Autonomous Robotics
Winter 2024

Abhishek Gupta

TAs: Karthikeya Vemuri, Arnav Thareja

Marius Memmel, Yunchu Zhang

Slides borrowed from many sources – Liyiming Ke, Sergey Levine

Class Outline

State Estimation
- Robotic System Design
- Filtering
- Localization
- SLAM

Control
- Feedback Control
- PID Control
- MPC
- LQR

Planning
- Search
- Heuristic Search
- Motion Planning
- Lazy Search

Learning
- Imitation Learning
- Policy Gradient
- Actor-Critic
- Model-Based RL

Logistics

- One paper presentation today
- Guest lecture 1 on March 1
- Project 5 (Final Project) will be released Feb 26.

Lecture Outline

Policy Gradient

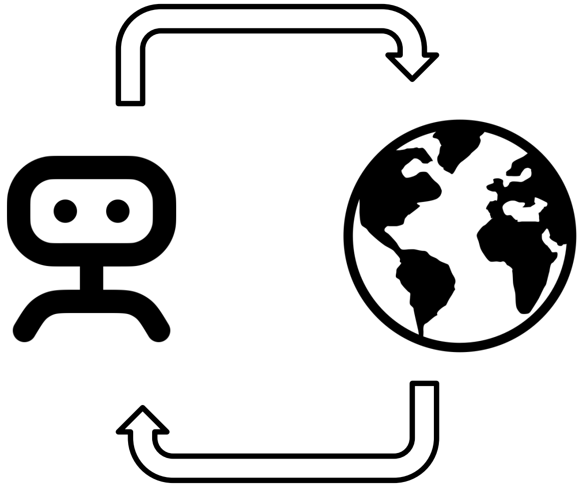
↓

Improving Policy Gradient

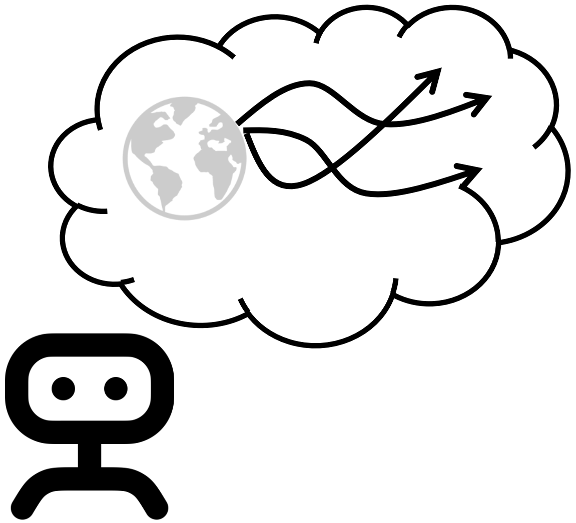
Ok so how can we learn policies?

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

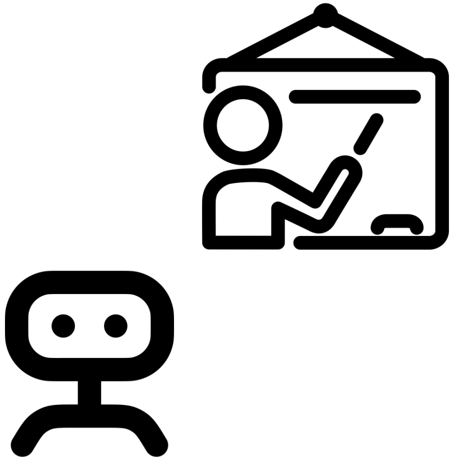
Model-free RL



Model-based RL



Imitation Learning



What if we just performed gradient ascent?

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

$$= \int p_\theta(\tau) R(\tau) d\tau$$

<u>Standard gradient descent (supervised learning)</u>

$$\nabla_\theta \mathbb{E}_{x \sim g(x)} \left[f_\theta(x) \right]$$

<u>REINFORCE gradient descent (RL)</u>

$$\nabla_\theta \mathbb{E}_{x \sim p_\theta(x)} \left[f(x) \right]$$

Gradient wrt expectation variable, not of integrand!

Taking the gradient of sum of rewards

$$J(\theta) = \int p_\theta(\tau) R(\tau) d(\tau)$$

$$\nabla_\theta J(\theta) = \nabla_\theta \int p_\theta(\tau) R(\tau) d(\tau)$$

$$= \int \nabla_\theta p_\theta(\tau) R(\tau) d(\tau) \quad = \int \frac{p_\theta(\tau)}{p_\theta(\tau)} \nabla_\theta p_\theta(\tau) R(\tau) d(\tau)$$

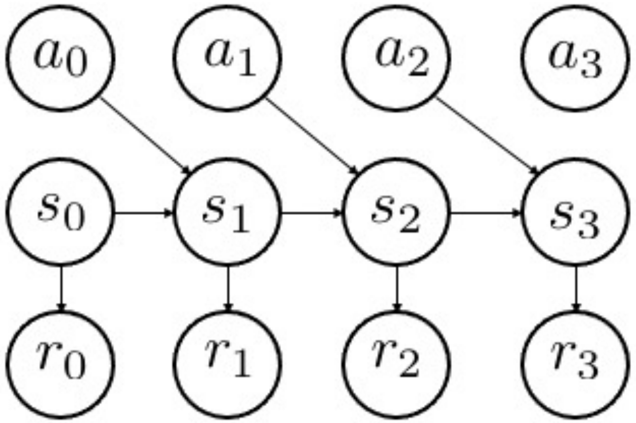
$$= \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) R(\tau) d(\tau) \quad = \mathbb{E}_{p_\theta(\tau)} \left[\nabla_\theta \log p_\theta(\tau) R(\tau)\right]$$

REINFORCE trick

Taking the gradient of return

Initial State Dynamics Policy

$$p_\theta(\tau) = p(s_0)\Pi_{t=0}^{T-1} p(s_{t+1}|s_t, a_t)\pi(a_t|s_t)$$



$$\log p_\theta(\tau) = \log p(s_0) + \sum_{t=0}^{T-1} \log p(s_{t+1}|s_t, a_t) + \log \pi(a_t|s_t)$$

$$\nabla_\theta \log p_\theta(\tau) = \nabla_\theta \log p(s_0) + \sum_{t=0}^{T-1} \nabla_\theta \log p(s_{t+1}|s_t, a_t) + \nabla_\theta \log \pi(a_t|s_t)$$

$$\nabla_\theta \log p_\theta(\tau) = \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)$$

Model Free!!

Taking the gradient of return

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\nabla_\theta \log p_\theta(\tau) \sum_{t=0}^{T} r(s_t, a_t) \right]$$

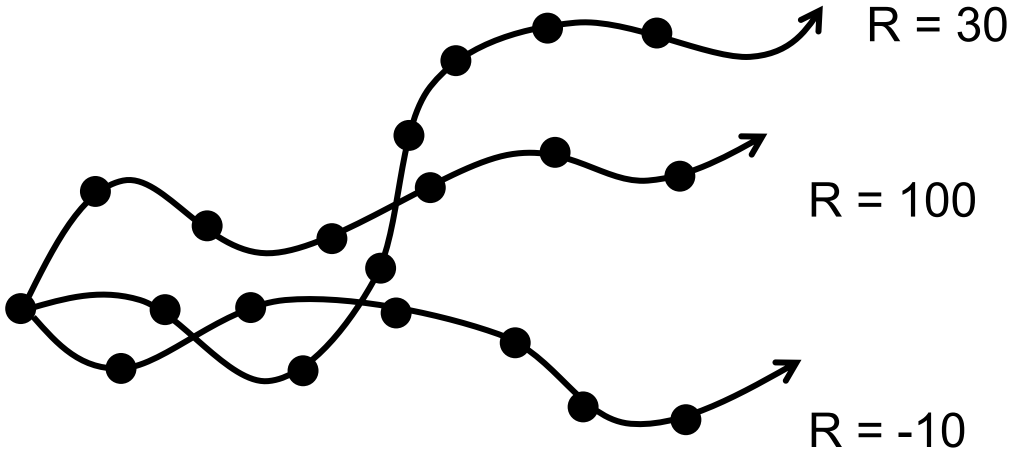
$$\nabla_\theta J(\theta) = \mathbb{E}_{\substack{s_0 \sim p(s_0) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t) \\ a_t \sim \pi(a_t|s_t)}} \left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=0}^{T} r(s_t, a_t) \right]$$

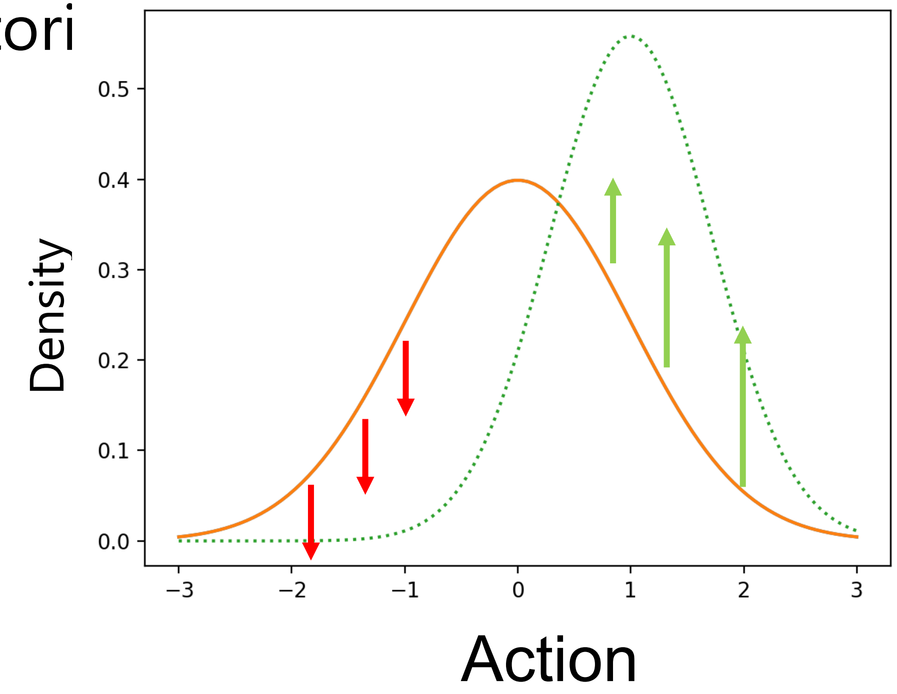
$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$ (approximating using samples)

What does this mean?

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

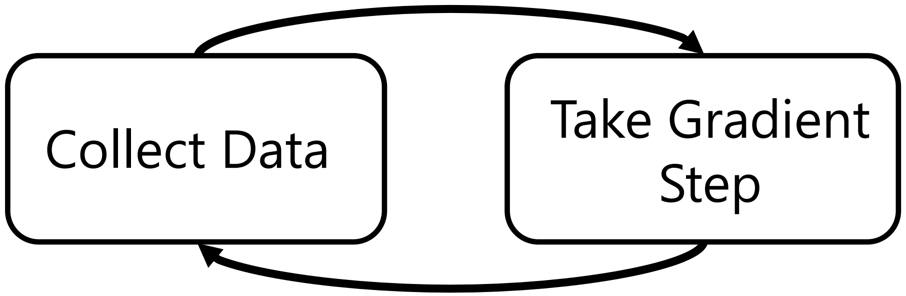
Increase the likelihood of actions in high return trajectori





Resulting Algorithm (REINFORCE)

$$\nabla_\theta J(\theta) = \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau)d\tau$$



REINFORCE algorithm:

On-policy

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Continuous Policy Gradient - Pseudocode

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Pseudocode example (with continuous actions):

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# q_values – (N*T) x 1 tensor of estimated state-action values
# Build the graph:
pred_mean, pred_cov = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = gaussian_log_likelihood(actions, mean= pred_mean, cov = pred_cov)
weighted_negative_likelihoods = tf.multiply(negative_likelihoods, returns)
loss = tf.reduce_mean(weighted_negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

Discrete Policy Gradient - Pseudocode

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right)\left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

Pseudocode example (with discrete actions):

```
# Given:
# actions - (N*T) x Da tensor of actions
# states - (N*T) x Ds tensor of states
# Build the graph:
logits = policy.predictions(states) # This should return (N*T) x Da tensor of action logits
negative_likelihoods = tf.nn.softmax_cross_entropy_with_logits(labels=actions,
logits=logits)
loss = tf.reduce_mean(negative_likelihoods)
gradients = loss.gradients(loss, variables)
```

How is this related to supervised learning?

Reinforcement Learning

$$\nabla_\theta J(\theta) = \int p_\theta(\tau)\nabla_\theta \log p_\theta(\tau)d\tau$$

$$\approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

Supervised Learning

$$\max_\theta \mathbb{E}_{(x,y)\sim\mathcal{D}}\left[\log p_\theta(y|x)\right]$$

$$\approx \frac{1}{N}\sum_{i}\nabla_\theta \log p_\theta(y^i|x^i)$$

PG = select good data + increase likelihood of selected data

Lecture Outline

Policy Gradient

↓

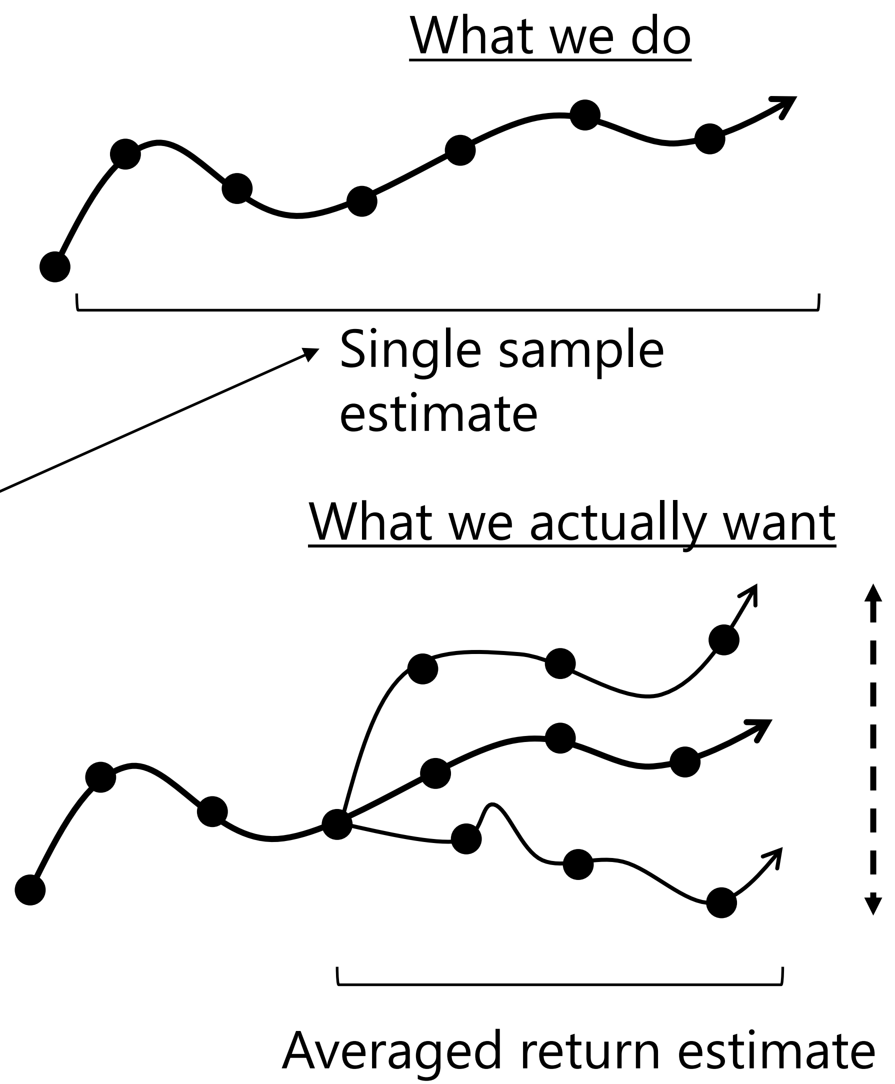
Improving Policy Gradient

What makes policy gradient challenging?

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

High variance estimator!!

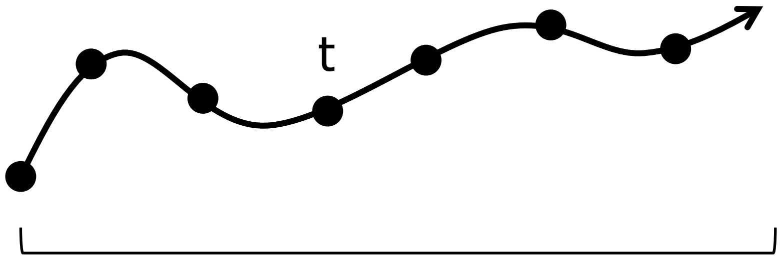


Hard to tell what matters without many samples

Variance Reduction with Causality

Idea: Trajectory returns depend on past and future, but we only care about the future, since actions cannot affect the past. Instead, consider **"return-to-go"**

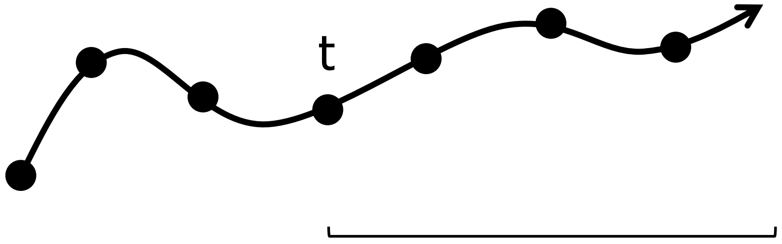
$$\approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\underbrace{\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)}_{\text{Includes } t' < t}$$



Full trajectory return

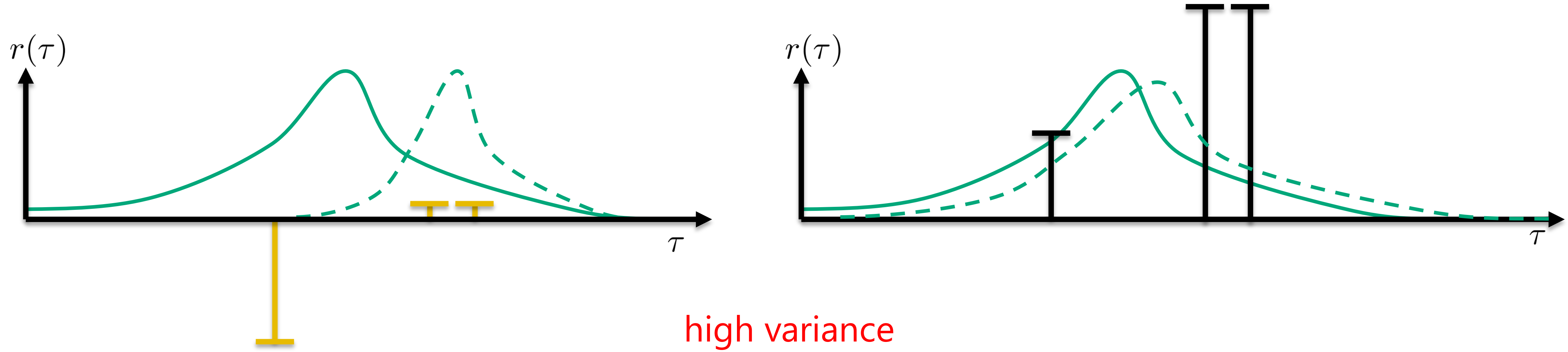
Ignore past terms

$$\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=t}^{T} r(s_t^i, a_t^i)$$



Return to go

Can we reduce variance further?



high variance

Arbitrarily uncentered, scaled returns can lead to huge variance:

→ Imagine all rewards were positive, every action would be pushed up, some more than others

→ What if instead, we pushed down some actions and pushed up some others (even if rewards are positive)

Credit: Sergey Levine

Variance Reduction with a Baseline

Idea: We can reduce variance by subtracting a current state dependent function from the policy gradient return

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\left[\sum_{t'=t}^{T} r(s_{t'}^i, a_{t'}^i) - b(s_t)\right]$$

Baseline: Centers the returns, reduces variance

But does this increase bias??

Variance Reduction with a Baseline

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) \left[\sum_{t'=t}^{T} r(s_{t'}, a_{t'}) - b(s_t) \right] ds_t \, da_t$$

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) \left[\sum_{t'=t}^{T} r(s_{t'}, a_{t'}) \right] ds_t \, da_t - \int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) b(s_t) \, ds_t \, da_t$$

Let us show this is 0!

Variance Reduction with a Baseline

$$\int\int p(s_t, a_t)\nabla_\theta \log \pi_\theta(a_t|s_t)\,[b(s_t)]\,ds_t da_t = \int\int p(s_t)\pi_\theta(a_t|s_t)\nabla_\theta \log \pi_\theta(a_t|s_t)\,[b(s_t)]\,ds_t da_t$$

$$= \int p(s_t)b(s_t) \int \pi_\theta(a_t|s_t)\nabla_\theta \log \pi_\theta(a_t|s_t) da_t ds_t$$

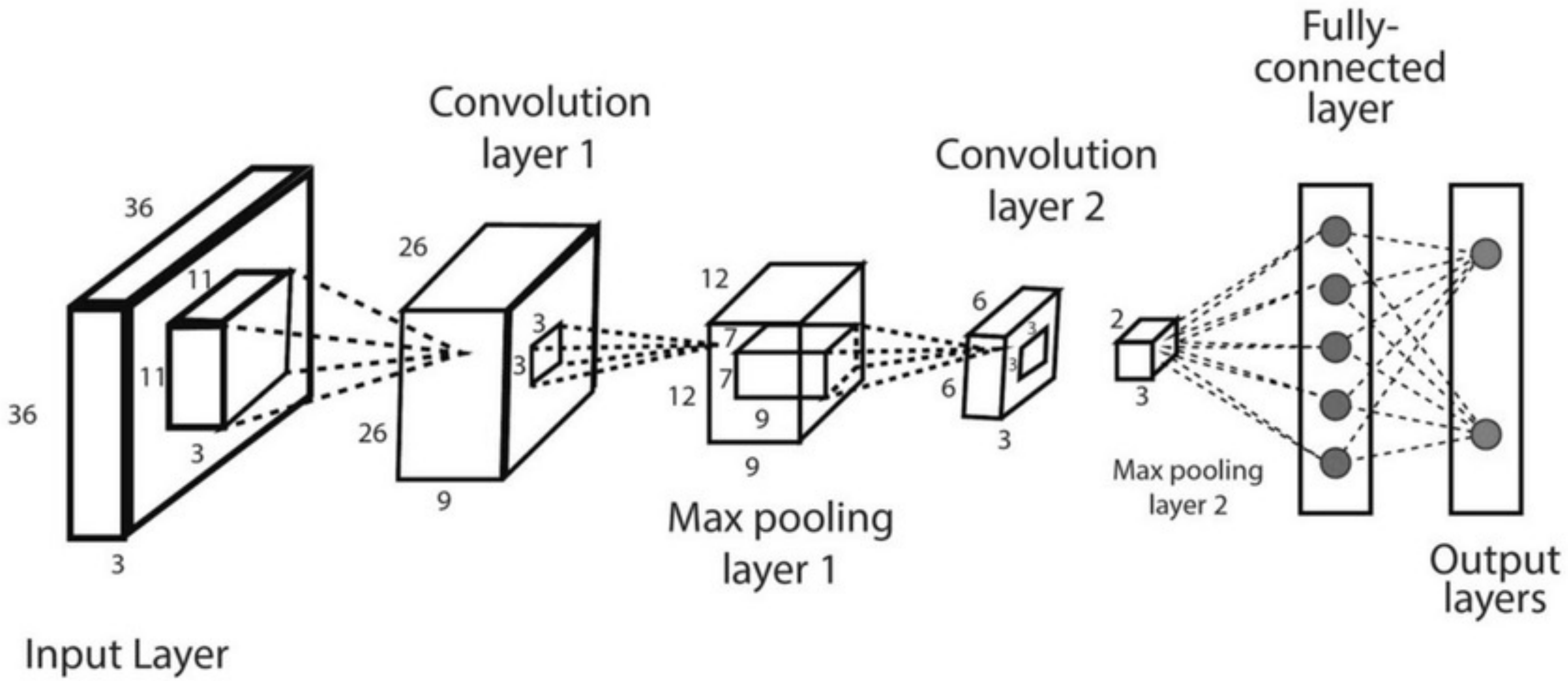
$$= \int p(s_t)b(s_t) \int \nabla_\theta \pi_\theta(a_t|s_t) da_t ds_t$$

$$= \int p(s_t)b(s_t)\nabla_\theta \int \pi_\theta(a_t|s_t) da_t ds_t = \int p(s_t)b(s_t)\nabla_\theta(1) ds_t = 0$$

Unbiased!

Learning Baselines

Baselines are typically learned as deep neural nets from $R^s \rightarrow R^1$



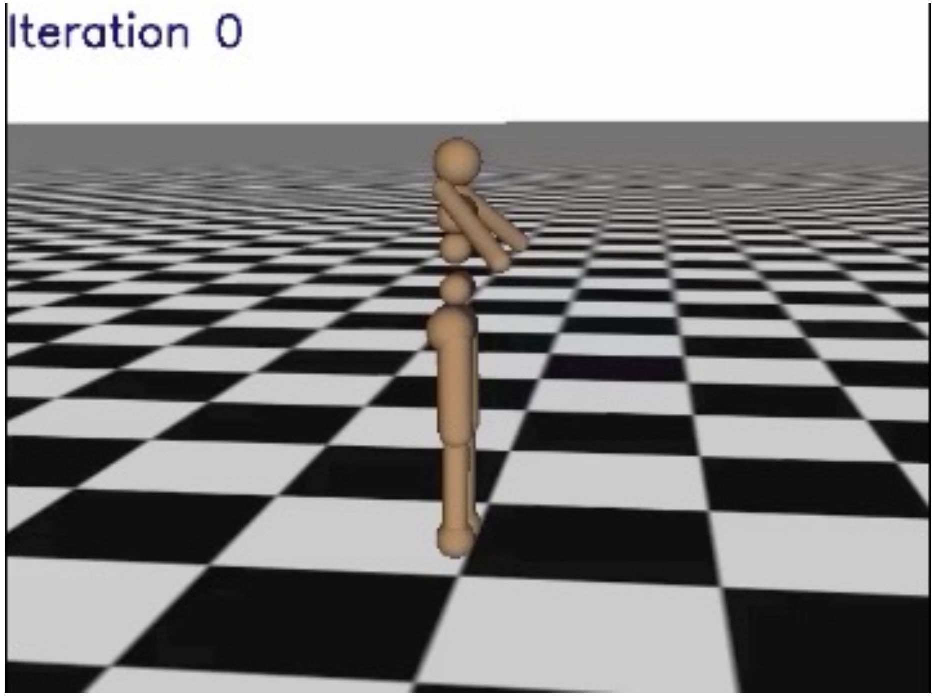
$$\frac{1}{N}\sum_{j=1}^{N}\|\hat{V}(s_t^j, a_t^j) - \sum_{t=1}^{H} r(s_t^j, a_t^j)\|$$

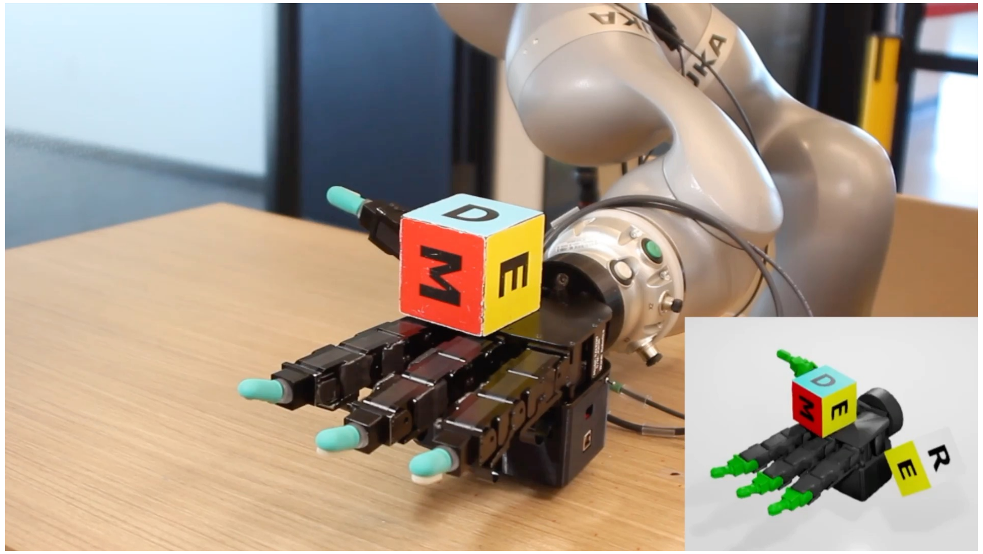
Minimize with Monte-carlo regression at every iteration, club with policy loss

$$A(s_t, a_t) = \sum_{t'=t}^{T} r(s'_t, a'_t) - V(s_t)$$

Allows us to define advantages

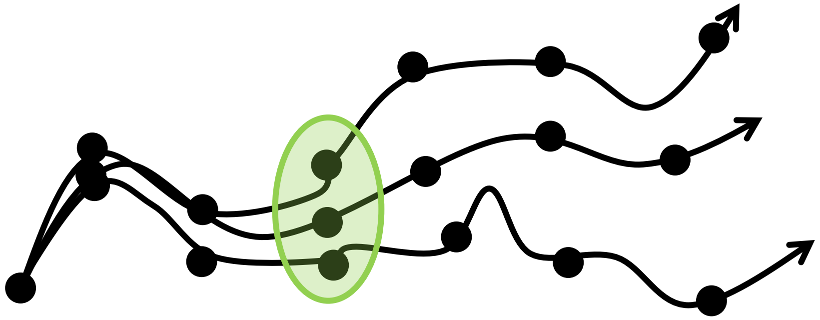
Policy Gradient in Action





How to further improve policy gradient?

<u>Lower variance further through function approximation</u>



Function approximator bundles return estimates across states

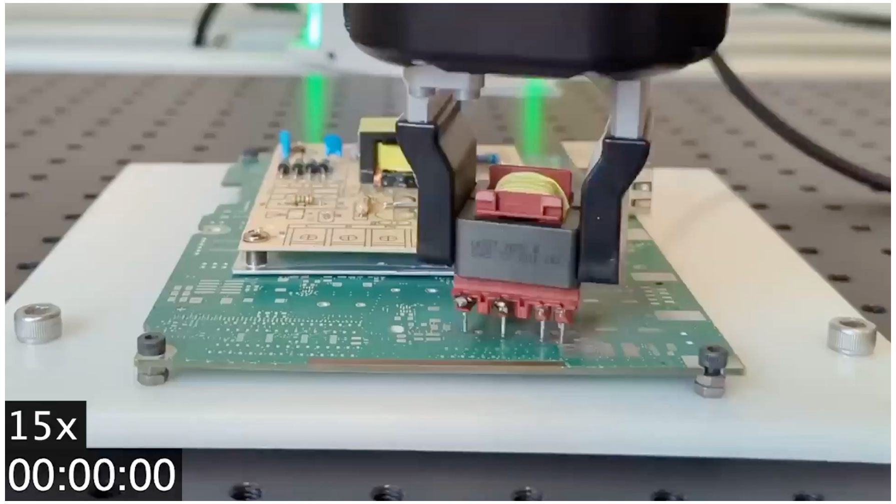
<u>Control Step Size</u>

Prevent excessive step size

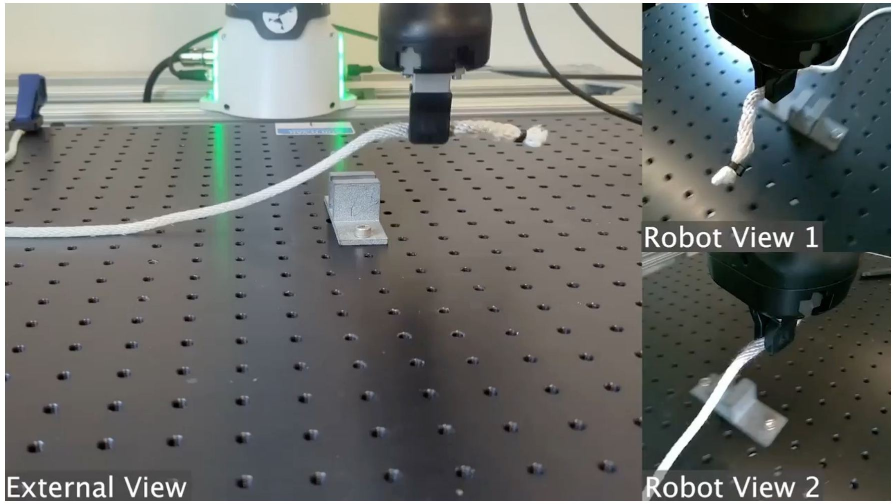
Policy Gradient in Action

With small improvements in estimation - can work on robots!

Smith et al



Luo et al



Luo et al

Class Outline

State Estimation
- Robotic System Design
- Filtering
- Localization
- SLAM

Control
- Feedback Control
- PID Control
- MPC
- LQR

Planning
- Search
- Heuristic Search
- Motion Planning
- Lazy Search

Learning
- Imitation Learning
- Policy Gradient
- Actor-Critic
- Model-Based RL