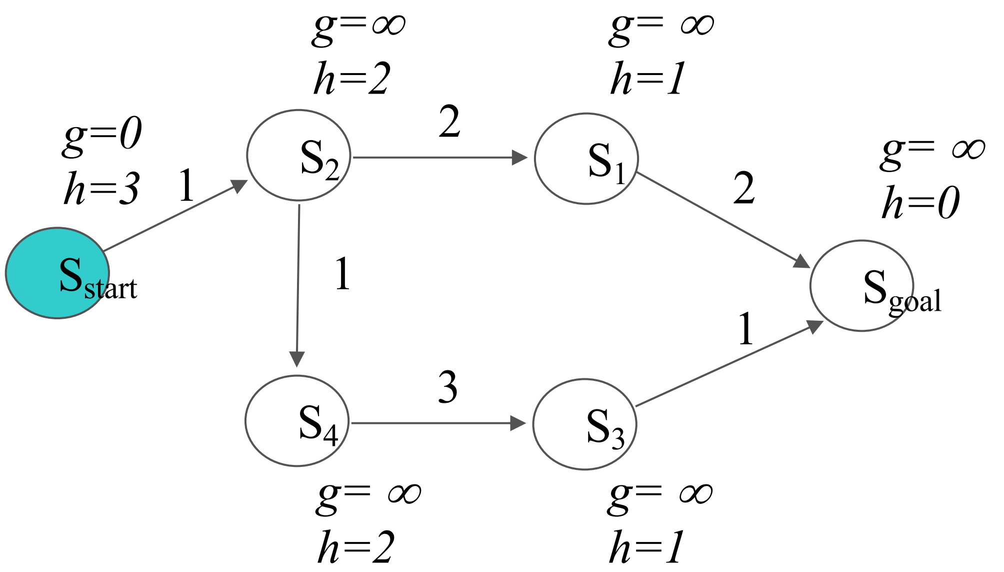
      $g(s') = g(s) + c(s,s')$;
      insert s' into *OPEN*;

$CLOSED = \{\}$
$OPEN = \{s_{start}\}$
next state to expand: s_{start}



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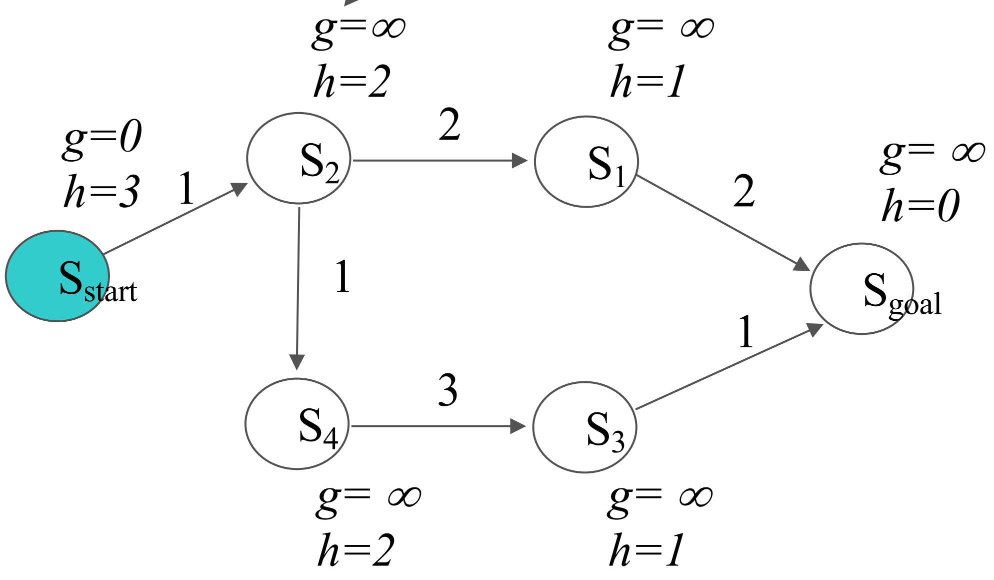
    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s')$;
      insert s' into $OPEN$;

$g(s_2) > g(s_{start}) + c(s_{start},s_2)$



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A* Search

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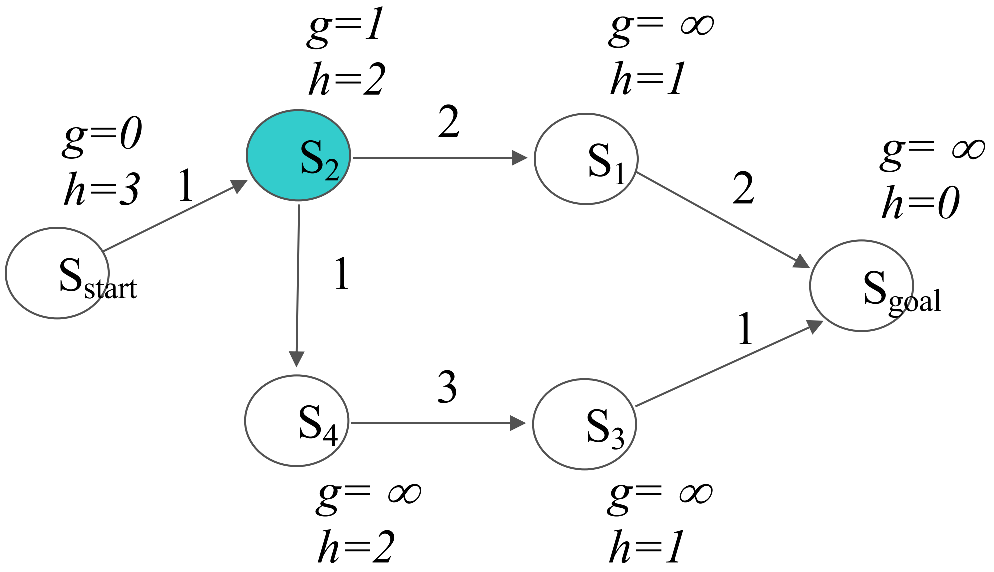
      insert s' into *OPEN*;

$CLOSED = \{s_{start}\}$
$OPEN = \{s_2\}$
next state to expand: s_2

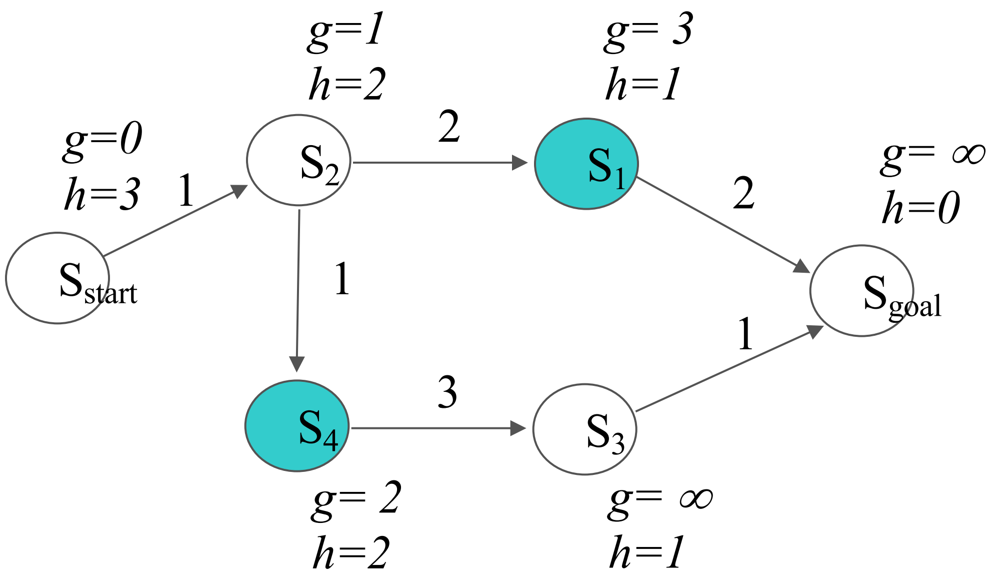


A* Search

- Computes optimal g-values for relevant states

while(s_{goal} is not expanded)
  remove s with the smallest $[f(s) = g(s)+h(s)]$ from *OPEN*;
  insert s into *CLOSED*;
  for every successor s' of s such that s' not in *CLOSED*
    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s');$
      insert s' into *OPEN*;

$CLOSED = \{s_{start}, s_2\}$
$OPEN = \{s_1, s_4\}$
next state to expand: s_1



A* Search

- Computes optimal g-values for relevant states

while(s_{goal} is not expanded)
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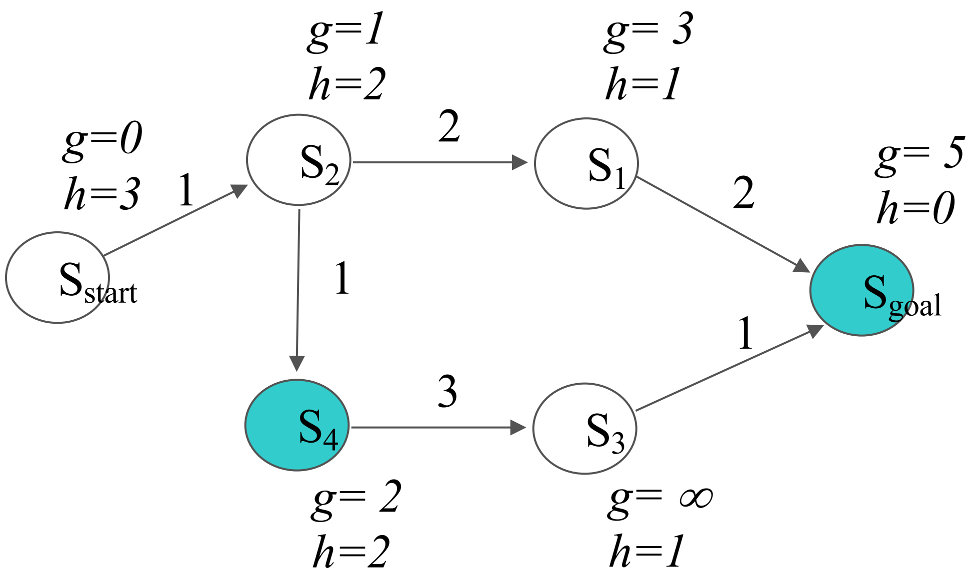
  insert s into *CLOSED*;
  for every successor s' of s such that s' not in *CLOSED*
    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s')$;
      insert s' into *OPEN*;

$CLOSED = \{s_{start}, s_2, s_1\}$
$OPEN = \{s_4, s_{goal}\}$
next state to expand: s_4



A* Search

- Computes optimal g-values for relevant states

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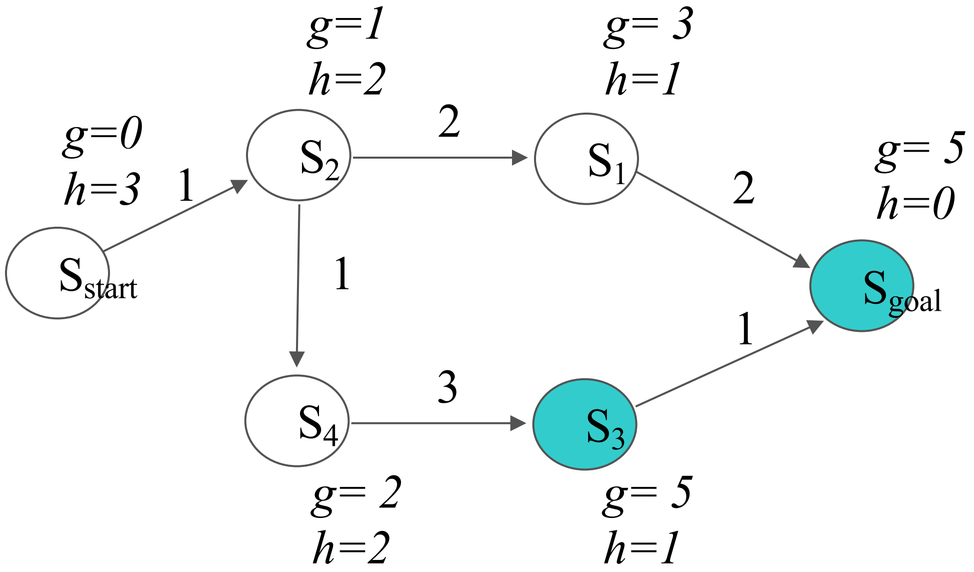
      $g(s') = g(s) + c(s,s')$;
      insert s' into *OPEN*;

$CLOSED = \{s_{start}, s_2, s_1, s_4\}$
$OPEN = \{s_3, s_{goal}\}$
next state to expand: s_{goal}



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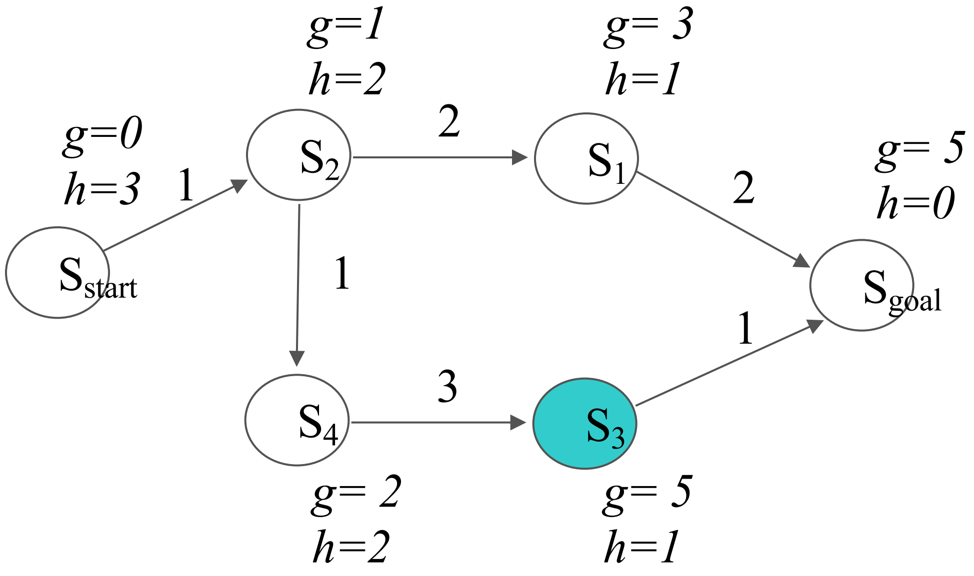
      $g(s') = g(s) + c(s,s');$
      insert s' into *OPEN*;

$CLOSED = \{s_{start}, s_2, s_1, s_4, s_{goal}\}$
$OPEN = \{s_3\}$
done



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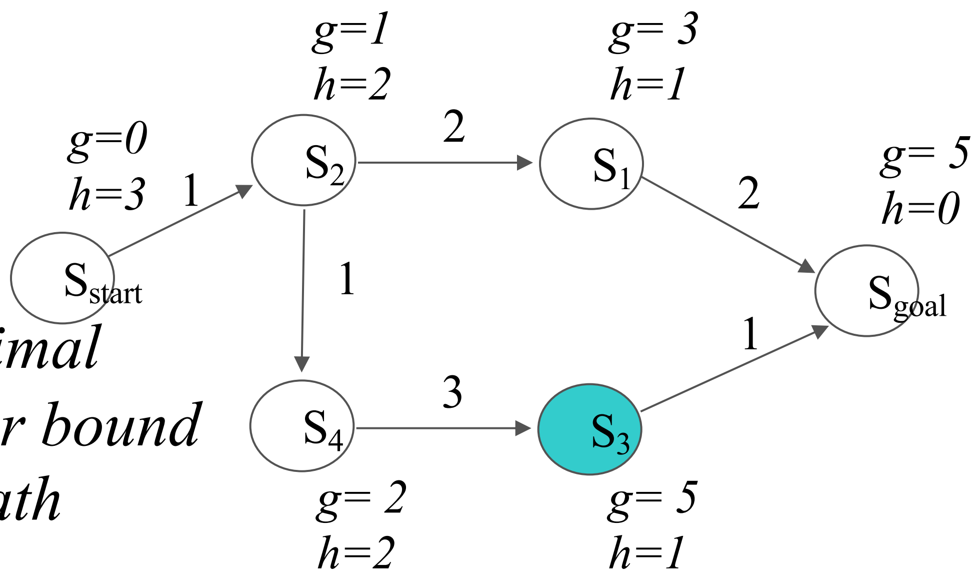

for every expanded state g(s) is optimal
for every other state g(s) is an upper bound
we can now compute a least-cost path

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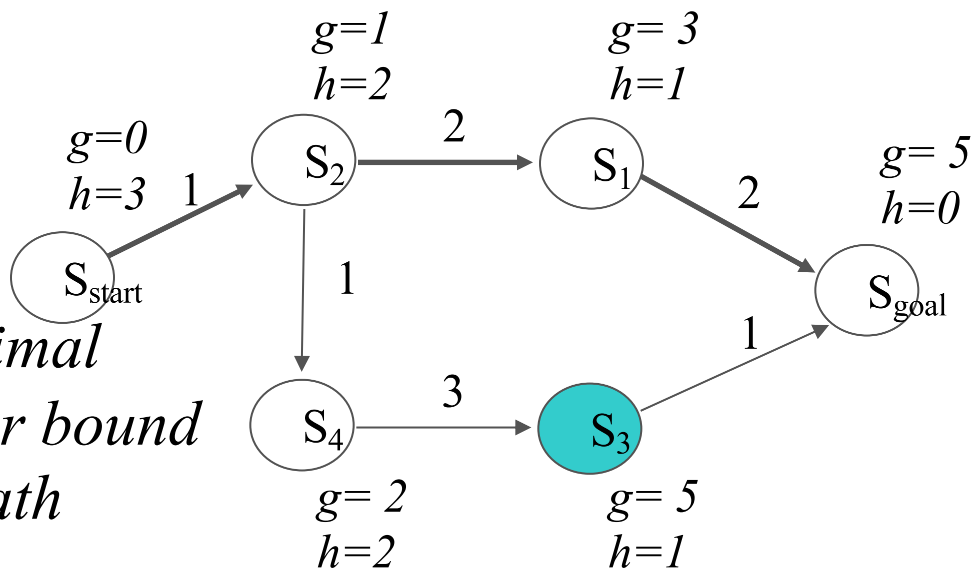

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for every other state g(s) is an upper bound
we can now compute a least-cost path

Properties of heuristics

What properties should *h(s)* satisfy? How does it affect search?

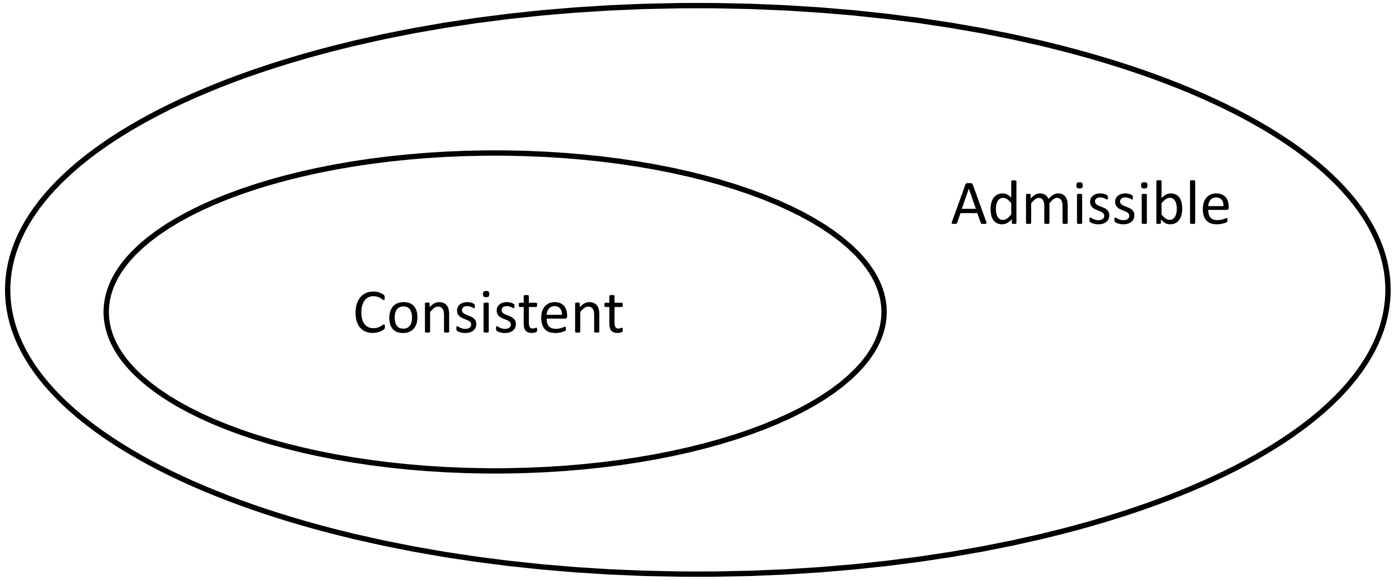
Admissible: $h(s) <= h^*(s)$ $h(goal) = 0$

If this true, the path returned by A* is optimal

Consistency: $h(s) <= c(s,s') + h(s')$ $h(goal) = 0$

If this true, A* is optimal AND efficient (will not re-expand a node)

Admissible vs Consistent



Theorem: ALL consistent heuristics are admissible, not vice versa!

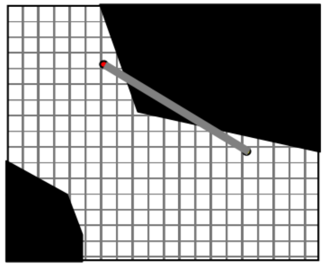
Takeaway:

Heuristics are great because they focus search on relevant states

AND

still give us optimal solution

Design of Informative Heuristics



- For grid-based navigation:
  - Euclidean distance
  - Manhattan distance: $h(x,y) = abs(x-x_{goal}) + abs(y-y_{goal})$
  - Diagonal distance: $h(x,y) = max(abs(x-x_{goal}), abs(y-y_{goal}))$
  - More informed distances???

Which heuristics are admissible for
4-connected grid?
8-connected grid?

Courtesy Max Likhachev

Design of Informative Heuristics

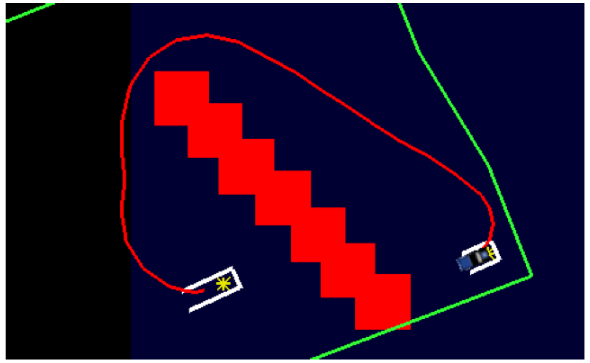
- For lattice-based 3D (x,y,Θ) navigation:

Any ideas?

Courtesy Max Likhachev

Design of Informative Heuristics

- For lattice-based 3D (x,y,Θ) navigation:



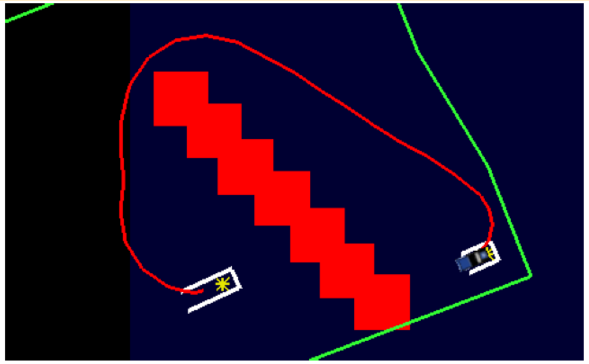
  - 2D (x,y) distance accounting for obstacles (single Dijkstra's on 2D grid cell starting at goalcell will give us these values)

Any problems where it will be highly uninformative?

Courtesy Max Likhachev

Design of Informative Heuristics

- For lattice-based 3D (x,y,Θ) navigation:



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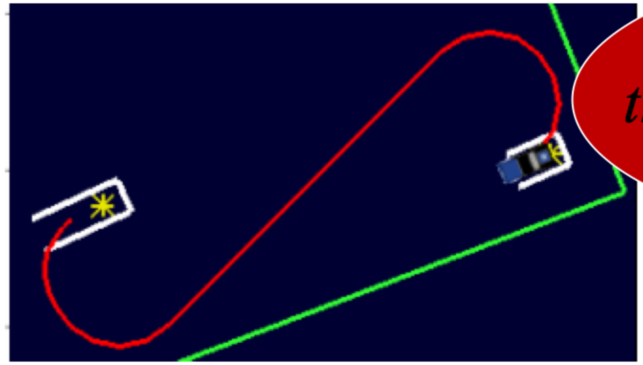


Any heuristic functions that will guide search well in this example?

Courtesy Max Likhachev

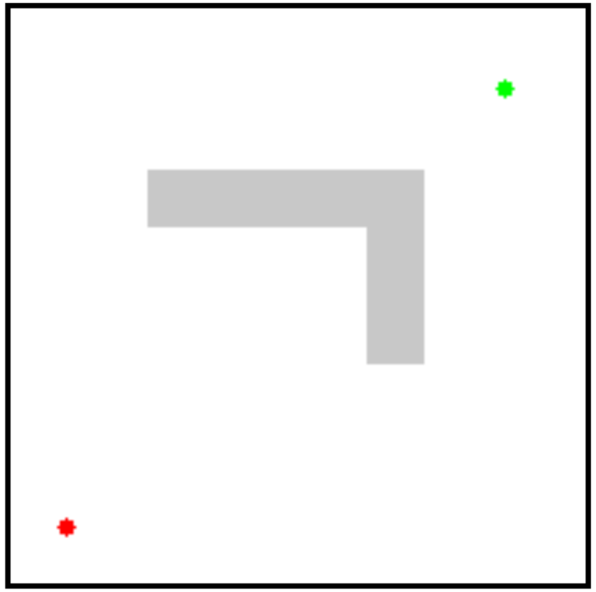
Design of Informative Heuristics

- Arm planning in 3D:

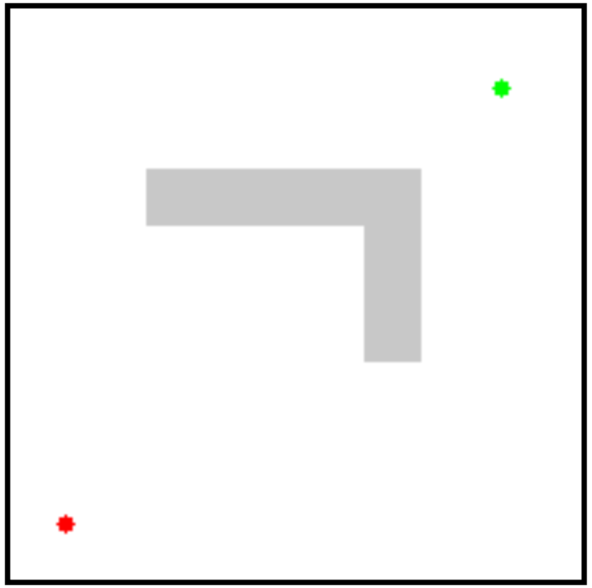
Any ideas?

Courtesy Max Likhachev

Is admissibility always what we want?



Admissible



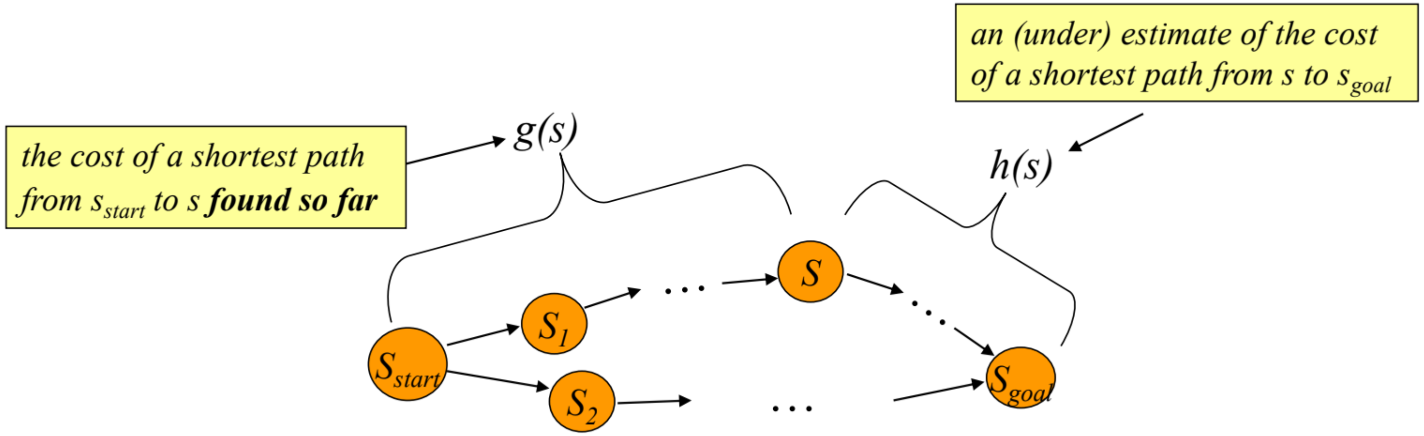
Inadmissible

Can inadmissible heuristics help us with this tradeoff?

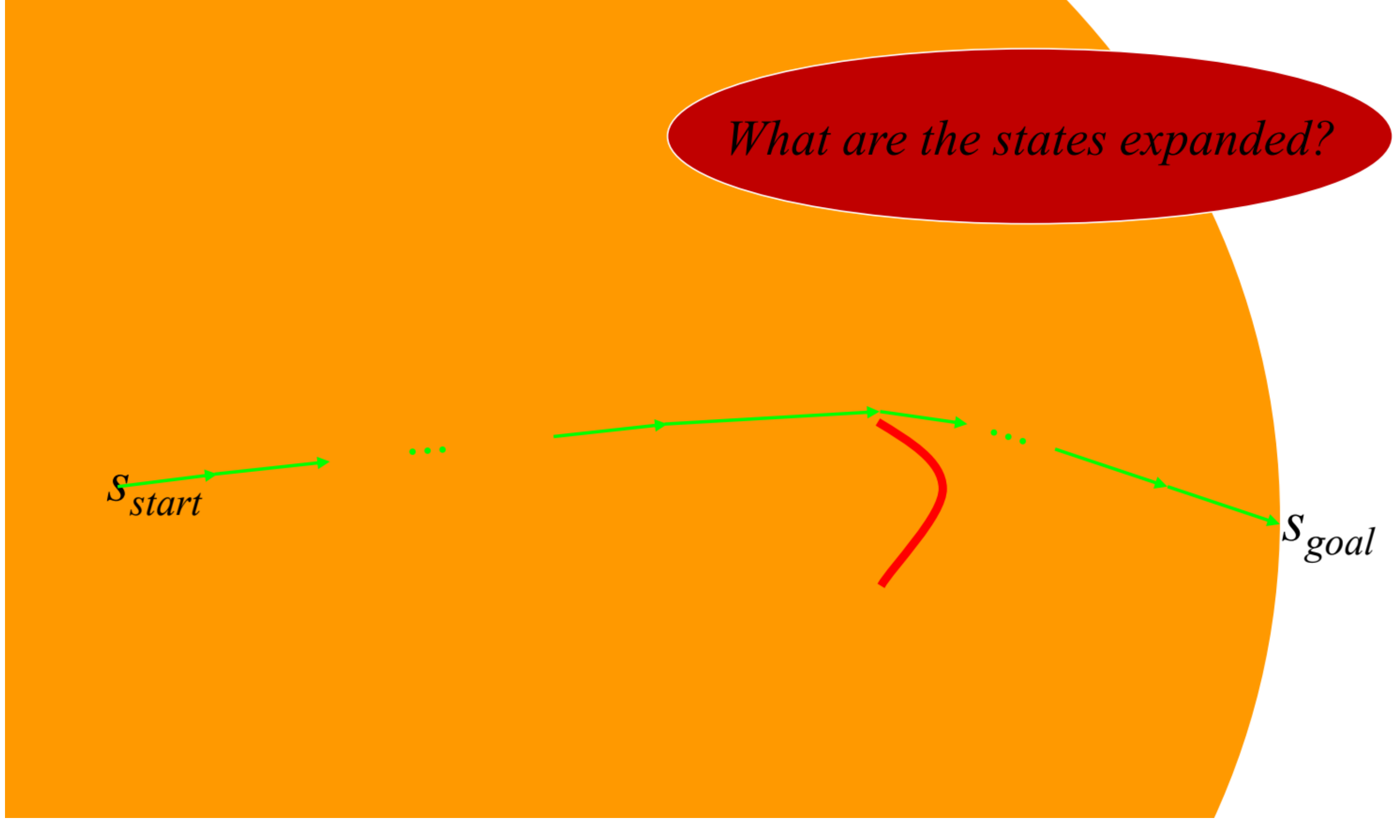
Solution Quality ⟷ Number of states expanded

- A* Search: expands states in the order of $f = g+h$ values
- Dijkstra's: expands states in the order of $f = g$ values

- **Weighted A*:** expands states in the order of $f = g+\varepsilon h$ values, $\varepsilon > 1$ = bias towards states that are closer to goal



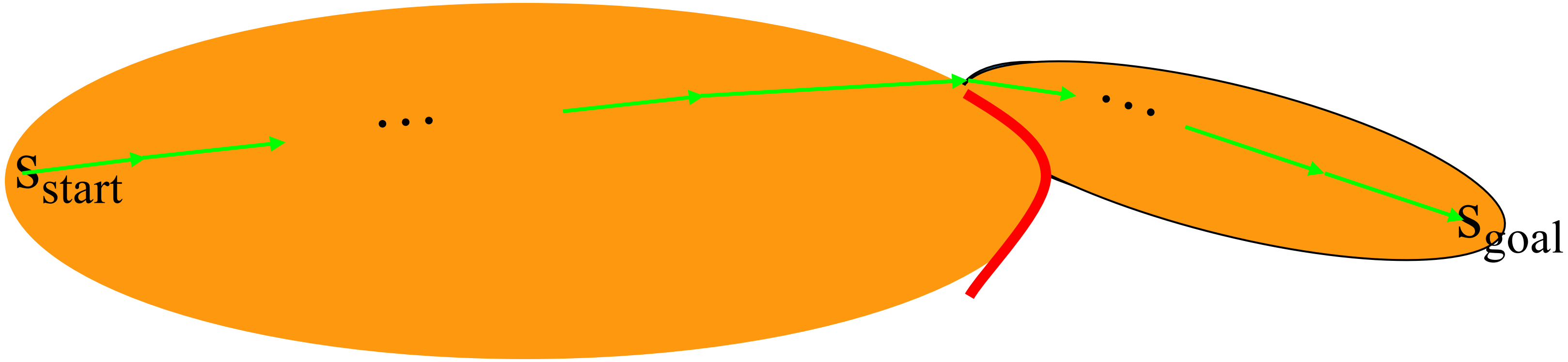
- Dijkstra's: expands states in the order of $f = g$ values



Courtesy Max Likhachev

Effect of the Heuristic Function

- A* Search: expands states in the order of $f = g+h$ values

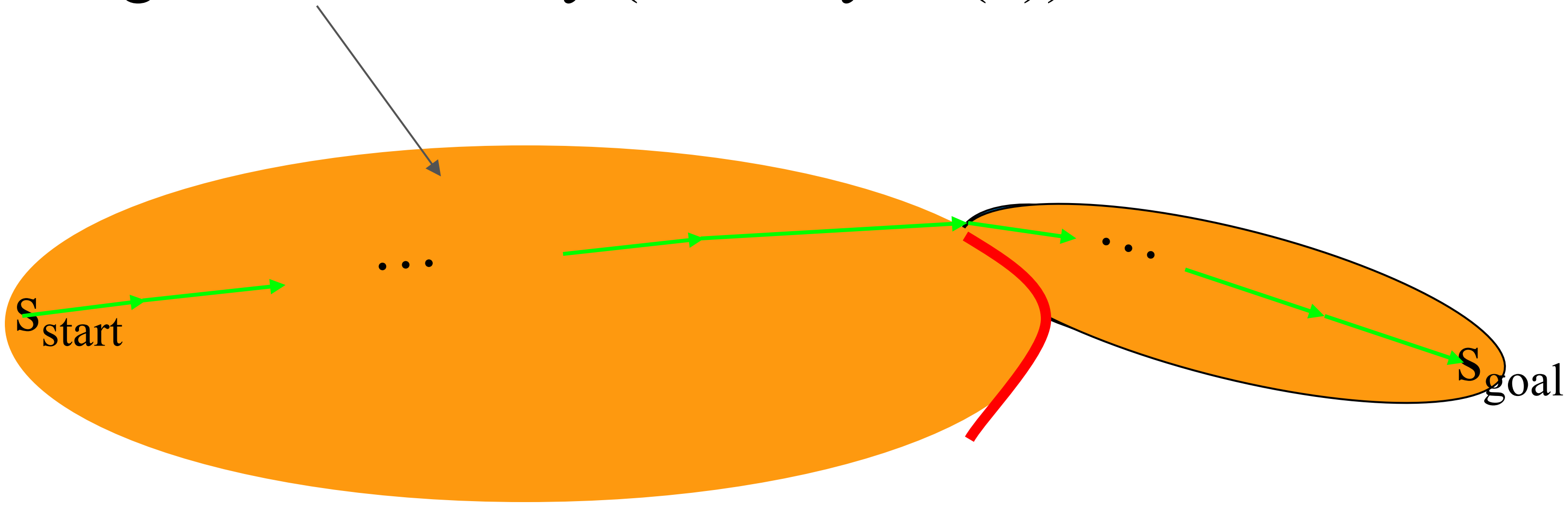


Courtesy Max Likhachev

Effect of the Heuristic Function

- A* Search: expands states in the order of *f* = *g*+*h* values

for large problems this results in A* quickly running out of memory (memory: O(n))

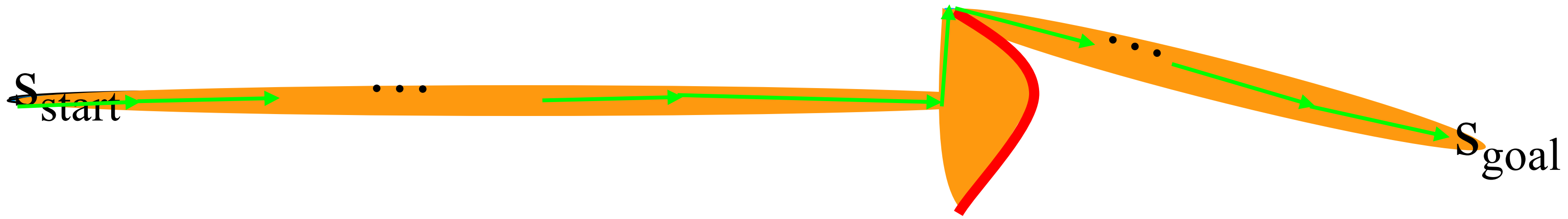


Courtesy Max Likhachev

Effect of the Heuristic Function

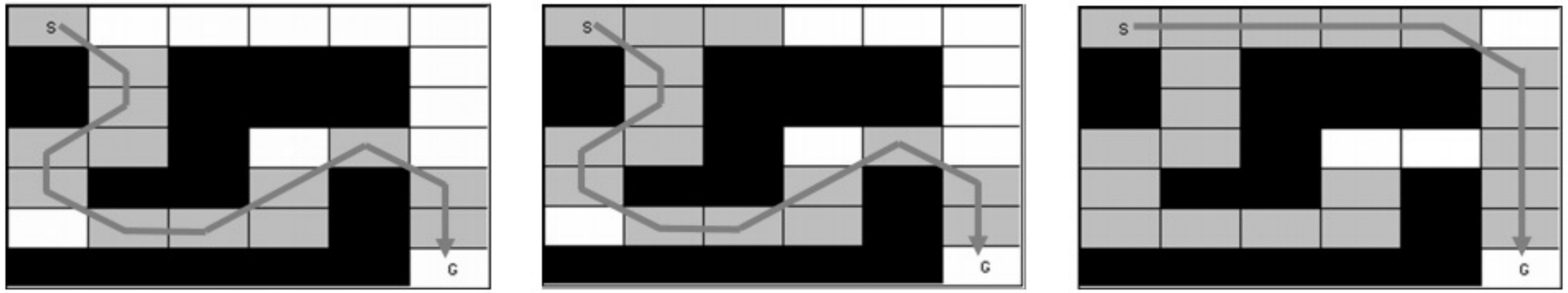
- Weighted A* Search: expands states in the order of $f = g+\varepsilon h$ values, $\varepsilon > 1$ = bias towards states that are closer to goal

solution is always ε-suboptimal:
cost(solution) ≤ ε·cost(optimal solution)



Courtesy Max Likhachev

Effect of the Heuristic Function



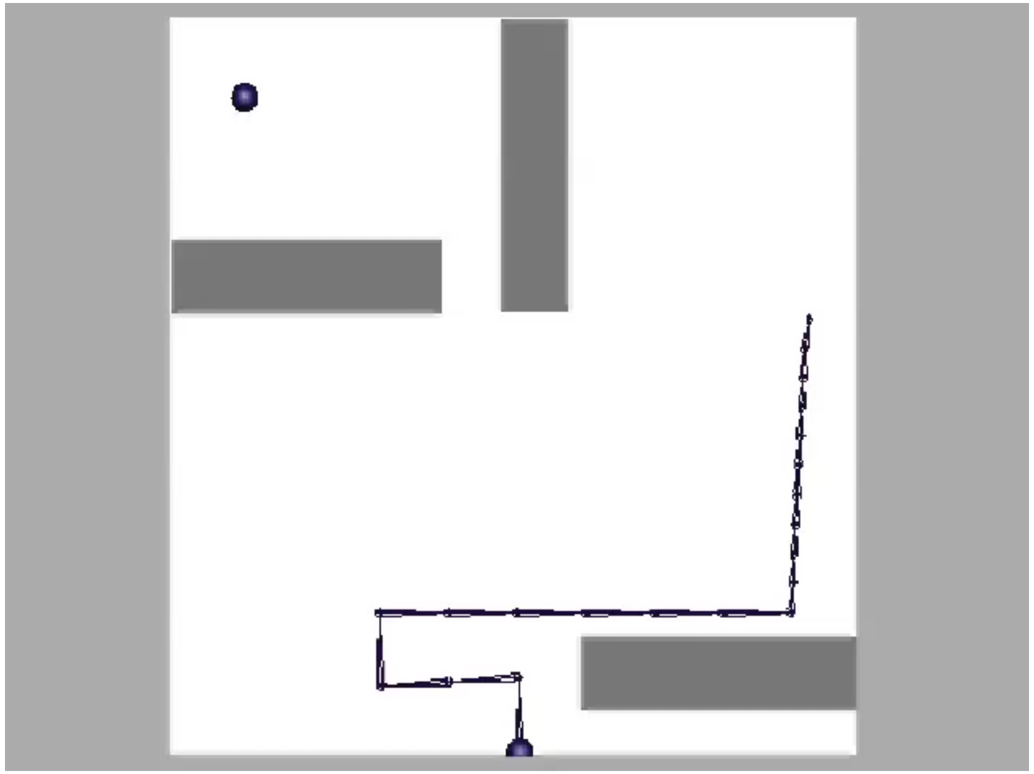
$\epsilon = 2.5$ $\epsilon = 1.5$ $\epsilon = 1.0$ (optimal search)

Courtesy Max Likhachev

Effect of the Heuristic Function

- Weighted A* Search: expands states in the order of $f = g+\varepsilon h$ values, $\varepsilon > 1$ = bias towards states that are closer to goal

20DOF simulated robotic arm
state-space size: over 10^{26} states

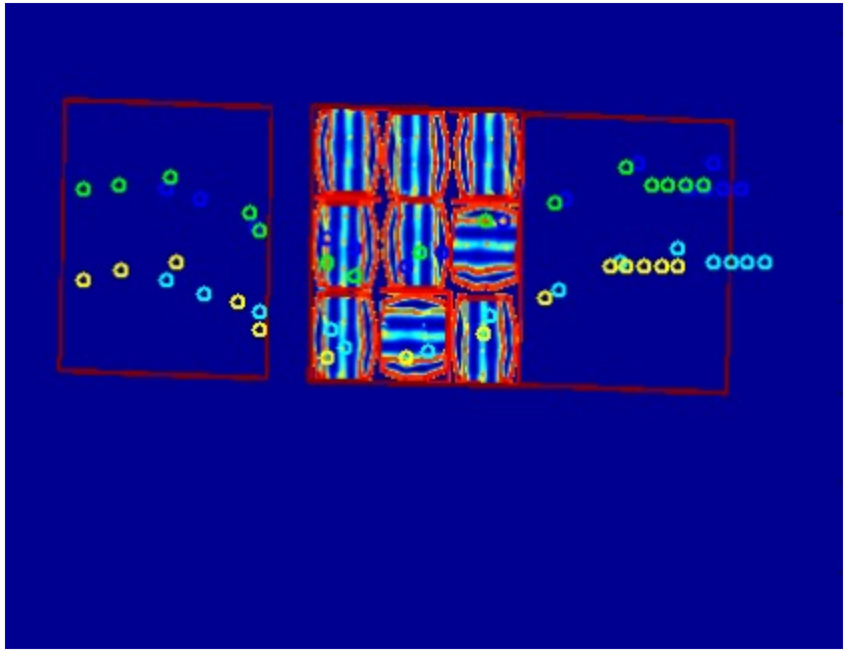


planning with ARA* (anytime version of weighted A*)

Courtesy Max Likhachev

Effect of the Heuristic Function

- planning in 8D (<x,y> for each foothold)
- heuristic is Euclidean distance from the center of the body to the goal location
- cost of edges based on kinematic stability of the robot and quality of footholds



Uses R* - A randomized version of weighted A*
Joint work between Max Likhachev, Subhrajit Bhattacharya, Joh Bohren, Sachin Chitta, Daniel D. Lee, Aleksandr Kushleyev, and Paul Vernaza

Lecture Outline

Best-First Search

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Heuristics and A*

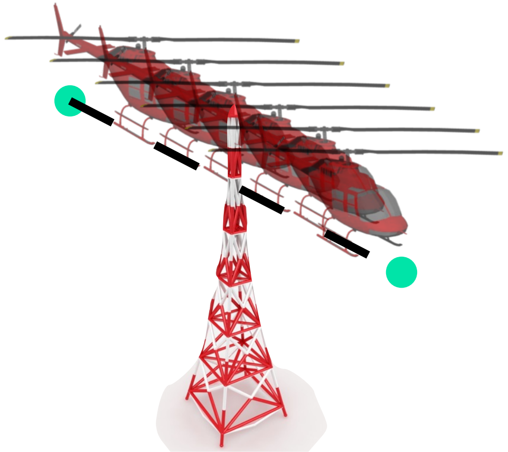
↓

Lazy A*

But is the number of expansions really what we want to minimize in motion planning?

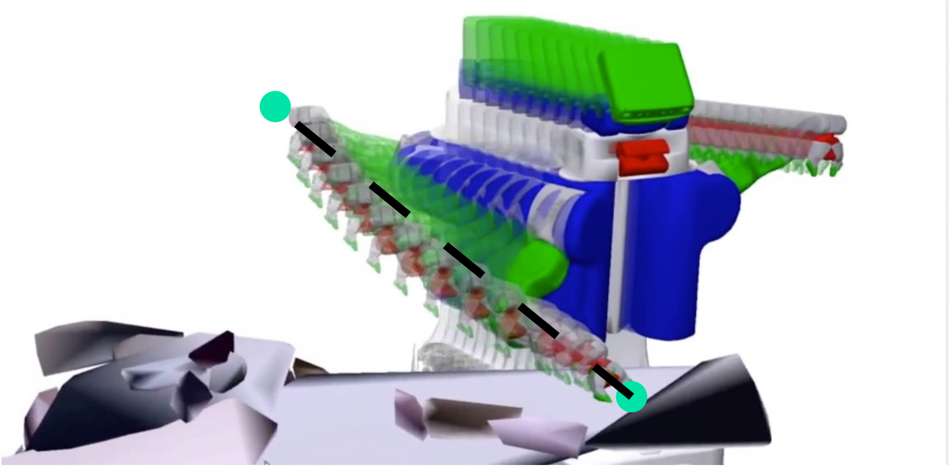
What is the most expensive step?

Edge evaluation is expensive



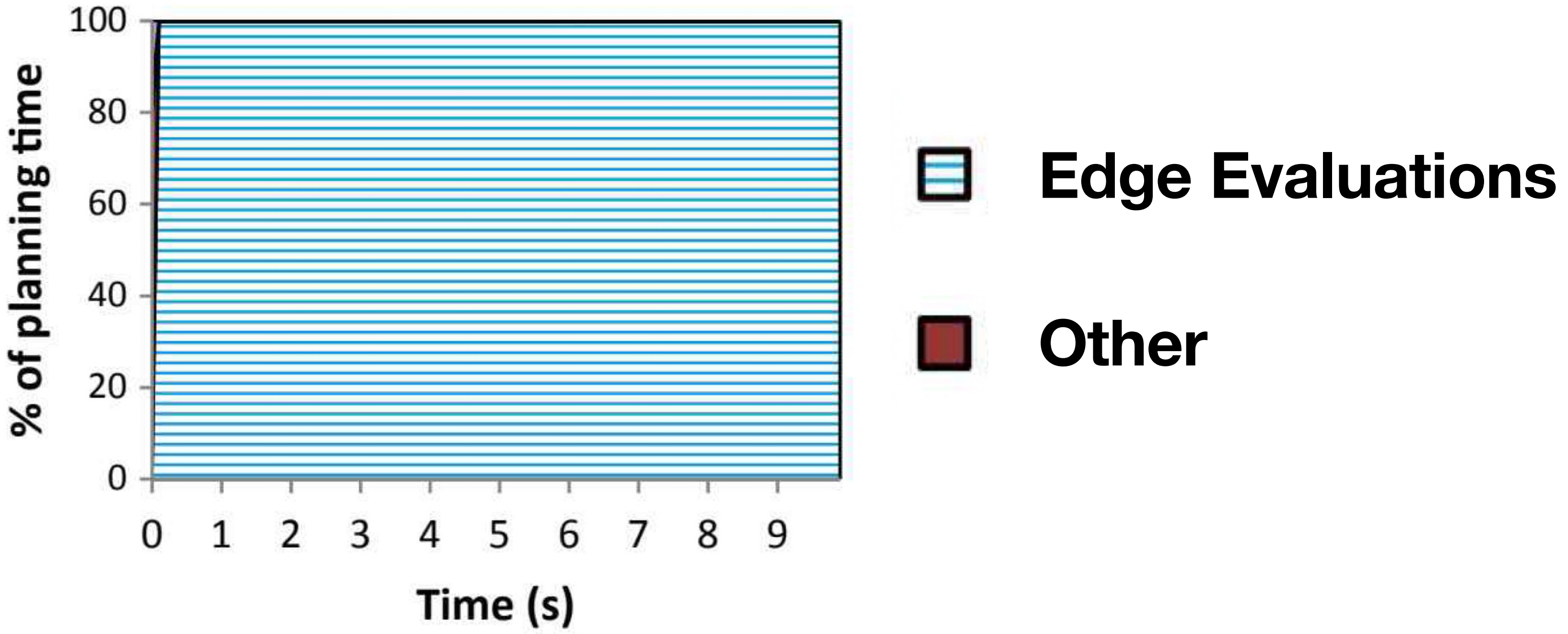
Check if helicopter intersects with tower

(Schulman et al. ’14)



Check if manipulator intersects with table

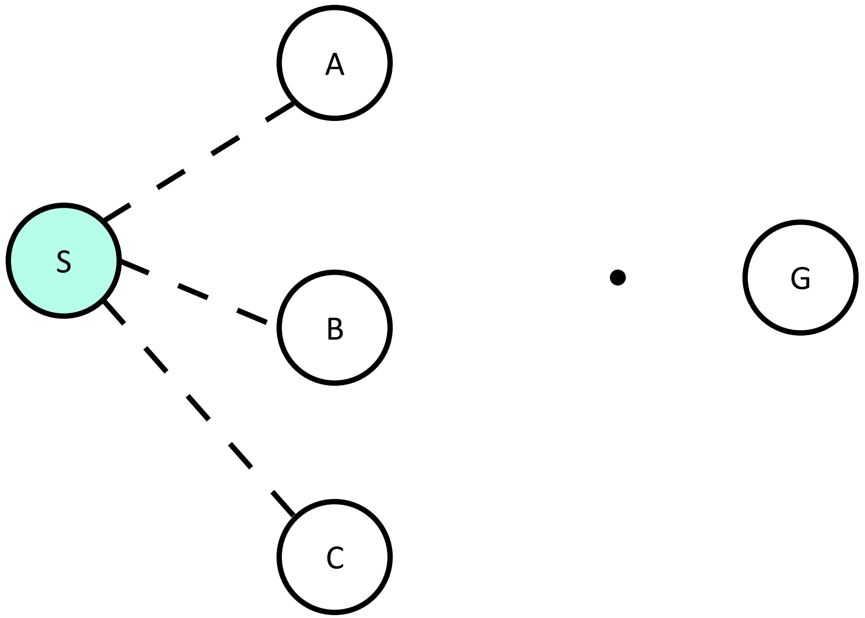
Edge evaluation dominates planning time



Hauser, Kris., Lazy collision checking in asymptotically-optimal motion planning. *ICRA* 2015

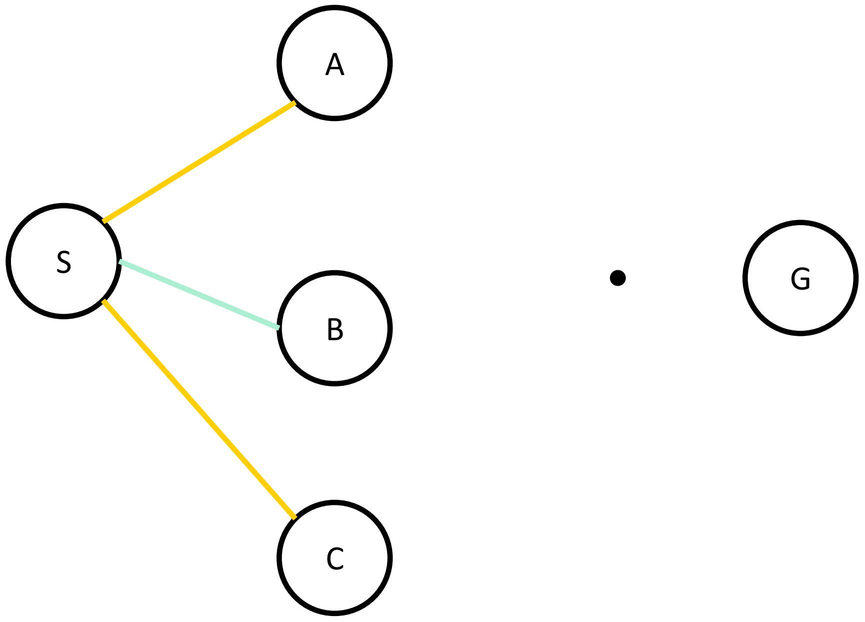
Let's revisit Best First Search

Element (Node)	**Priority Value (f-value)**
Node S	f(S)



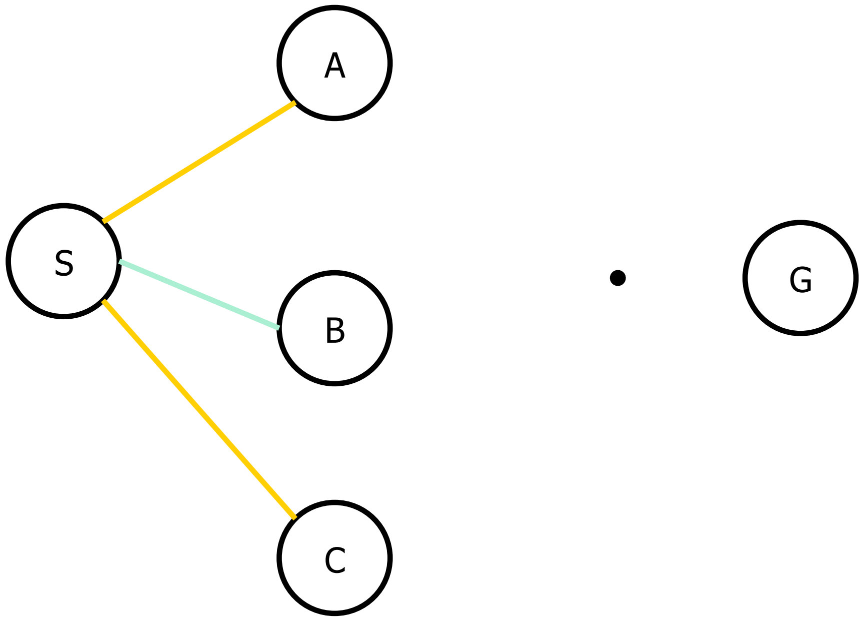
Let’s revisit Best First Search

Element (Node)	Priority Value (f-value)
~~Node S~~	~~f(S)~~
Node A	f(A)
Node C	f(C)



What if we never use C? Wasted collision check!

Element (Node)	Priority Value (f-value)
~~Node S~~	~~f(S)~~
Node A	f(A)
Node C	f(C)



The provable virtue of laziness:

Take the thing that's expensive (collision checking)

and

procrastinate as long as possible
till you have to evaluate it!

Lazy (weighted) A*

Cohen, Phillips, and Likhachev 2014

Key Idea:

1. When expanding a node, don't collision check edge to successors

   (be optimistic and assume the edge will be valid)

2. When expanding a node, collision check the edge to parent

   (expansion means this node is good and worth the effort)

3. Important: OPEN list will have multiple copies of a node

   (multiple candidate parents since we haven't collision check)

Lazy A*

Cohen, Phillips, and Likhachev 2014

Non lazy A*

while(s_{goal} is not expanded)
  remove *s* with the smallest
  [$f(s) = g(s)+h(s)$] from *OPEN*;

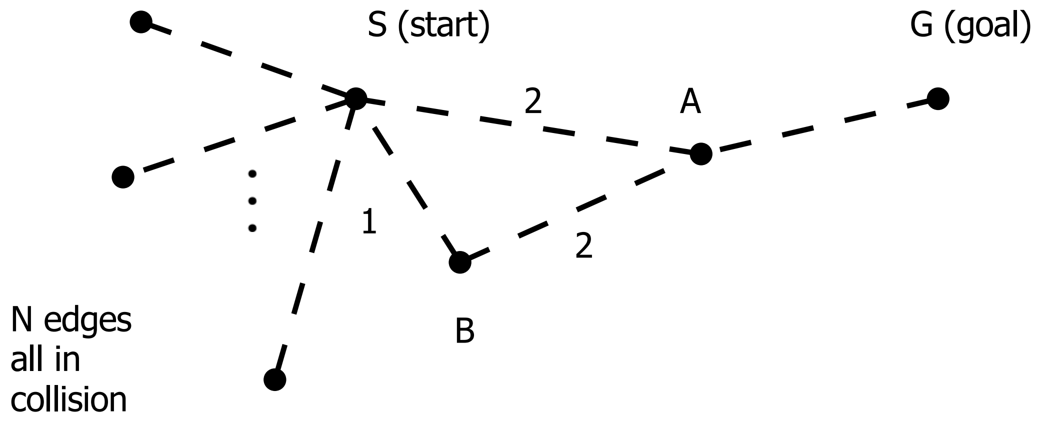
  insert *s* into *CLOSED*;
  for every successor *s'* of *s* such
    that *s'* not in *CLOSED*
    if *edge* (s,s') in collision
      $c(s,s') = \infty$
    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s')$;
      insert *s'* into *OPEN*;

Lazy A*

while(s_{goal} is not expanded)
  remove *s* with the smallest
  [$f(s) = g(s)+h(s)$] from *OPEN*;
  if s is in CLOSED
    continue;
  if *edge(parent(s), s)* in collision
    continue;
  insert *s* into *CLOSED*;
  for every successor *s'* of *s* such
    that *s'* not in *CLOSED*
    no collision checking of edge
    if $g(s') > g(s) + c(s,s')$
      $g(s') = g(s) + c(s,s')$;
      insert *s'* into *OPEN*; // multiple copies

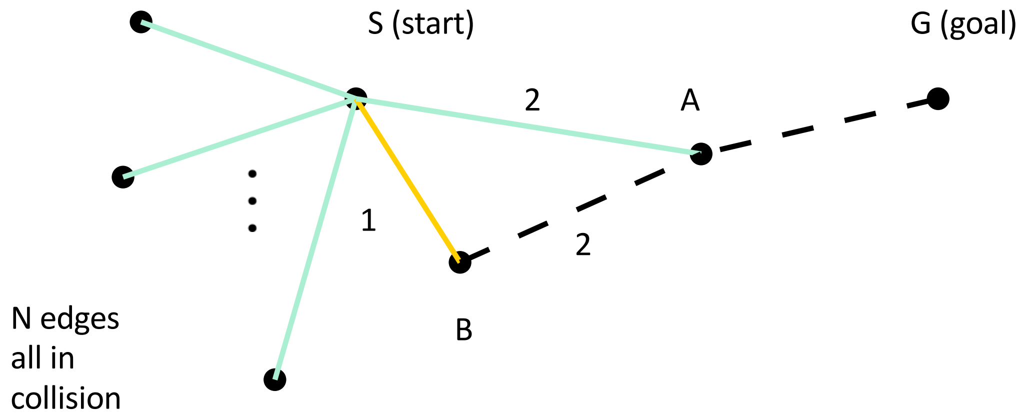
A*

Let’s say S-A is in collision and true shortest path is S-B-A-G



A*

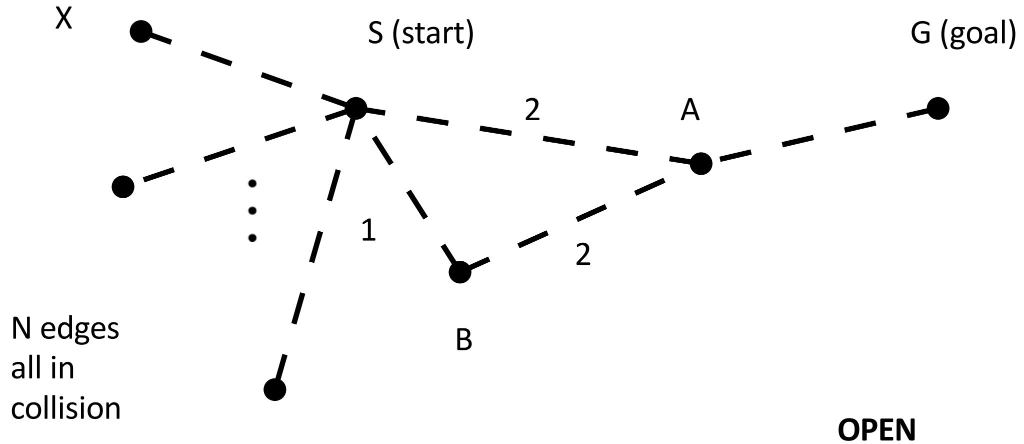
Let's say S-A is in collision and true shortest path is S-B-A-G



A* will
collision check
all N+2 edges!

Lazy A*

Let's say S-A is in collision and true shortest path is S-B-A-G

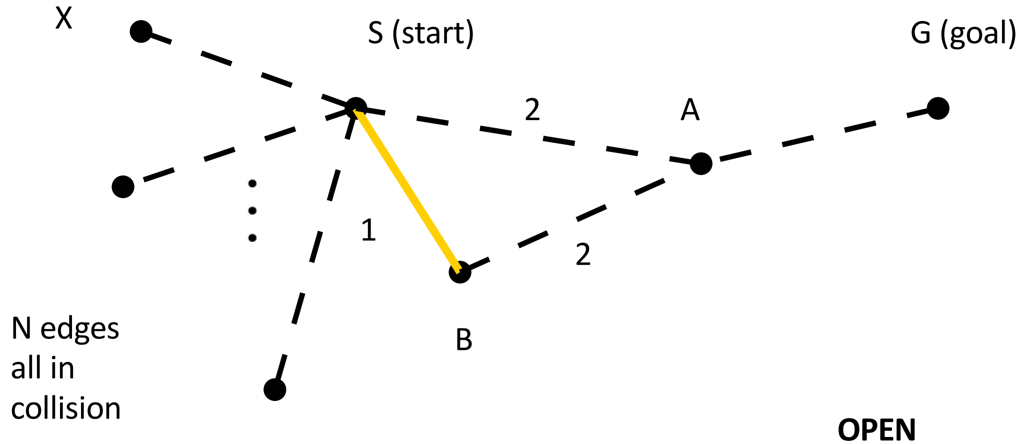


Lets set $f(s) = g(s)$

	OPEN	CLOSED	CollChecked
f = 1	B (from S)	S	
f = 2	A (from S)		
f = 1000	X (from S)		
	.....		

Lazy A*

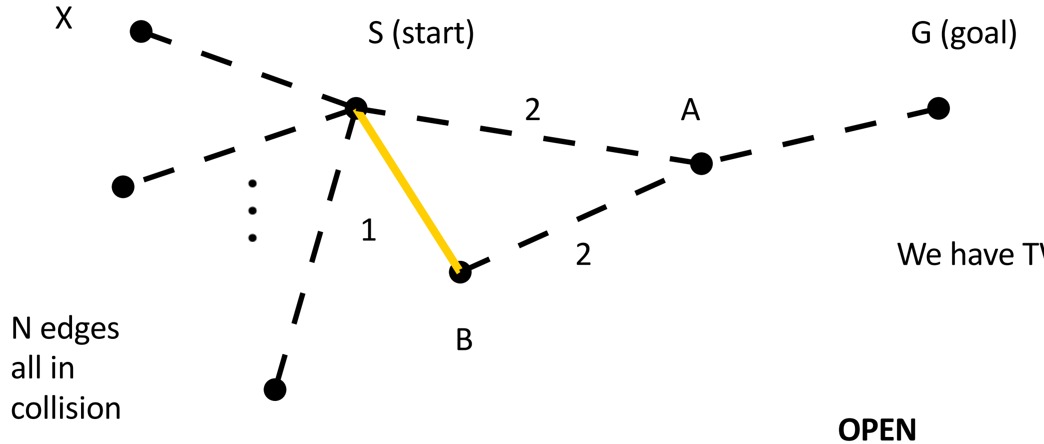
Let’s say S-A is in collision and true shortest path is S-B-A-G



	OPEN	CLOSED	CollChecked
f = 2	A (from S)	S	S-B
f = 3	A (from B)	B	
	.....		

Lazy A*

Let's say S-A is in collision and true shortest path is S-B-A-G



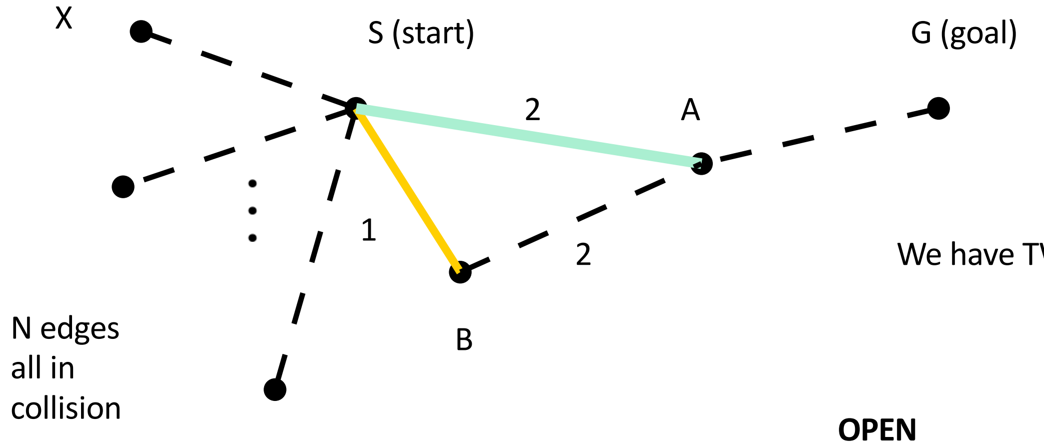
We have TWO copies of A in OPEN!

	OPEN	CLOSED	CollChecked
f = 2	A (from S)	S	S-B
f = 3	A (from B)	B	

.....

Lazy A*

Let's say S-A is in collision and true shortest path is S-B-A-G

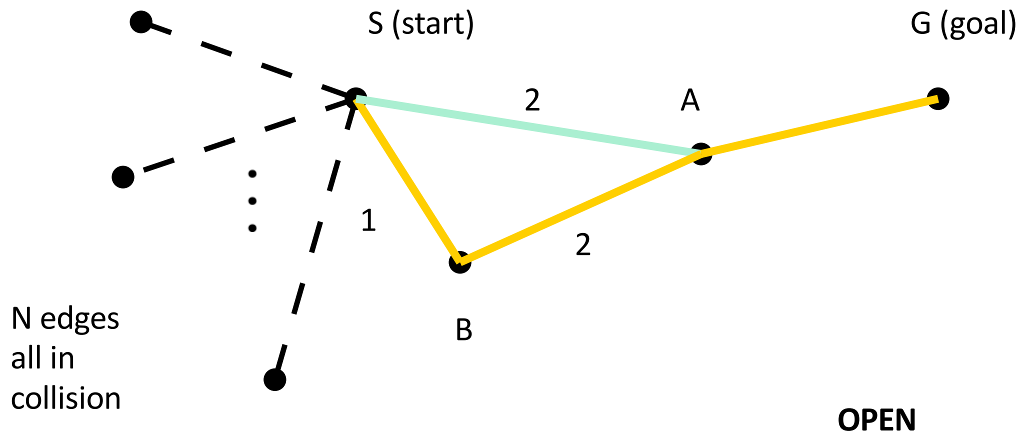


We have TWO copies of A in OPEN!

	OPEN	CLOSED	CollChecked
f = 3	A (from B)	S	S-B
		B	S-A
	.....	A	

Lazy A*

Let's say S-A is in collision and true shortest path is S-B-A-G



OPEN	CLOSED	CollChecked
	S	S-B
	B	S-A
	A	B-A
	G	A-G

Lecture Outline

Best-First Search

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Heuristics and A*

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Lazy A*

Class Outline

State Estimation

- Robotic System Design
- Filtering
- Localization
- SLAM

Control

- Feedback Control
- PID Control
- MPC
- LQR

Planning

- Search
- Heuristic Search
- Motion Planning
- Lazy Search

Learning

- Imitation Learning
- Policy Gradient
- Actor-Critic
- Model-Based RL