Autonomous Robotics
Winter 2024

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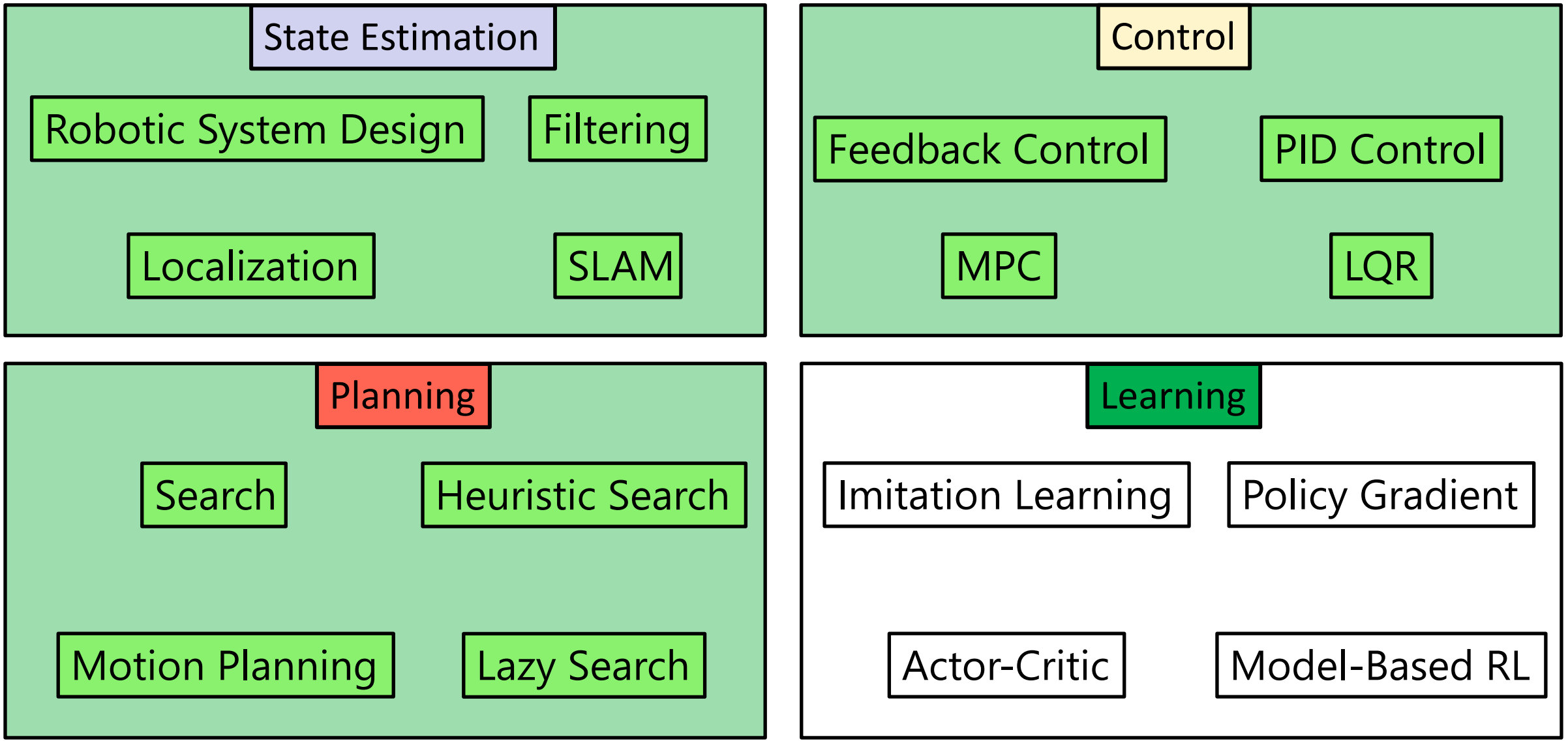
Slides borrowed from many sources – Liyiming Ke, Sergey Levine

Class Outline



Logistics

- One paper presentation Feb 26
- Guest lecture 1 moved to March 1
- Project 5 (Final Project) will be released Feb 26.

Lecture Outline

Imitation Learning Recap + Compounding Error

Imitation Learning: Improvements – Multimodality

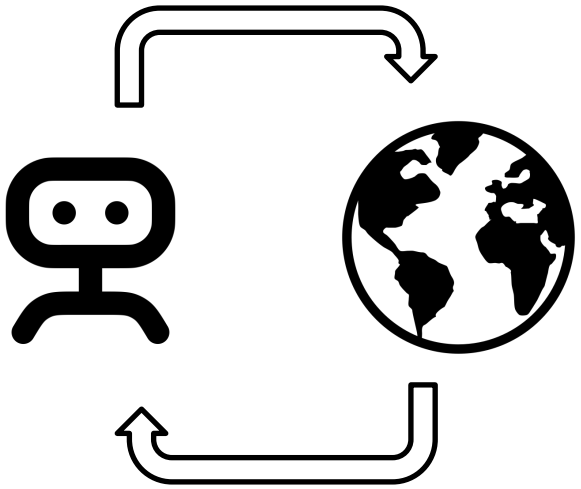
Policy Gradient

Improving Policy Gradient

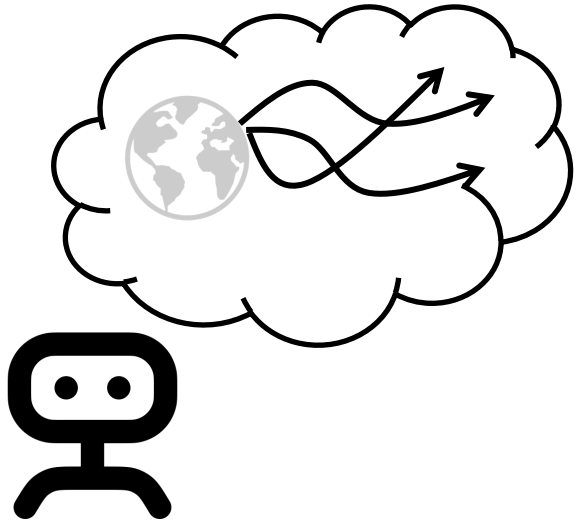
Ok so how can we learn policies?

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

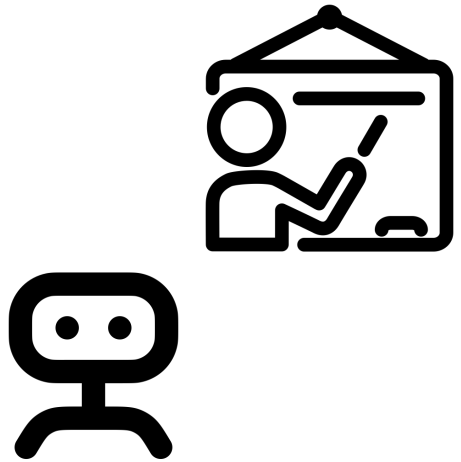
Model-free RL



Model-based RL



Imitation Learning



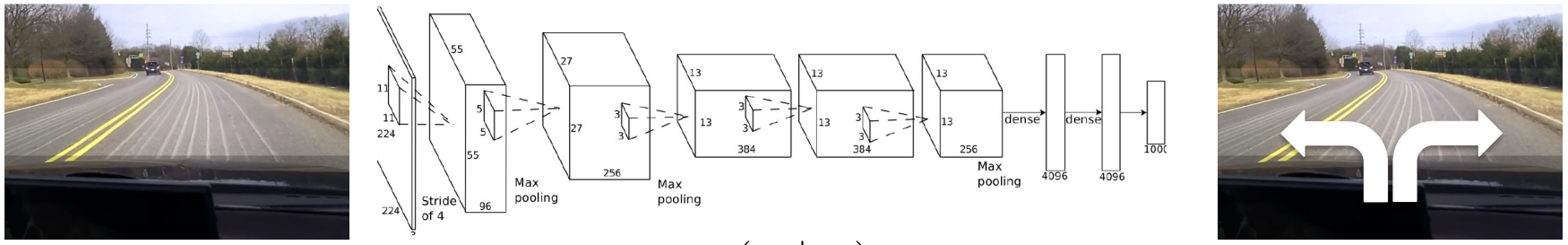
Idea 1: Imitation Learning via Behavior Cloning

Given: Demonstrations of optimal behavior

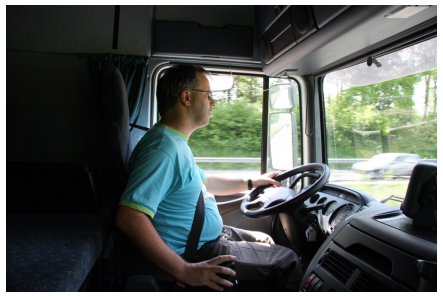
Goal: Train a policy to mimic the demonstrator

$$\arg\max_{\theta} \mathbb{E}_{(s^*,a^*)\sim\mathcal{D}} \left[\log \pi_\theta(a^*|s^*)\right]$$

Idea: Treat imitation learning as a supervised learning problem!



$\mathbf{o}_t$ $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ $\mathbf{a}_t$



$\mathbf{o}_t$

$\mathbf{a}_t$

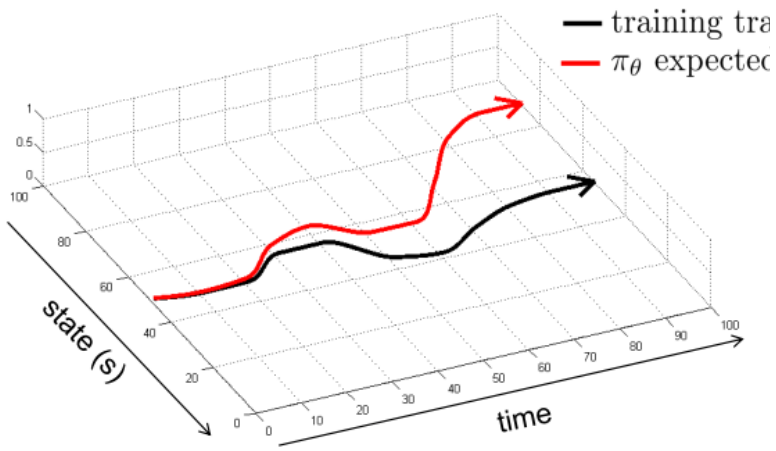
training data

supervised learning

$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$

So does behavior cloning really work?

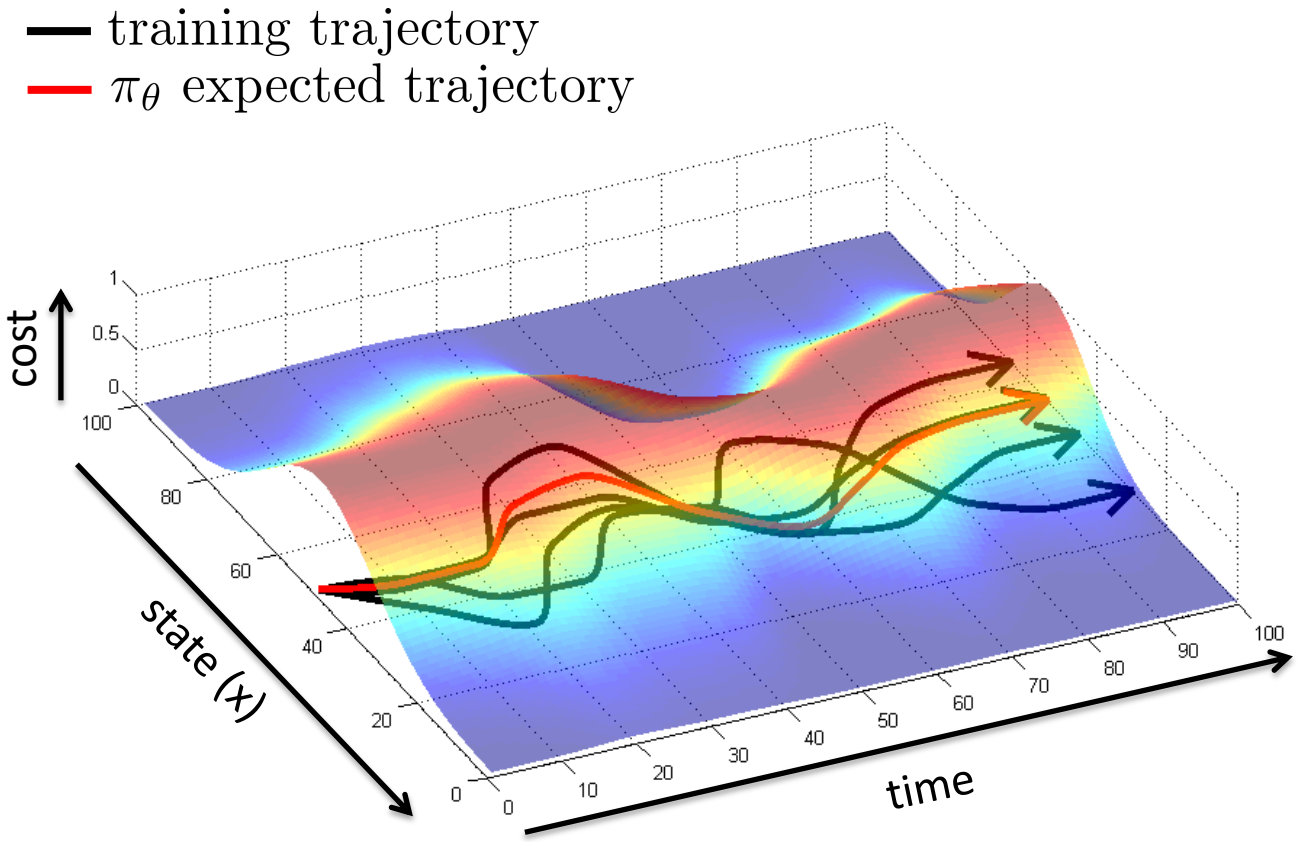
- Imitation Learning ≠ Supervised Learning



$$\arg\max_\theta \mathbb{E}_{(s^*,a^*)\sim\mathcal{D}}\left[\log \pi_\theta(a^*|s^*)\right] \qquad \mathbb{E}_{(s,a)\sim\rho(\pi)}\left[1(a=a^*)\right]$$

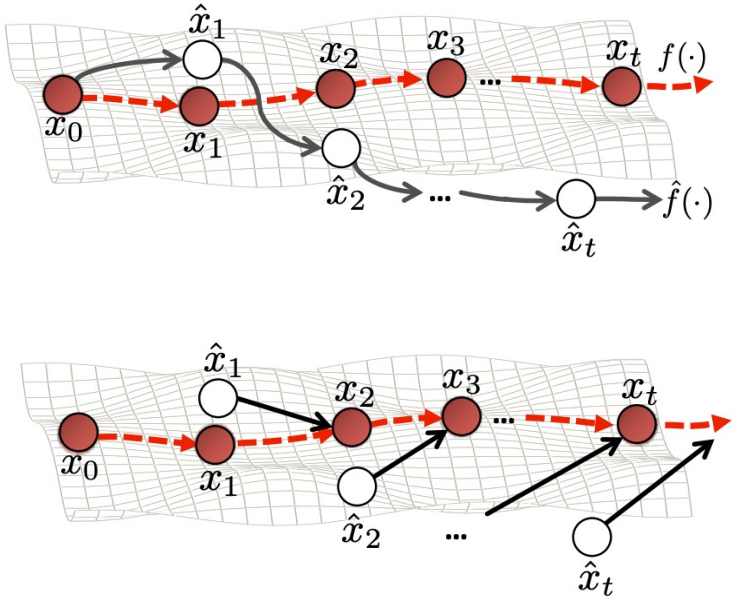
Not the same!

What is the general principle?



stability

Corrective labels that bring you back to the data



Concrete Instantation: DAgger

can we make $p_{\text{data}}(\mathbf{o}_t) = p_{\pi_\theta}(\mathbf{o}_t)$?

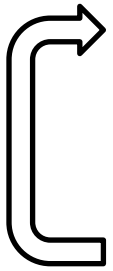
idea: instead of being clever about $p_{\pi_\theta}(\mathbf{o}_t)$, be clever about $p_{\text{data}}(\mathbf{o}_t)$!

DAgger: Dataset Aggregation

goal: collect training data from $p_{\pi_\theta}(\mathbf{o}_t)$ instead of $p_{\text{data}}(\mathbf{o}_t)$

how? just run $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$

but need labels $\mathbf{a}_t$!



1. train $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ from human data $\mathcal{D} = \{\mathbf{o}_1, \mathbf{a}_1, \ldots, \mathbf{o}_N, \mathbf{a}_N\}$
2. run $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ to get dataset $\mathcal{D}_\pi = \{\mathbf{o}_1, \ldots, \mathbf{o}_M\}$
3. Ask human to label $\mathcal{D}_\pi$ with actions $\mathbf{a}_t$
4. Aggregate: $\mathcal{D} \leftarrow \mathcal{D} \cup \mathcal{D}_\pi$

Ross et al. '11

How might we fix this?

"Generate" corrective labels automatically

1. train $\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$ from human data $\mathcal{D} = \{\mathbf{o}_1, \mathbf{a}_1, \ldots, \mathbf{o}_N, \mathbf{a}_N\}$
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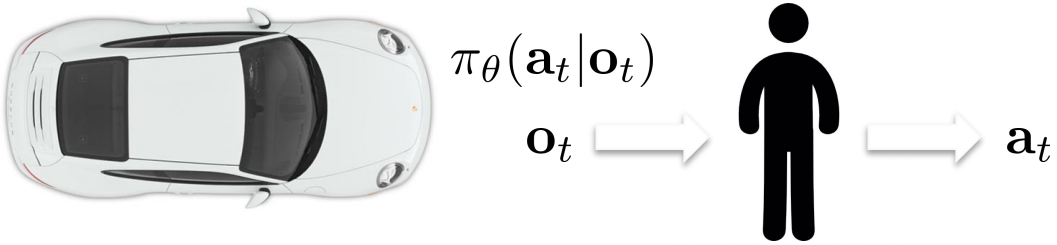
Do at data collection time



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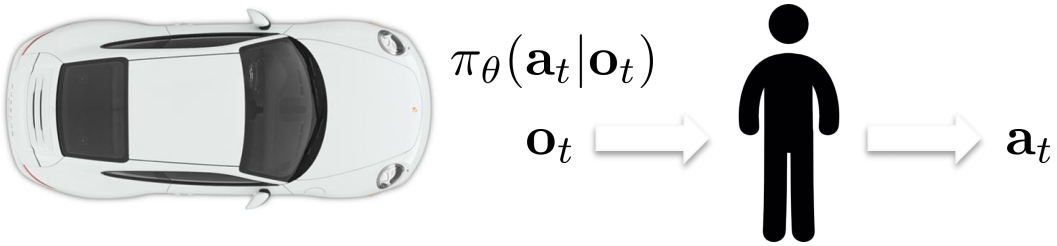
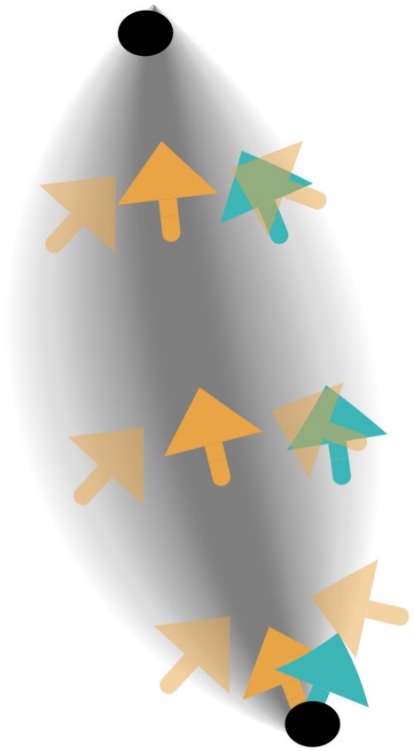


Ross et al. '11

Noising the Data Collection Process

Key idea: force the human to correct for noise **during** training



Supervisor

Robot

Noise Injection

$$\hat{\psi}_{k+1} = \underset{\psi}{\operatorname{argmin}}\, E_{p(\xi|\pi_{\theta^*},\psi_k)} - \sum_{t=0}^{T-1} \log\left[\pi_{\theta^*}\left(\pi_{\hat{\theta}}(\mathbf{x_t})|\mathbf{x_t},\psi\right)\right]$$

Maximize likelihood

Under noise during data collection

DART: Noise Injection for Robust Imitation Learning, Laskey et al CoRL ‘17

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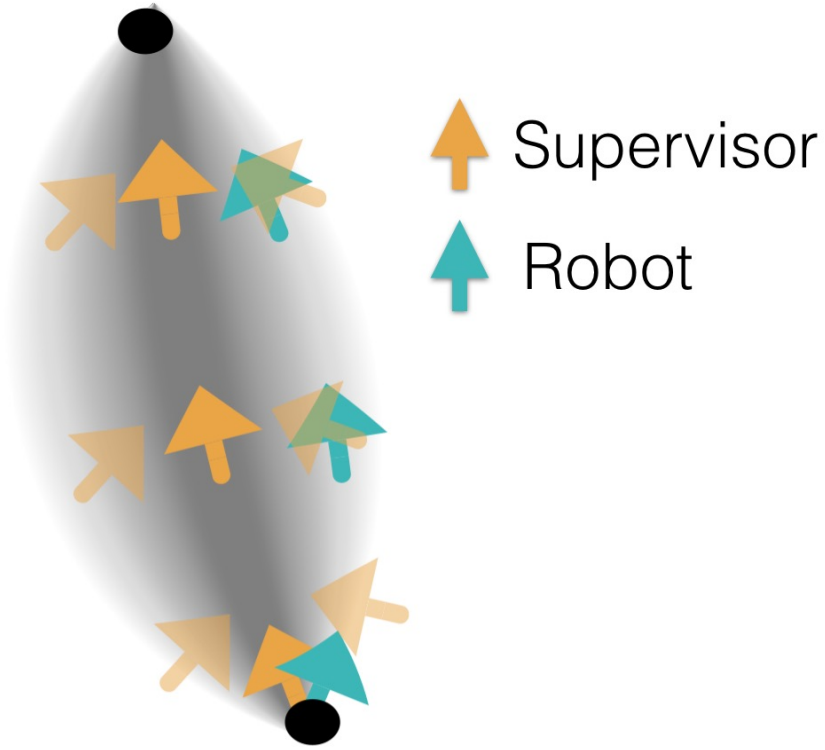
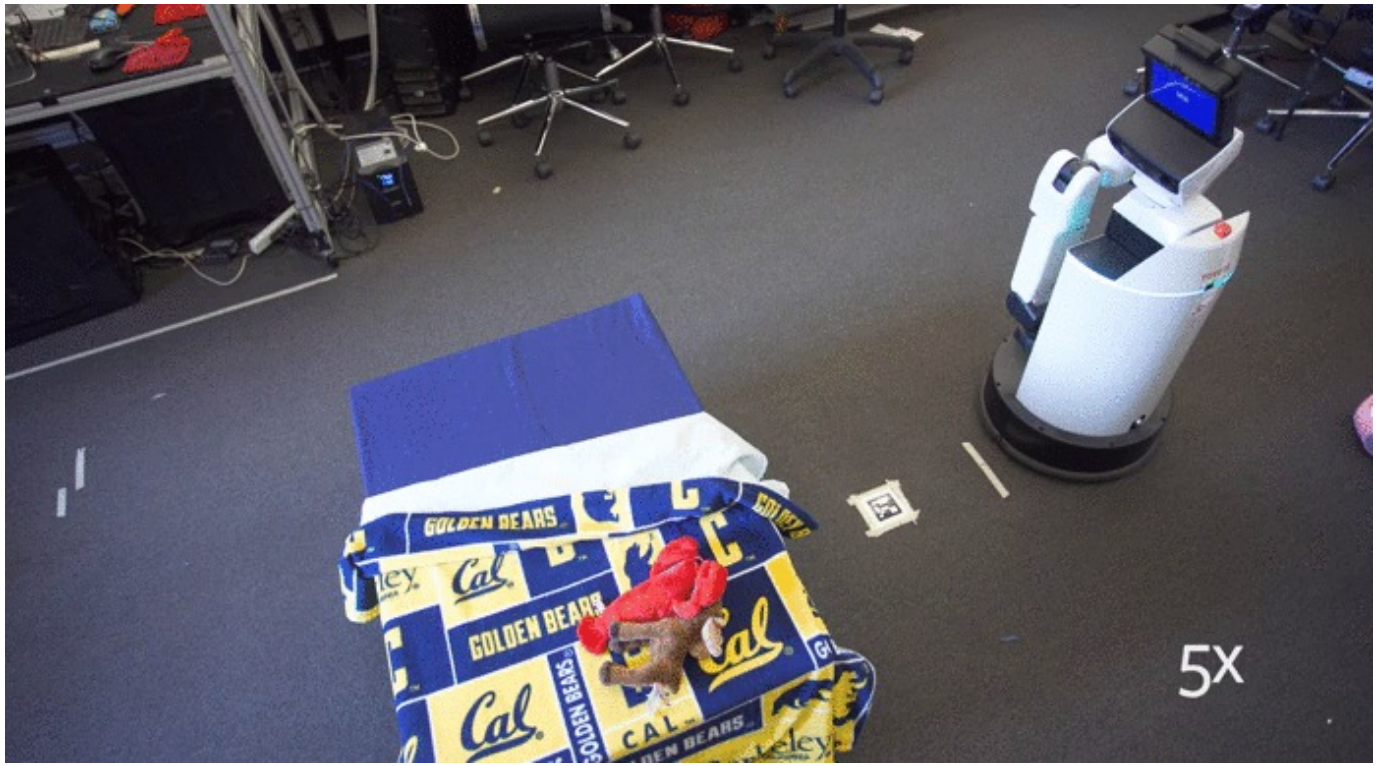


Noise Injection



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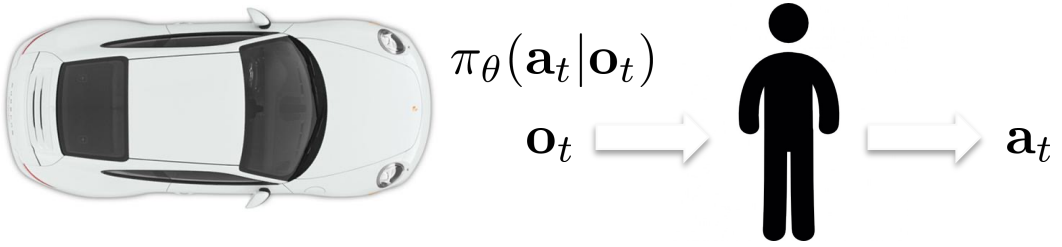
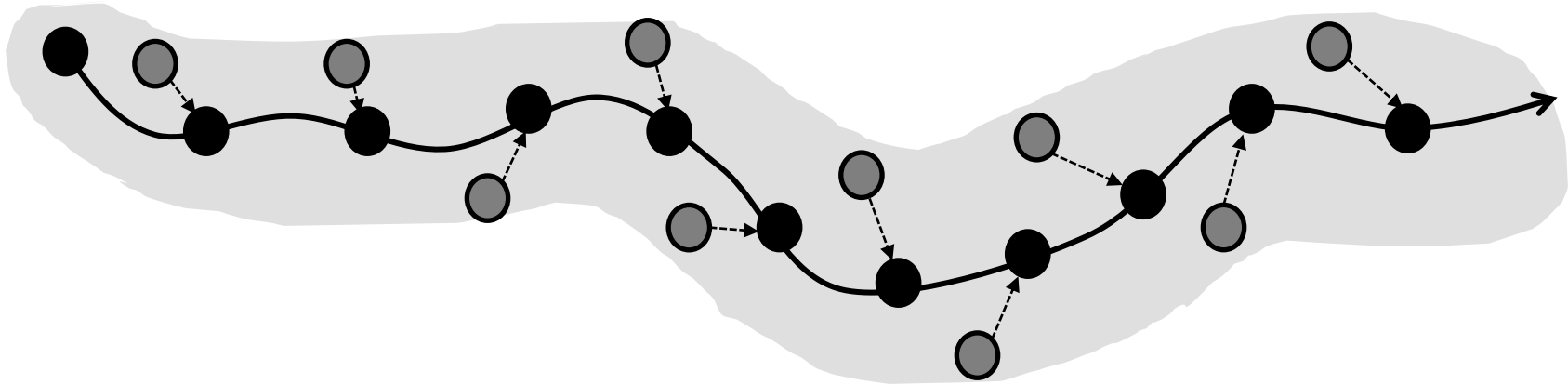


Ross et al. '11

Generating Corrective Labels for Imitation Learning with Learned Dynamics



$\hat{f}_\phi$

minimizing MSE on expert data
+ spectral norm

When can we trust learned dynamics $\hat{f}_\phi$?

↓

Trust models “around” training data

$$\|s^*_{t+1} - \hat{f}_\phi(s_t, a_t)\| \leq \epsilon$$

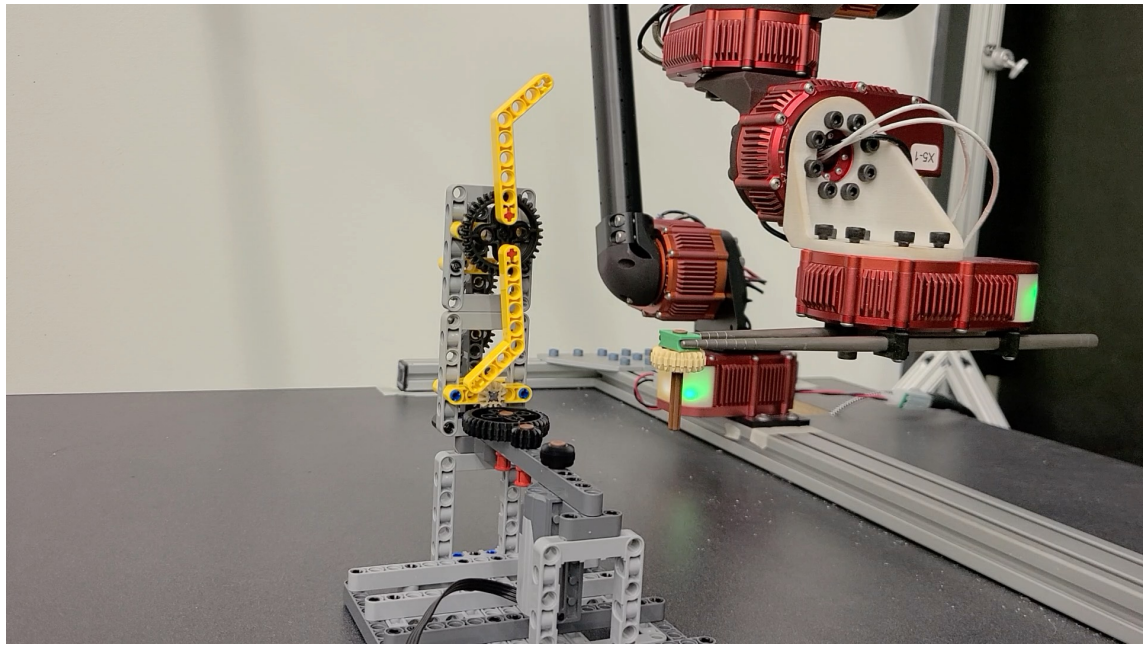
Find states (s_t), actions (a_t) that lead back to optimal states under ~~true~~ learned dynamics, **where learned dynamics can be trusted**

CCIL: Continuity-based data augmentation for corrective imitation learning, Ke et al ICLR ‘24

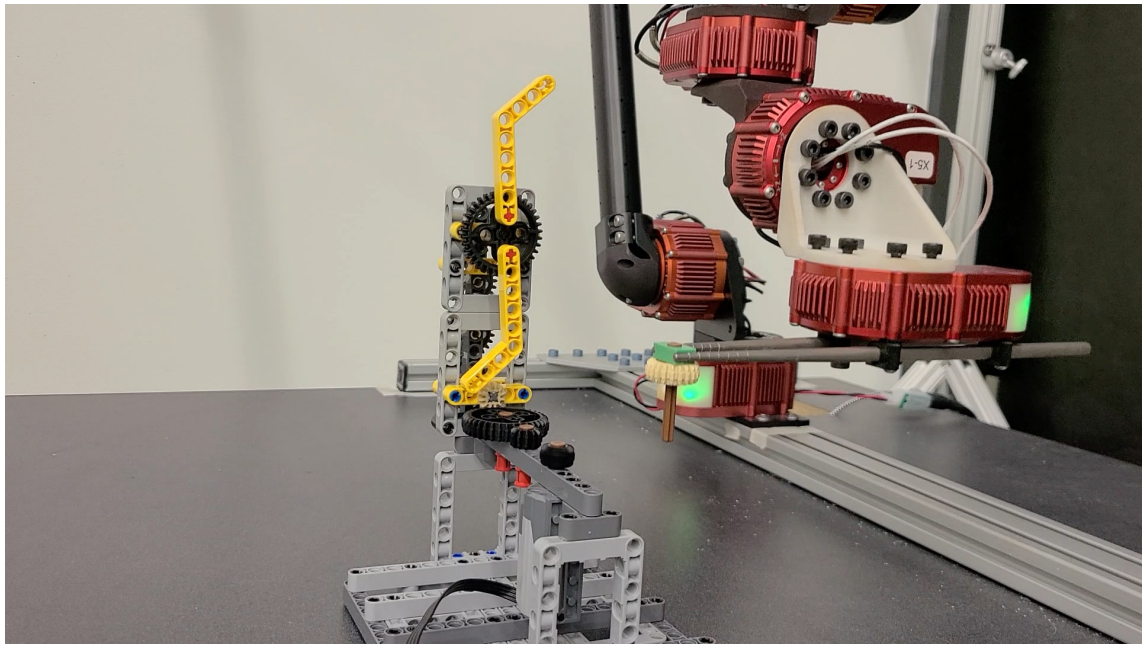
Overall Learning Pipeline with Corrective Labels

Learn models of dynamics and find states (s_t), actions (a_t) that lead back to optimal states under this learned dynamics

Standard behavior cloning



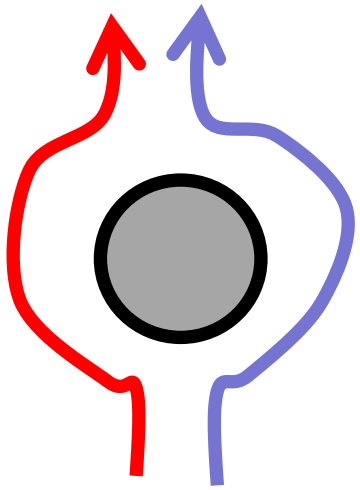
Corrective labels

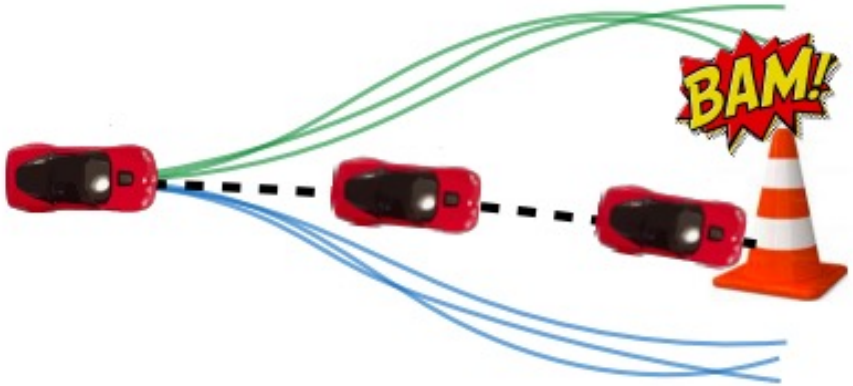


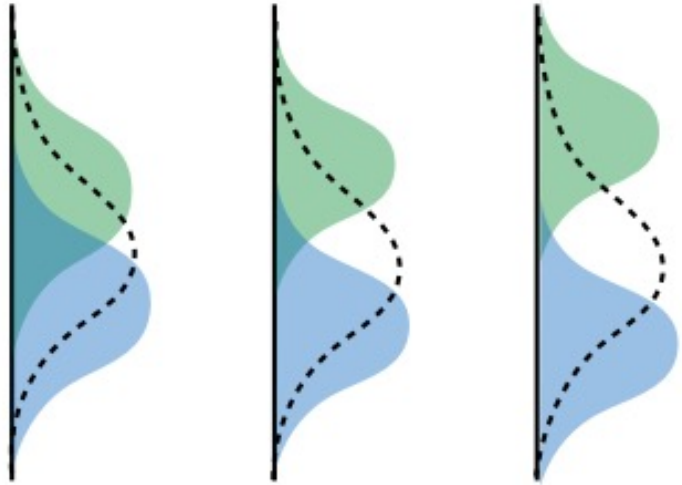
So does this solve all the issues in imitation?

Why might we fail to fit the expert?

Multimodal behavior.. amongst other reasons



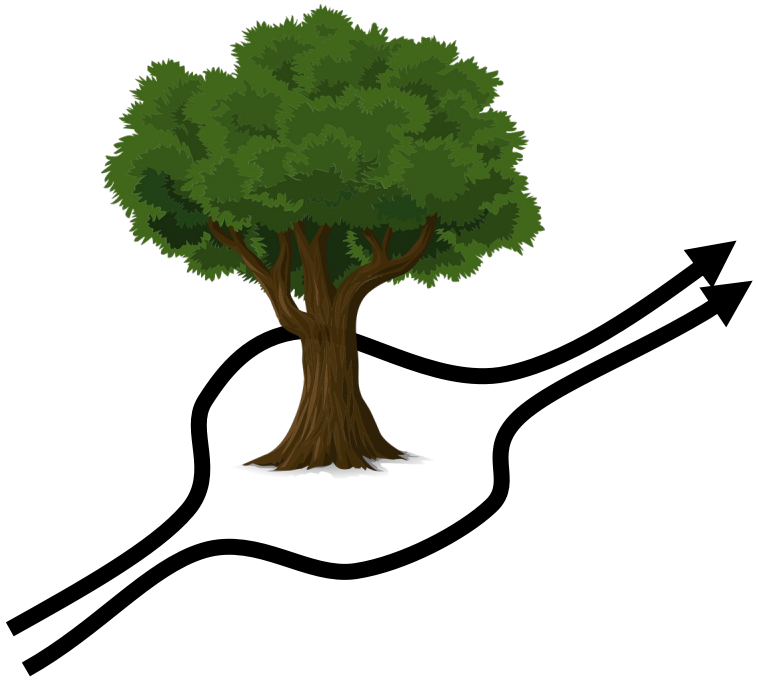


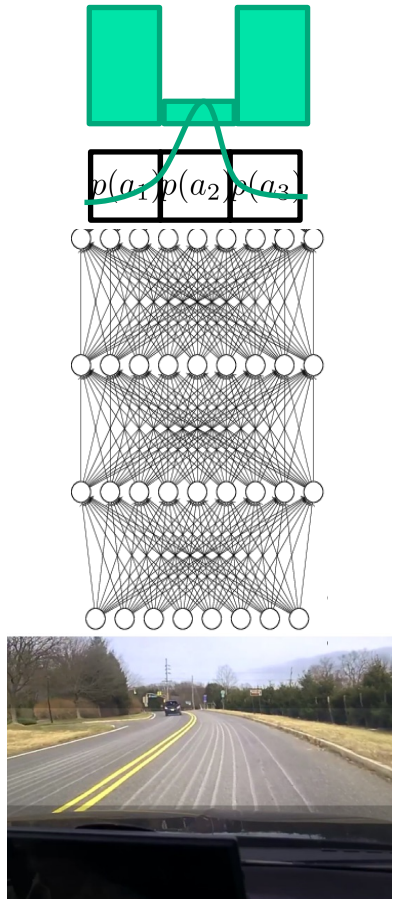


Not a matter of network size! It’s about distributional expressivity

Why might we fail to fit the expert?

Multimodal behavior → use more **expressive** probability distributions





1. Output mixture of Gaussians
2. Latent variable models
3. Autoregressive discretization
4. Diffusion models
5. …

Why might we fail to fit the expert?

1. Output mixture of Gaussians
2. Latent variable models
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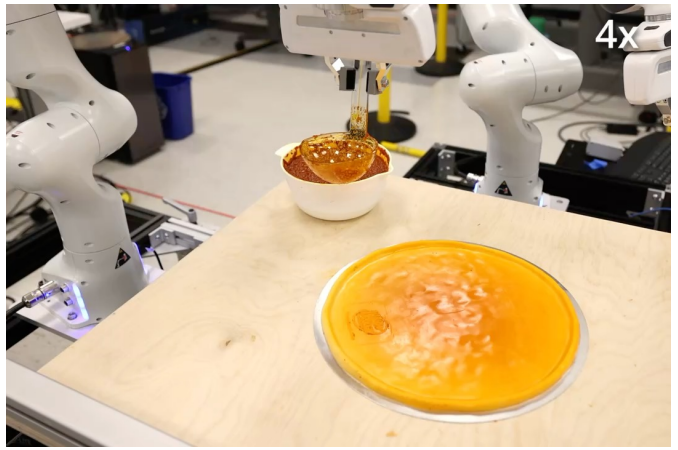
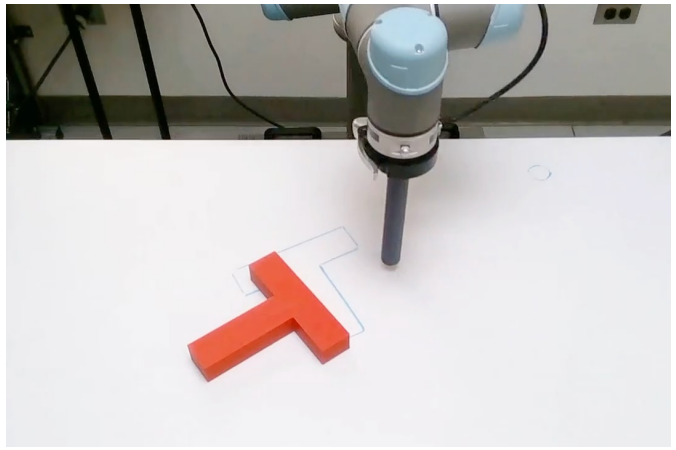
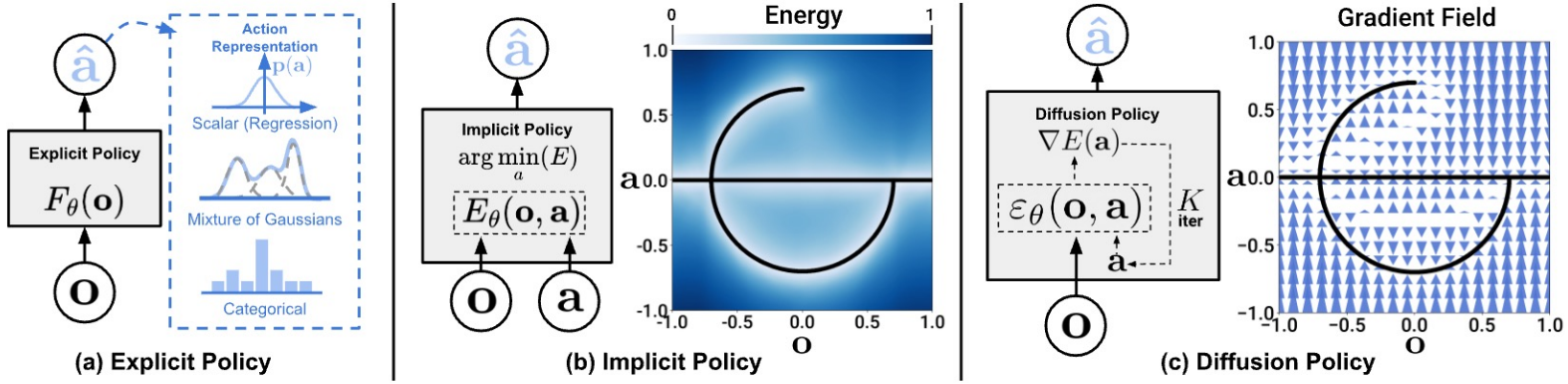






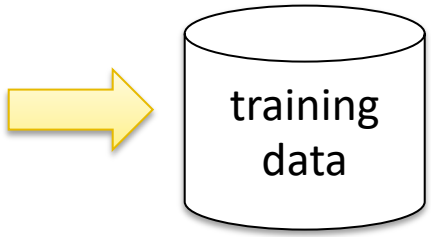
Some cool imitation videos

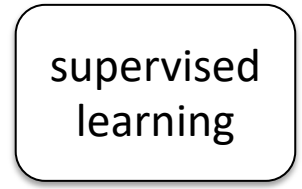
1x and tesla humanoid robots

ALOHA and CherryBot Fine Manipulation

Perspectives on Imitation – don't believe everything you see online

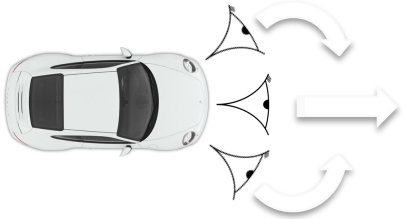
$\mathbf{o}_t$
$\mathbf{a}_t$



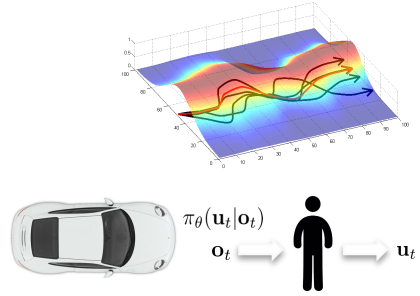


$\pi_\theta(\mathbf{a}_t|\mathbf{o}_t)$

- Pros:
  - Easy to use, no additional infra
  - Can sometimes be unreasonably effective



- Cons:
  - Challenges of compounding error, multimodality
  - Doesn't really generalize
  - Very expensive in terms of data collection!



Lecture Outline

A Formalism for Sequential Decision Making

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Imitation Learning: Behavior Cloning

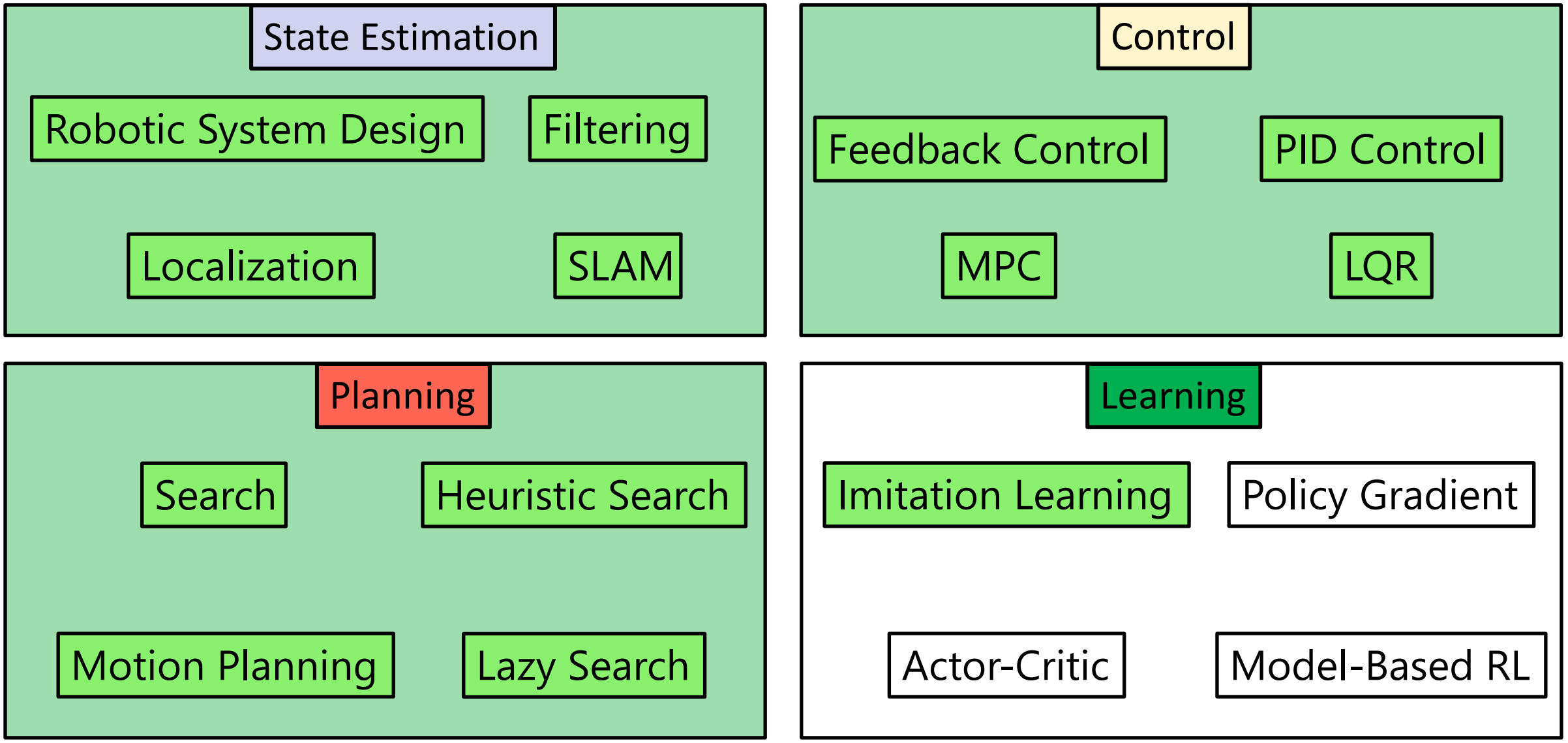
↓

Imitation Learning: Improvements – Compounding Error

↓

Imitation Learning: Improvements – Multimodality

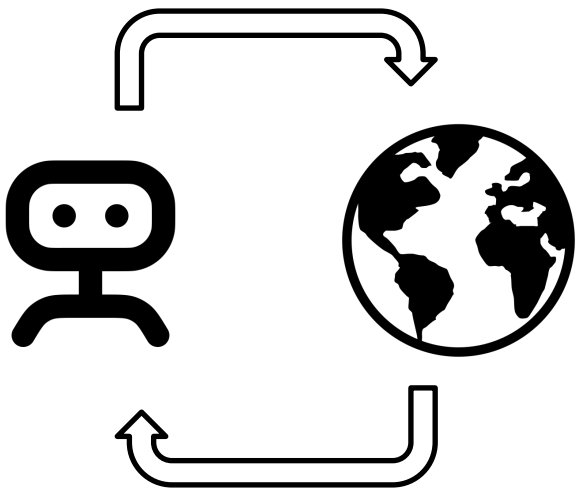
Class Outline



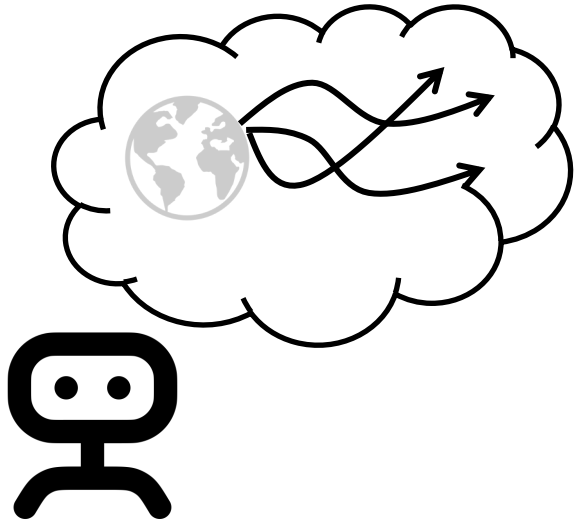
Ok so how can we learn policies?

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

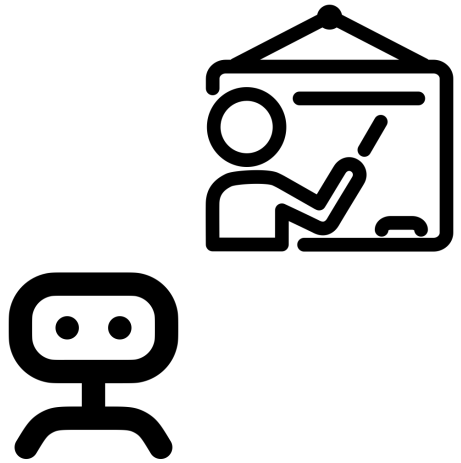
Model-free RL



Model-based RL



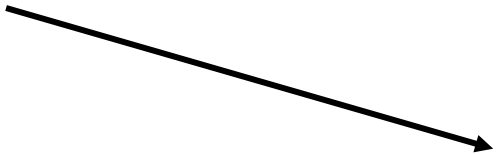
Imitation Learning



What if we just performed gradient ascent?

$$\max_{\theta} \mathbb{E}_{\tau \sim \pi_\theta} \left[\sum_{t=0}^{T} r(s_t, a_t) \right]$$

$$= \int p_\theta(\tau) R(\tau) d\tau$$



Standard gradient descent (supervised learning)

$$\nabla_\theta \mathbb{E}_{x \sim g(x)} \left[f_\theta(x) \right]$$

REINFORCE gradient descent (RL)

$$\nabla_\theta \mathbb{E}_{x \sim p_\theta(x)} \left[f(x) \right]$$

Gradient wrt expectation variable, not of integrand!

Taking the gradient of sum of rewards

$$J(\theta) = \int p_\theta(\tau) R(\tau) d(\tau)$$

$$\nabla_\theta J(\theta) = \nabla_\theta \int p_\theta(\tau) R(\tau) d(\tau)$$

$$= \int \nabla_\theta p_\theta(\tau) R(\tau) d(\tau) \quad = \int \frac{p_\theta(\tau)}{p_\theta(\tau)} \nabla_\theta p_\theta(\tau) R(\tau) d(\tau)$$

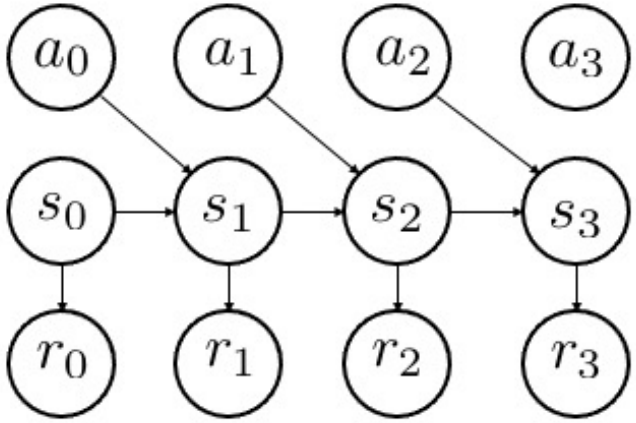
$$= \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) R(\tau) d(\tau) \quad = \mathbb{E}_{p_\theta(\tau)} \left[\nabla_\theta \log p_\theta(\tau) R(\tau) \right]$$

REINFORCE trick

Taking the gradient of return

Initial State Dynamics Policy

$$p_\theta(\tau) = p(s_0)\Pi_{t=0}^{T-1} p(s_{t+1}|s_t, a_t)\pi(a_t|s_t)$$



$$\log p_\theta(\tau) = \log p(s_0) + \sum_{t=0}^{T-1} \log p(s_{t+1}|s_t, a_t) + \log \pi(a_t|s_t)$$

$$\nabla_\theta \log p_\theta(\tau) = \nabla_\theta \log p(s_0) + \sum_{t=0}^{T-1} \nabla_\theta \log p(s_{t+1}|s_t, a_t) + \nabla_\theta \log \pi(a_t|s_t)$$

$$\nabla_\theta \log p_\theta(\tau) = \sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t|s_t)$$

Model Free!!

Taking the gradient of return

$$\nabla_\theta J(\theta) = \mathbb{E}_{\tau \sim p_\theta(\tau)} \left[\nabla_\theta \log p_\theta(\tau) \sum_{t=0}^{T} r(s_t, a_t) \right]$$

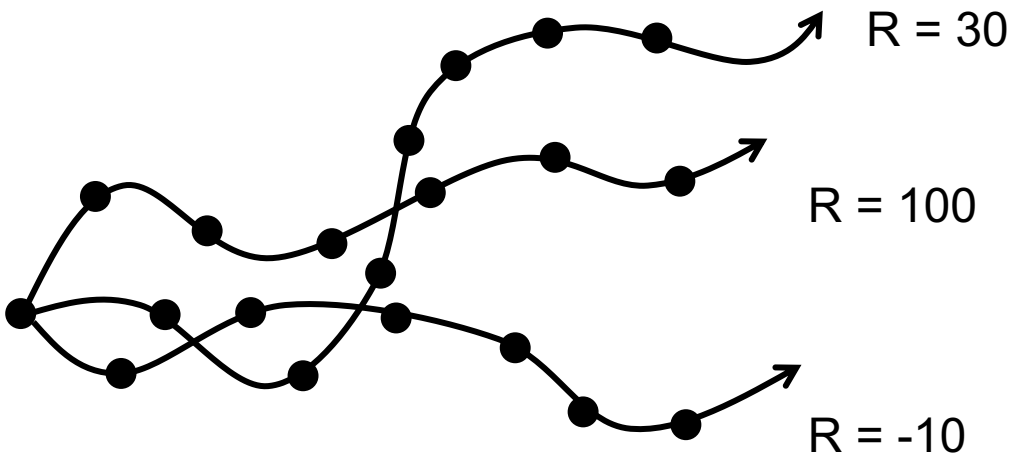
$$\nabla_\theta J(\theta) = \mathbb{E}_{\substack{s_0 \sim p(s_0) \\ s_{t+1} \sim p(s_{t+1}|s_t, a_t) \\ a_t \sim \pi(a_t|s_t)}} \left[\sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t|s_t) \sum_{t'=0}^{T} r(s_t, a_t) \right]$$

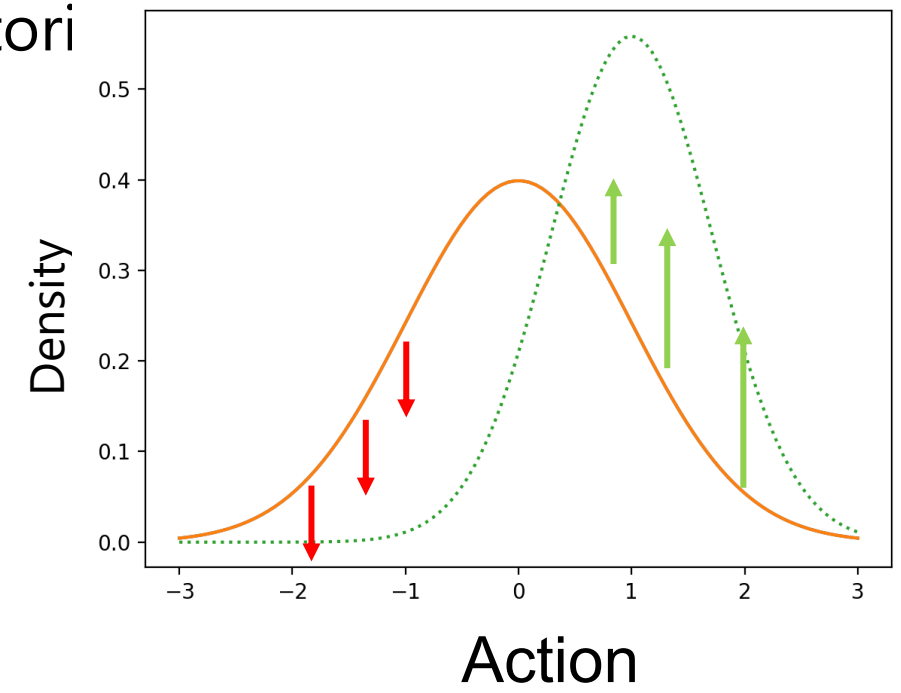
$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i|s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$ (approximating using samples)

What does this mean?

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau \approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

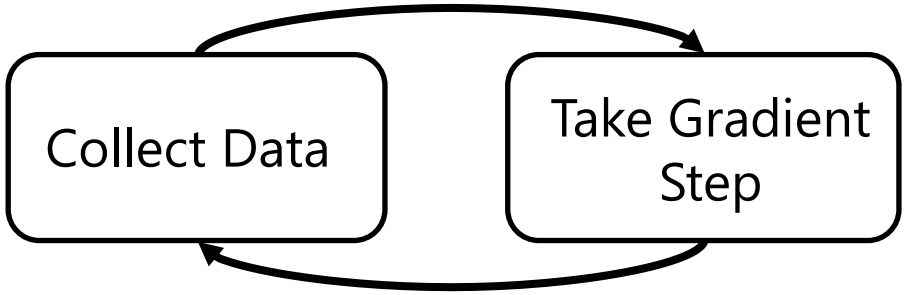
Increase the likelihood of actions in high return trajectori





Resulting Algorithm (REINFORCE)

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$



On-policy

REINFORCE algorithm:

1. sample $\{\tau^i\}$ from $\pi_\theta(\mathbf{a}_t|\mathbf{s}_t)$ (run it on the robot)
2. $\nabla_\theta J(\theta) \approx \sum_i \left(\sum_t \nabla_\theta \log \pi_\theta(\mathbf{a}_t^i|\mathbf{s}_t^i)\right) \left(\sum_t r(\mathbf{s}_t^i, \mathbf{a}_t^i)\right)$
3. $\theta \leftarrow \theta + \alpha \nabla_\theta J(\theta)$

How is this related to supervised learning?

Reinforcement Learning

$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

Supervised Learning

$$\max_\theta \mathbb{E}_{(x,y)\sim\mathcal{D}} \left[\log p_\theta(y|x)\right]$$

$$\approx \frac{1}{N} \sum_i \nabla_\theta \log p_\theta(y^i | x^i)$$

PG = select good data + increase likelihood of selected data

What makes policy gradient challenging?

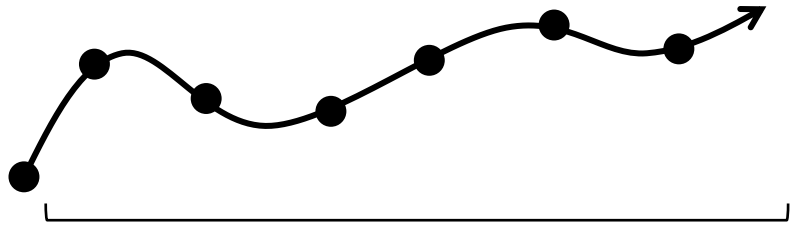
$$\nabla_\theta J(\theta) = \int p_\theta(\tau) \nabla_\theta \log p_\theta(\tau) d\tau$$

$$\approx \frac{1}{N} \sum_{i=0}^{N} \sum_{t=0}^{T} \nabla_\theta \log \pi_\theta(a_t^i | s_t^i) \sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)$$

High variance estimator!!

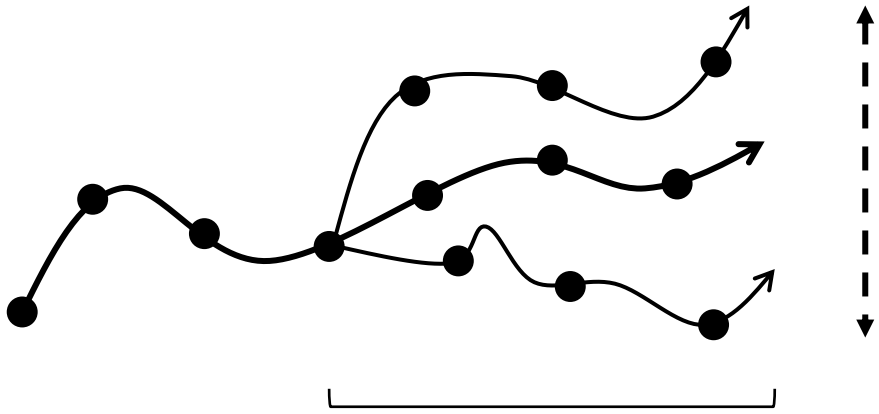
Hard to tell what matters without many samples

What we do



Single sample estimate

What we actually want

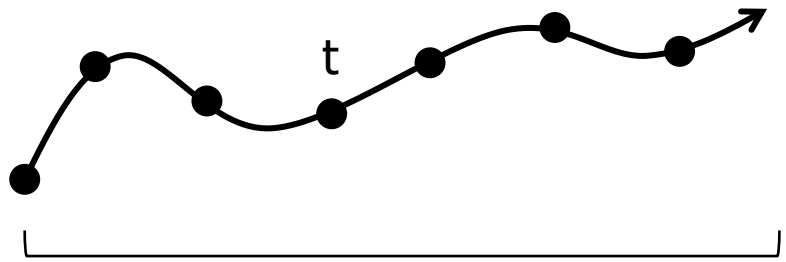


Averaged return estimate

Variance Reduction with Causality

Idea: Trajectory returns depend on past and future, but we only care about the future, since actions cannot affect the past. Instead, consider **"return-to-go"**

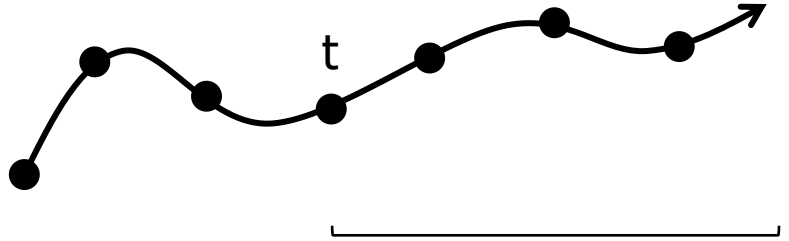
$$\approx \frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\underbrace{\sum_{t'=0}^{T} r(s_{t'}^i, a_{t'}^i)}_{\text{Includes } t' < t}$$



Full trajectory return

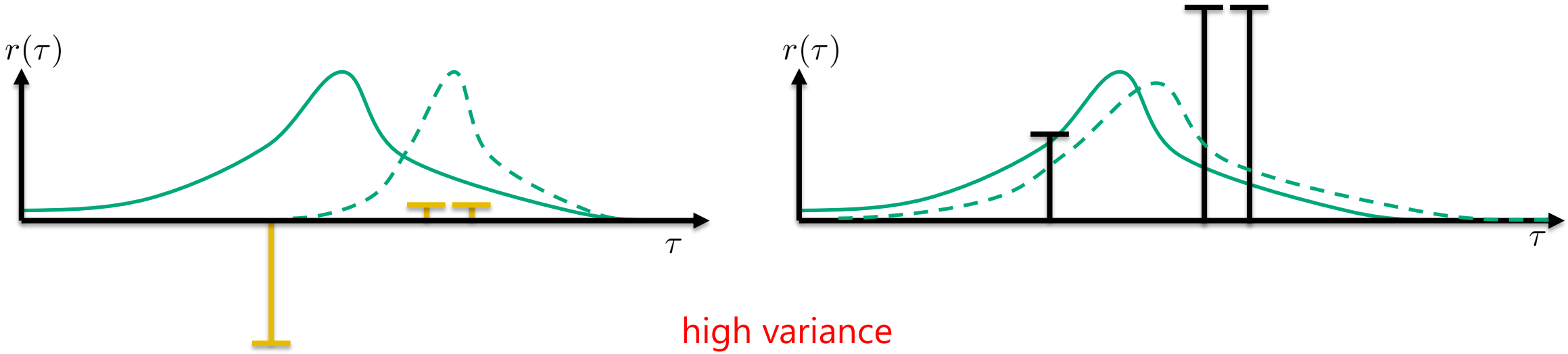
Ignore past terms

$$\frac{1}{N}\sum_{i=0}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\sum_{t'=t}^{T} r(s_t^i, a_t^i)$$



Return to go

Can we reduce variance further?



high variance

Arbitrarily uncentered, scaled returns can lead to huge variance:

→ Imagine all rewards were positive, every action would be pushed up, some more than others

→ What if instead, we pushed down some actions and pushed up some others (even if rewards are positive)

Credit: Sergey Levine

Variance Reduction with a Baseline

Idea: We can reduce variance by subtracting a current state dependent function from the policy gradient return

$$\frac{1}{N}\sum_{i=1}^{N}\sum_{t=0}^{T}\nabla_\theta \log \pi_\theta(a_t^i|s_t^i)\left[\sum_{t'=t}^{T} r(s_{t'}^i a_{t'}^i) - b(s_t)\right]$$

Baseline: Centers the returns, reduces variance

But does this increase bias??

Variance Reduction with a Baseline

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) \left[\sum_{t'=t}^{T} r(s_{t'}, a_{t'}) - b(s_t) \right] ds_t \, da_t$$

$$\int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) \left[\sum_{t'=t}^{T} r(s_{t'}, a_{t'}) \right] ds_t \, da_t - \int_{\mathcal{S}} \int_{\mathcal{A}} p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) b(s_t) \, ds_t \, da_t$$

Let us show this is 0!

Variance Reduction with a Baseline

$$\int \int p(s_t, a_t) \nabla_\theta \log \pi_\theta(a_t|s_t) \left[b(s_t)\right] ds_t da_t = \int \int p(s_t) \pi_\theta(a_t|s_t) \nabla_\theta \log \pi_\theta(a_t|s_t) \left[b(s_t)\right] ds_t da_t$$

$$= \int p(s_t) b(s_t) \int \pi_\theta(a_t|s_t) \nabla_\theta \log \pi_\theta(a_t|s_t) da_t ds_t$$

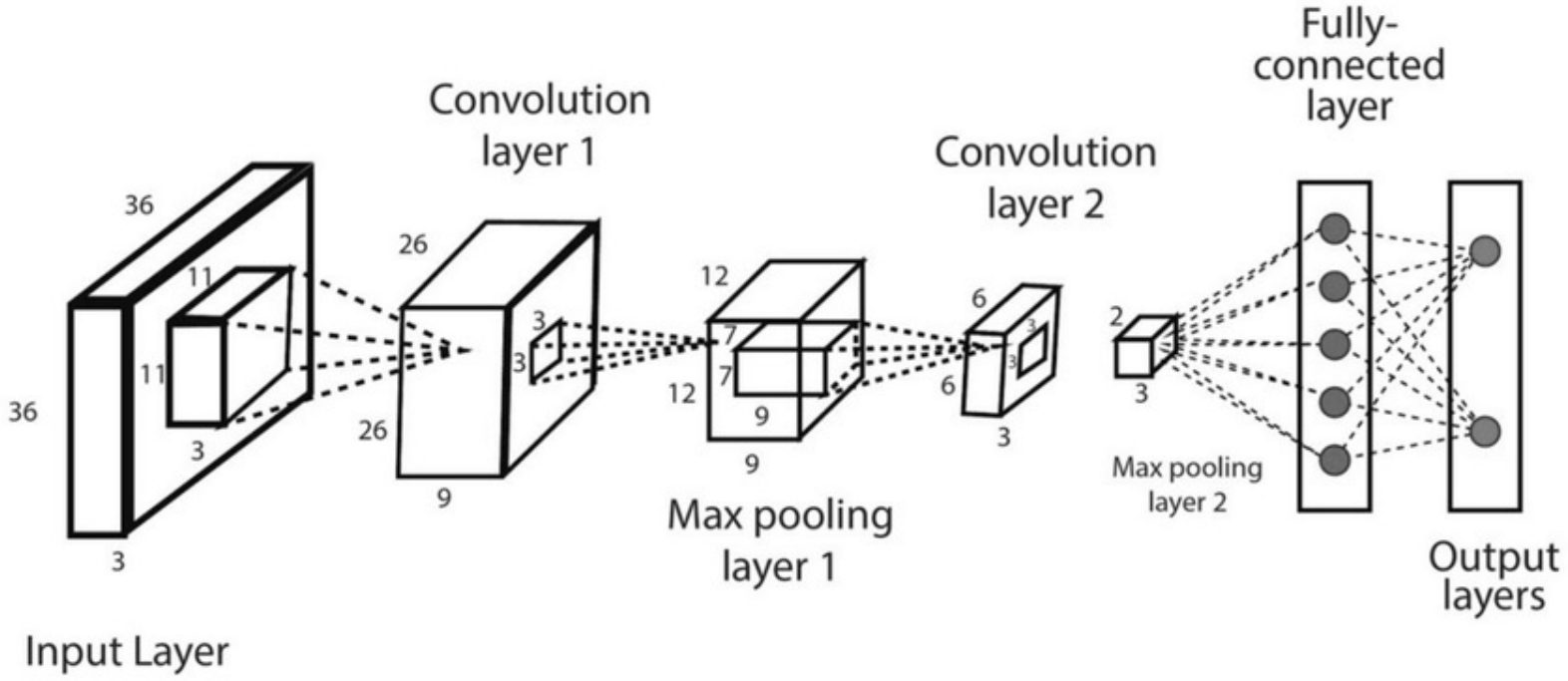
$$= \int p(s_t) b(s_t) \int \nabla_\theta \pi_\theta(a_t|s_t) da_t ds_t$$

$$= \int p(s_t) b(s_t) \nabla_\theta \int \pi_\theta(a_t|s_t) da_t ds_t = \int p(s_t) b(s_t) \nabla_\theta (1) ds_t = 0$$

Unbiased!

Learning Baselines

Baselines are typically learned as deep neural nets from $R^s \rightarrow R^1$



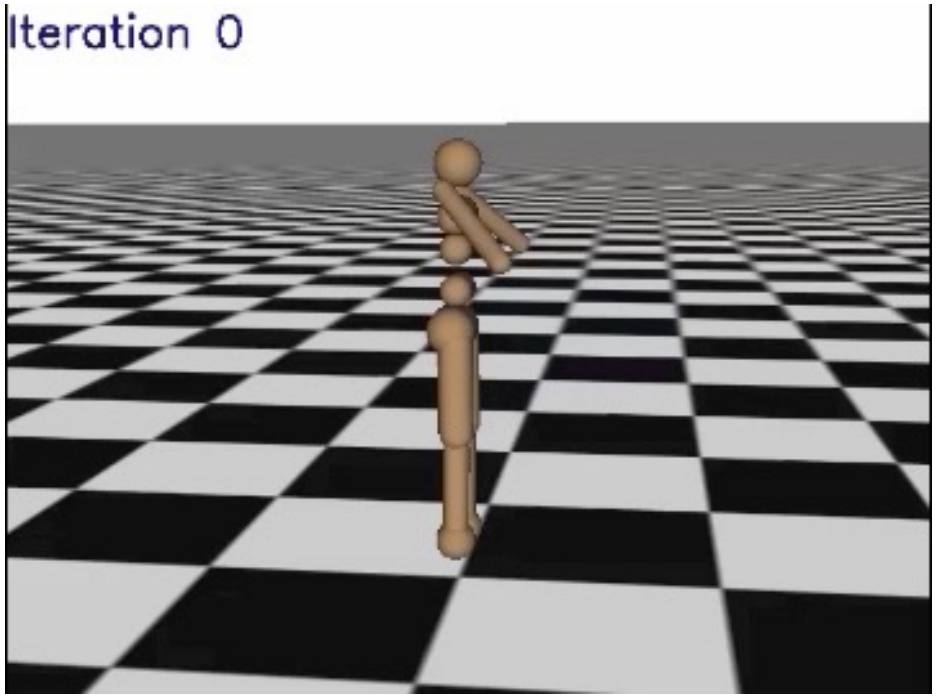
$$\frac{1}{N}\sum_{j=1}^{N}\|\hat{V}(s_t^j, a_t^j) - \sum_{t=1}^{H} r(s_t^j, a_t^j)\|$$

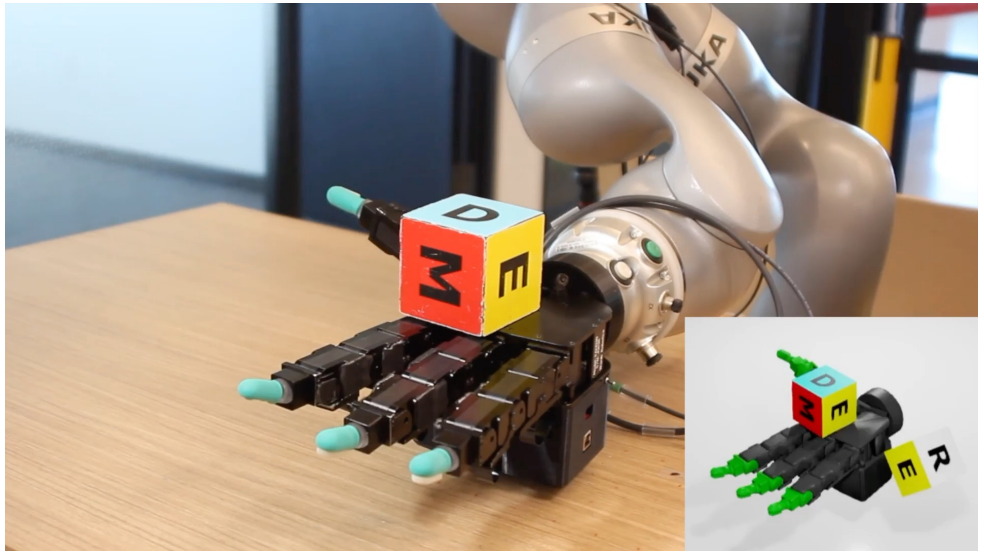
Minimize with Monte-carlo regression at every iteration, club with policy loss

$$A(s_t, a_t) = \sum_{t'=t}^{T} r(s_t', a_t') - V(s_t)$$

Allows us to define advantages

Policy Gradient in Action





Class Outline