Probabilistic Models II

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*Slides based on or adapted from Sanjiban Choudhury

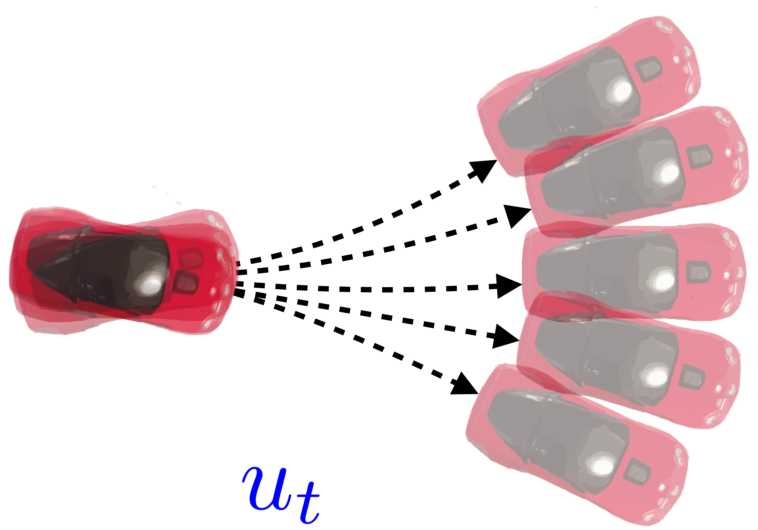
Logistics

- Lab0 due tonight at midnight
- Lab1 to be released today; due in 2 weeks
- No Class/OH Monday (MLK day)
- Probability recitation next Thursday (9am, CSE1 022)
- Exciting guest lectures planned
  - Prof. Dieter Fox
  - Prof. Sidd Srinivasa
  - Dr. Tapo Bhattacharjee
  - Demo by Starship Robotics

Probabilistic Models in Localization

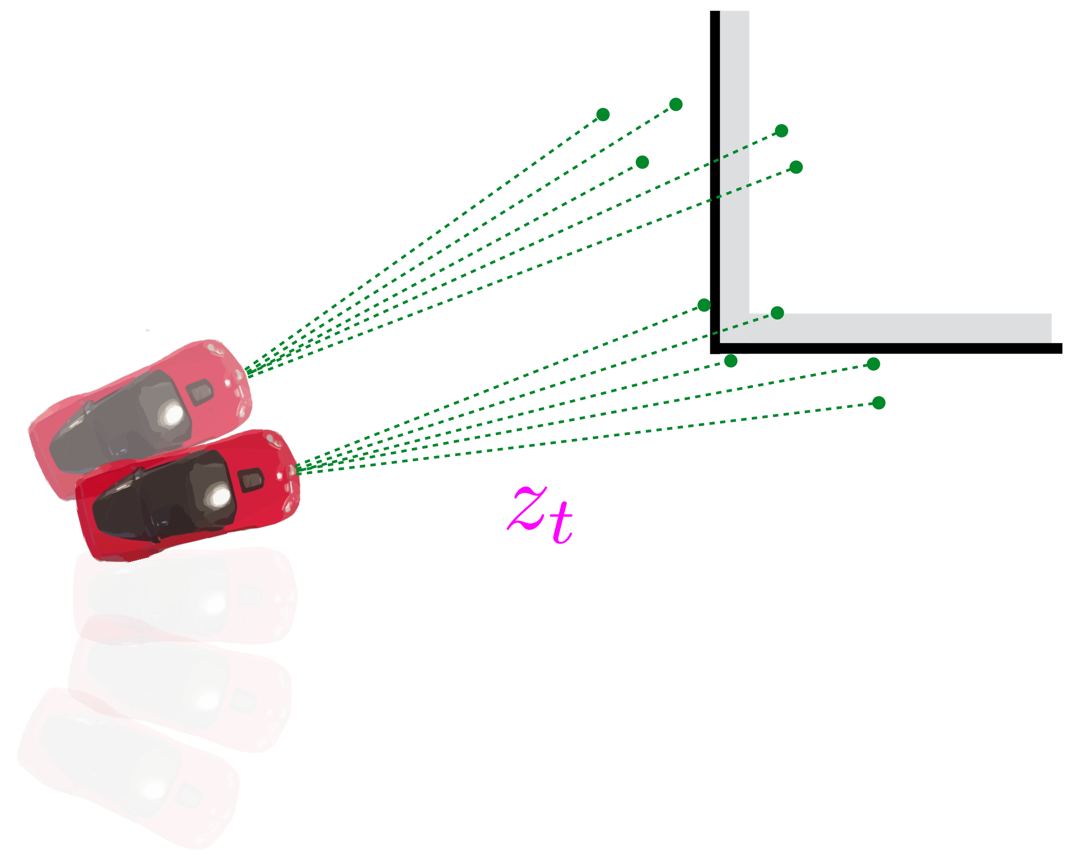
Motion model

$$P(x_t | u_t, x_{t-1})$$



Measurement model

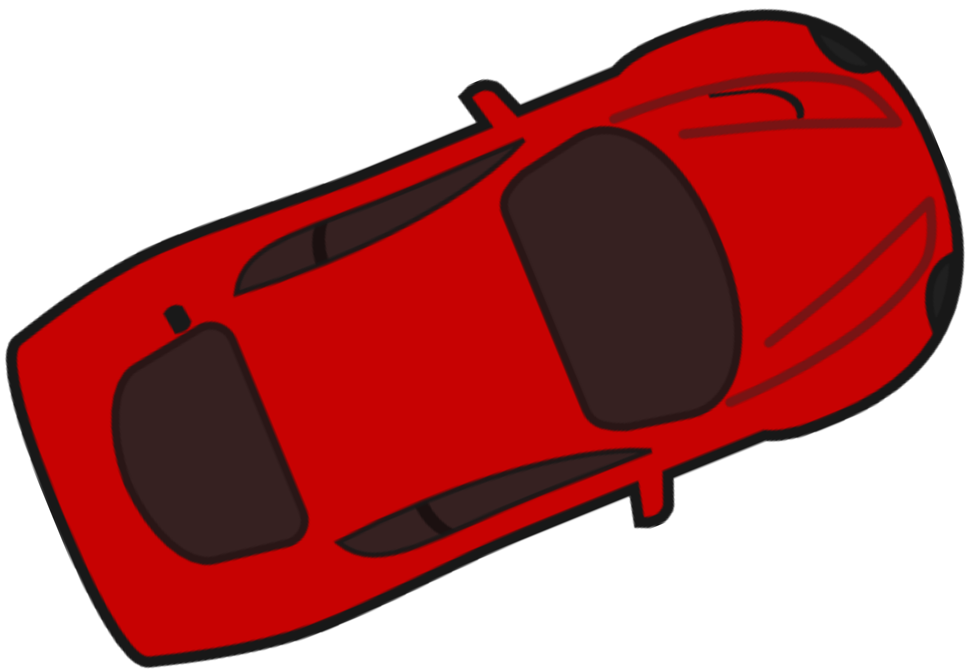
$$P(z_t | x_t)$$



Key Idea:

Simple model + Stochasticity

Simple Model



$$\dot{x} = V\cos(\theta)$$

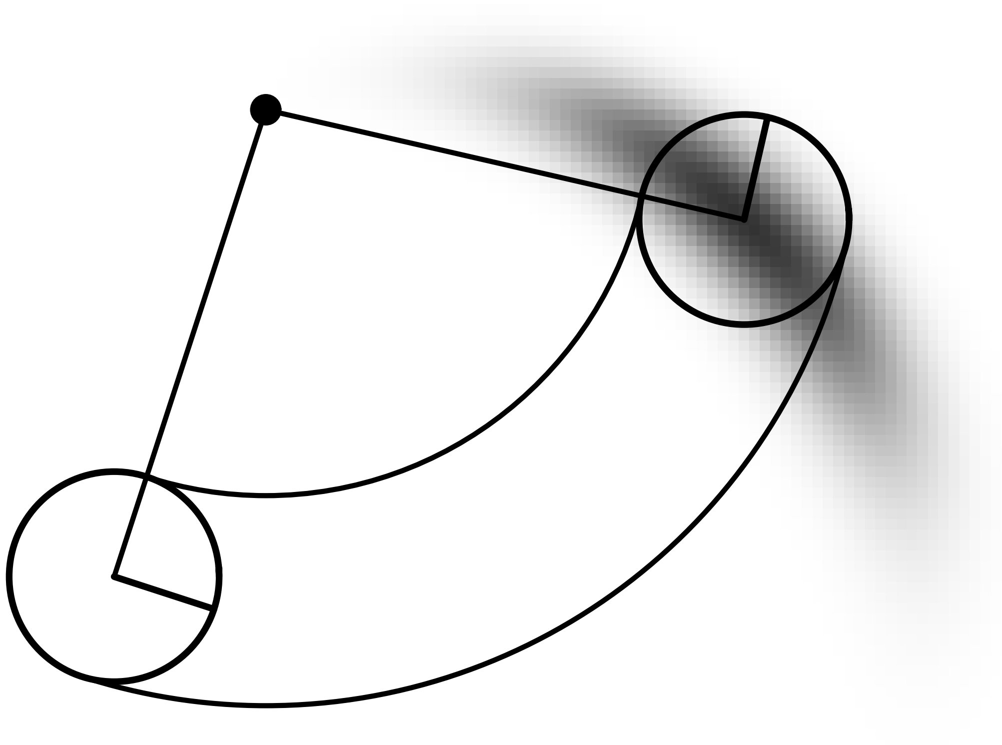
$$\dot{y} = V\sin(\theta)$$

$$\dot{\theta} = \omega = \frac{V\tan\delta}{L}$$

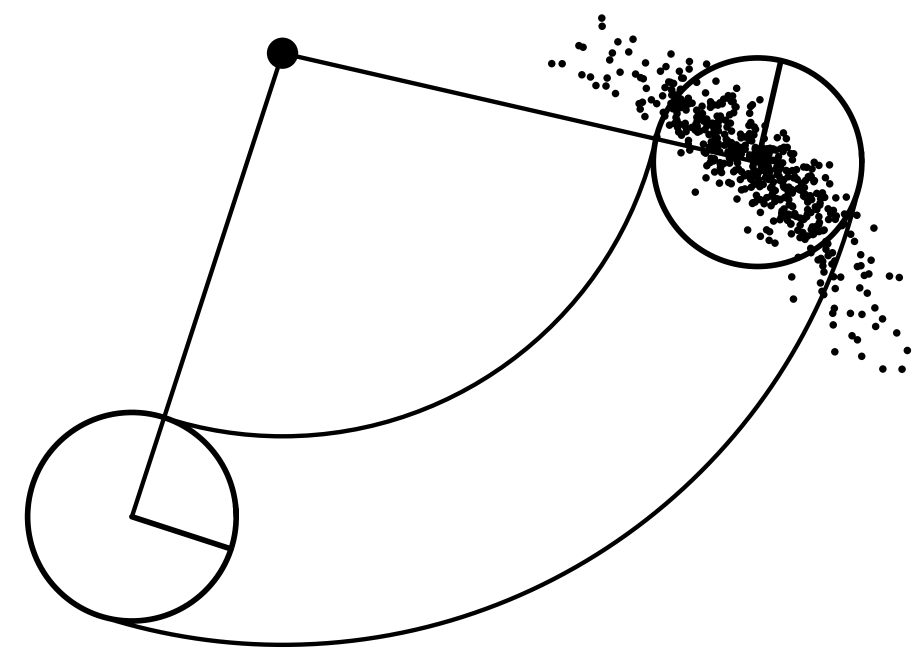
Why is it simple?
-Simplified physics
-Assumes perfect actuation
-Assumes perfect knowledge of design parameters

Accounting for Stochasticity

$$\dot{x} = f(u, x) + \text{Noise}$$



Probability density function

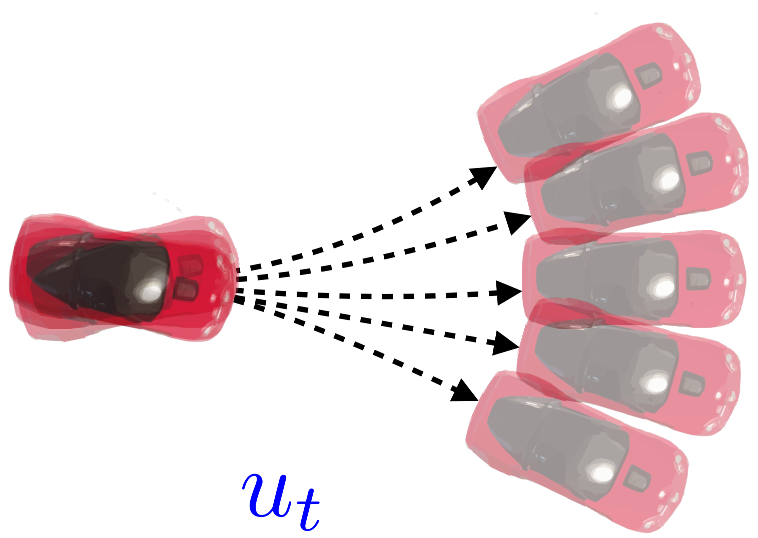


Samples from the pdf

Probabilistic models in localization

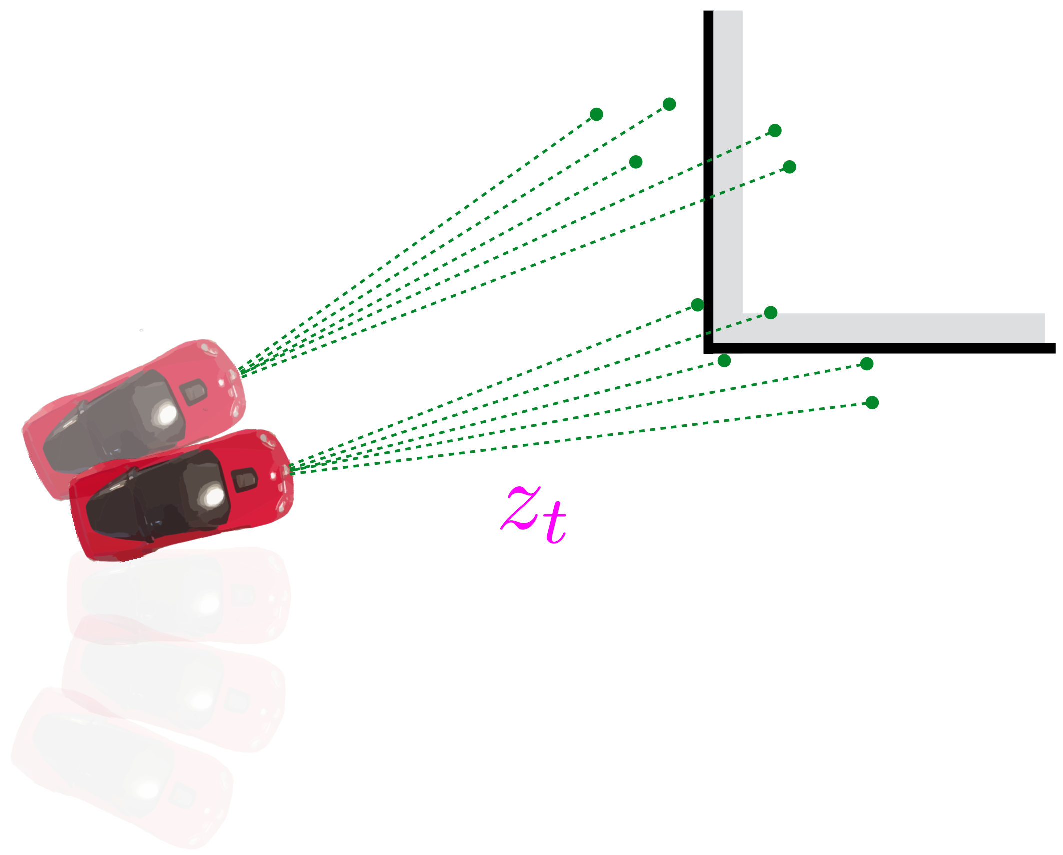
Motion model

$$P(x_t | u_t, x_{t-1})$$



Measurement model

$$P(z_t | x_t)$$



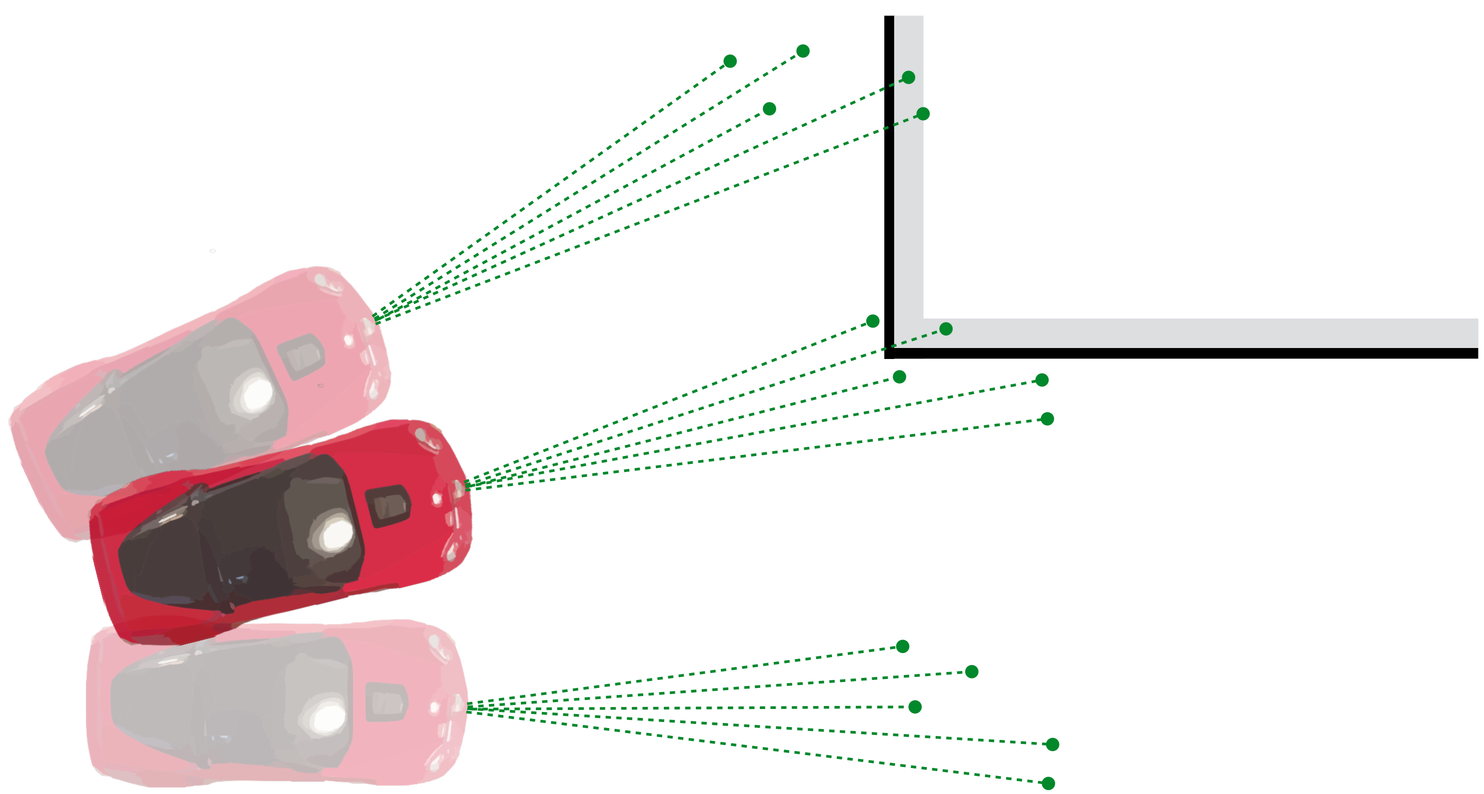
Measurement Model

$$P(z_t | x_t, m)$$

sensor reading

state

map



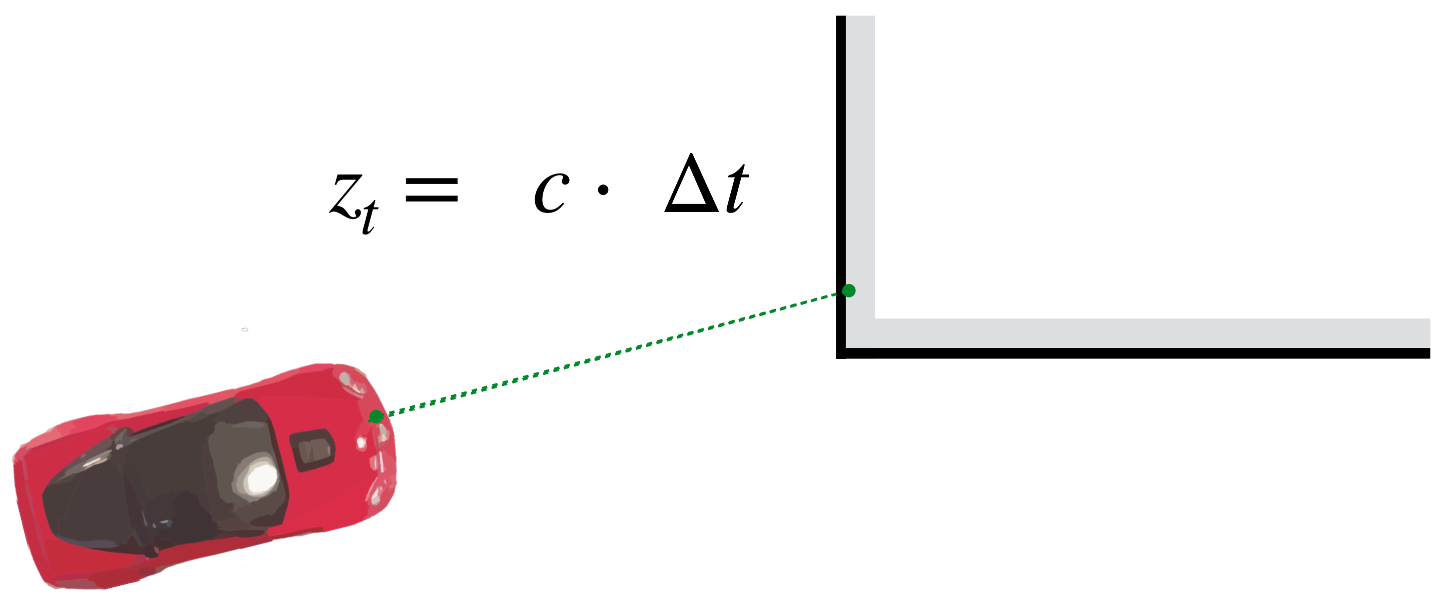
LiDAR

- **Li**ght + Ra**DAR**
- **Li**ght **D**etection **A**nd **R**anging
- A distance sensor
- Everywhere, self-driving cars ... your racecar
- Edward Hutchinson Synge (1930)

How does a LiDAR work?

- Source emits photons
- Photons bounce on objects
- Return to receiver
- Measure time between emission-reception
- Multiply by speed of light
- Get distance measurement

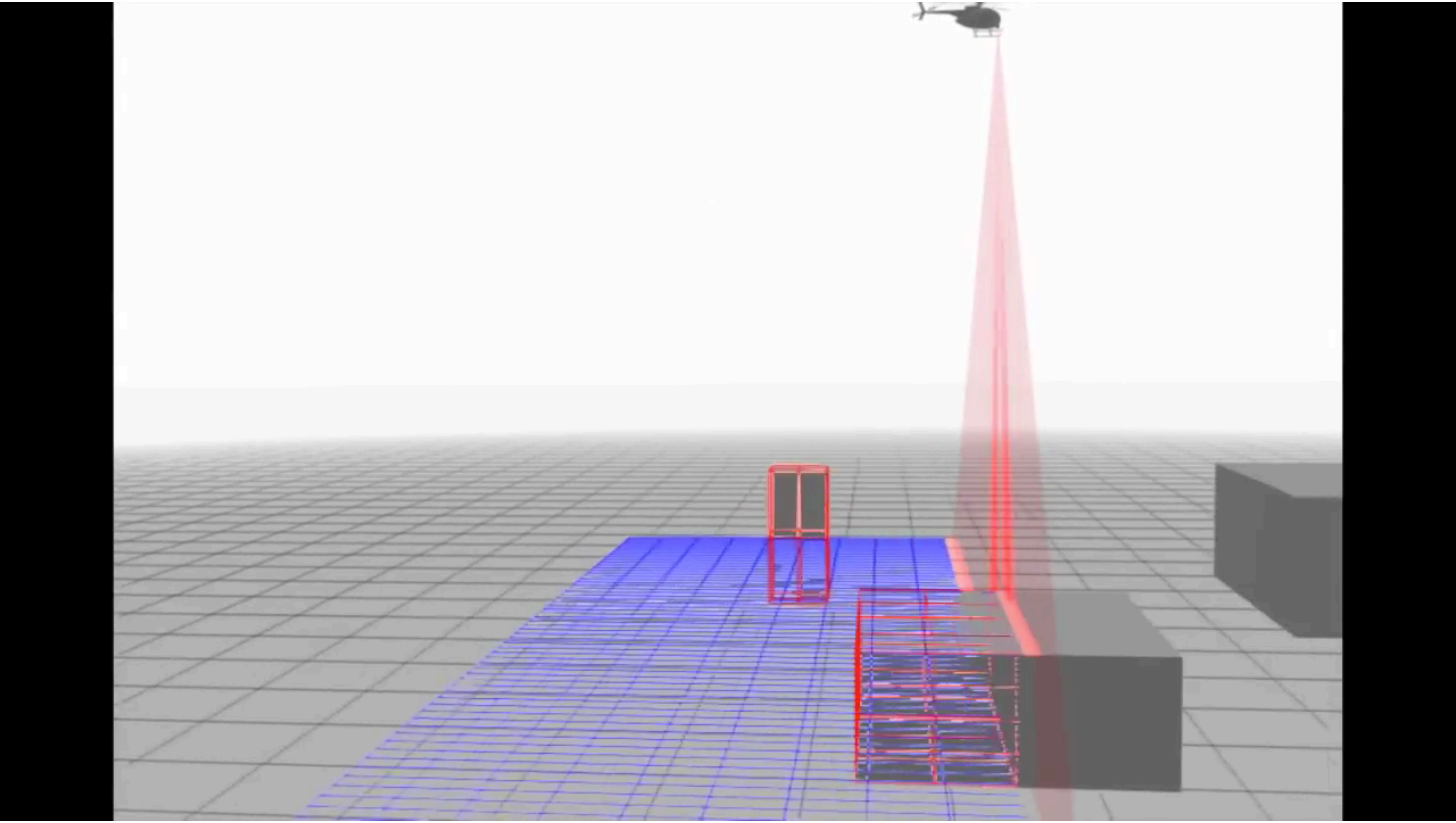
$$z_t = c \cdot \Delta t$$



How does a LiDAR work?

https://www.youtube.com/watch?v=NZKvf1cXe8s&t=67s

Working with Lasers in the Real World



(courtesy Lyle Chamberlain)

Some Pros & Cons

- Fast (~1M photons per second)
- Accurate
- Does not require external light
- Works remarkably well in the real world
- High resolution images
- Photons pass through glass
- Sensitive to weather conditions (rain, snow, ...)
- High-res LiDAR might be expensive

Three Questions You Should Ask

1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

Why is the Measurement Model Probabilistic?

Several sources of stochasticity

Three Questions You Should Ask

1. Why is the model probabilistic?

2. What defines a good model?

3. What model should I use for my robot?

What Defines a Good Model?

Good news: LiDAR is very precise!
A handful of measurements is enough to localize robot

However, has distinct modes of failures

Problem: Overconfidence in measurement can be catastrophic

Solution: Anticipate specific types of failures and add stochasticity accordingly.

Three questions you should ask

1. Why is the model probabilistic?

2. What defines a good model?

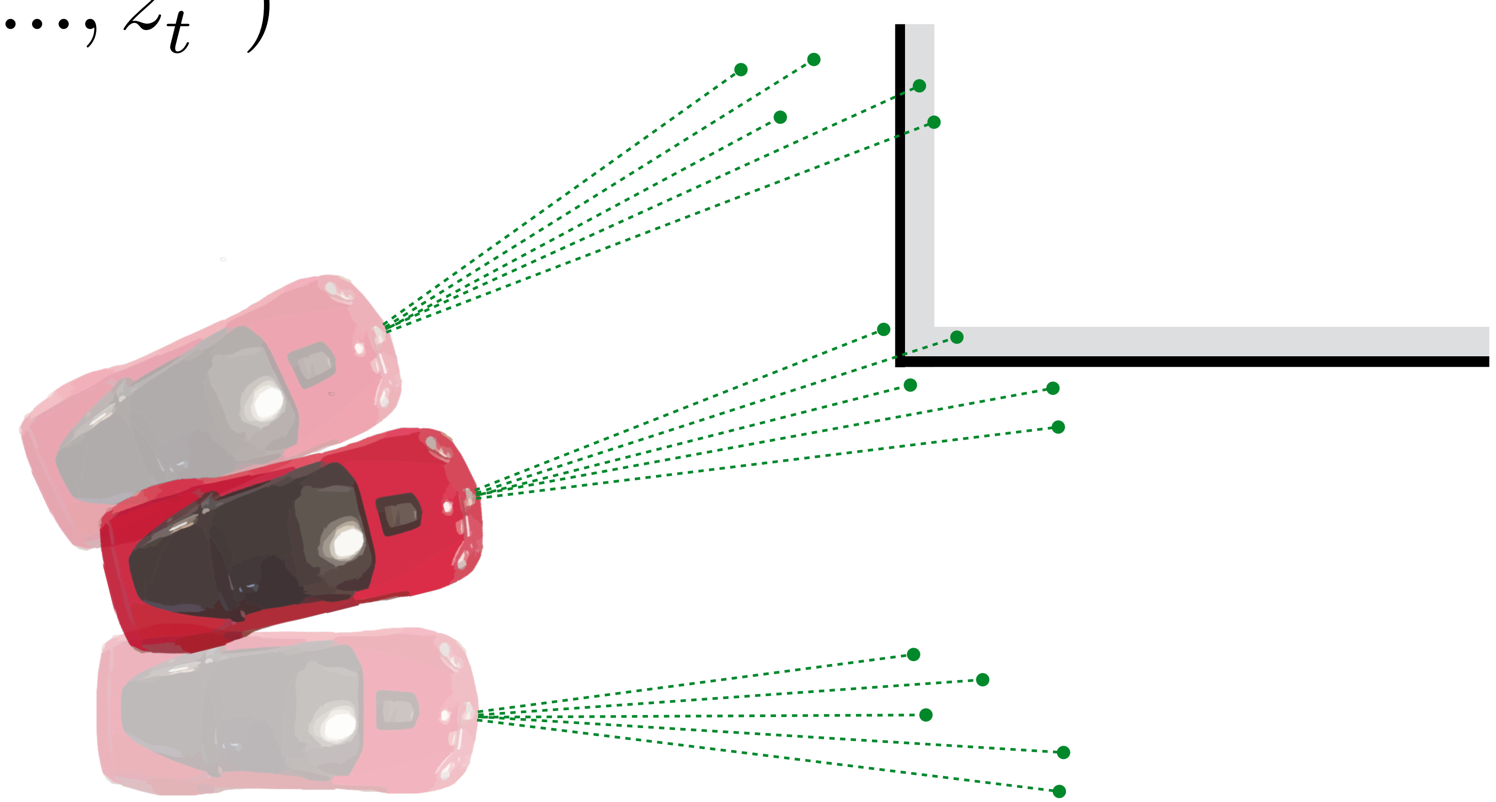
3. What model should I use for my robot?

Measurement Model for LiDAR

$$P(z_t | x_t, m)$$

laser scan — state — map

$$z_t = (z_t^1, ..., z_t^K)$$



Beam-based Model for LiDAR

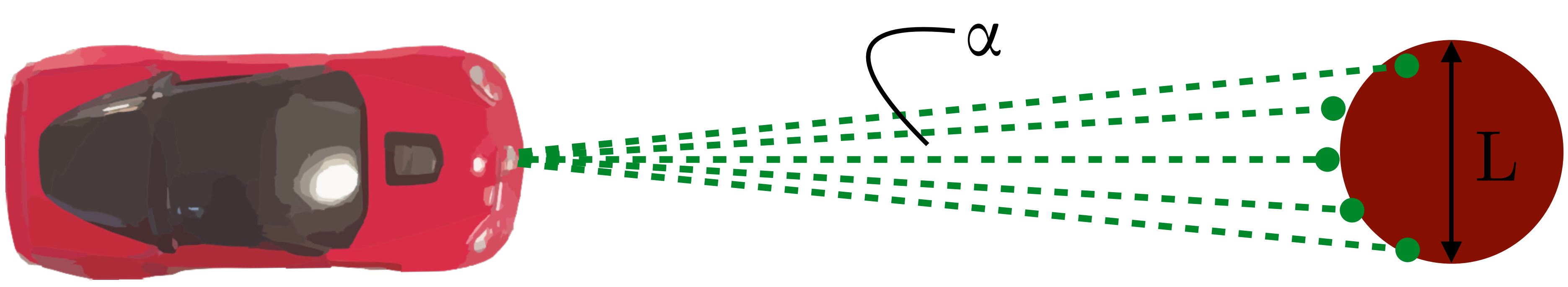
Assume individual beams are conditionally independent given map

$$P(z_t|x_t, m) = \prod_{i=1}^{K} P(z_t^k|x_t, m)$$

laser scan

individual beams

When is this assumption invalid?



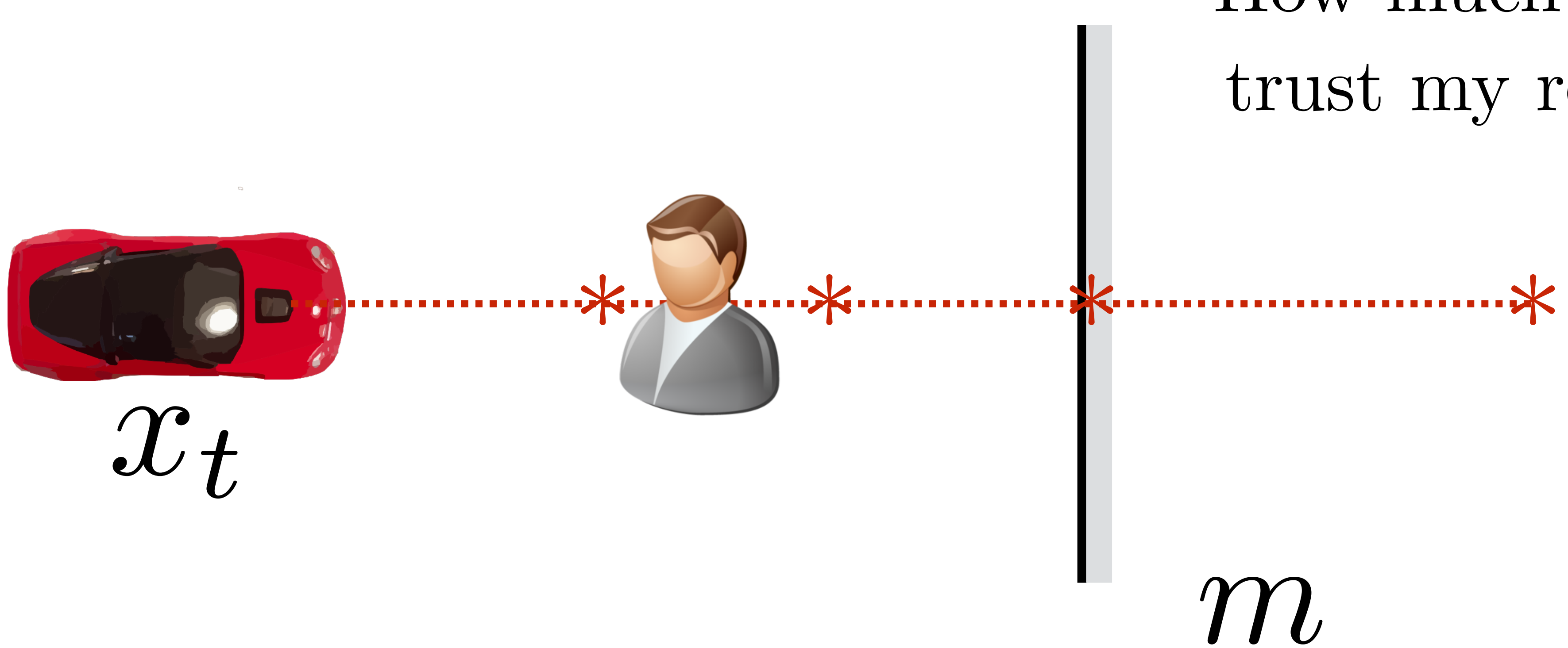
Example: Small α + Large L =Beams correlated

Measurement Model for Single Beam

$$P(z_t^k | x_t, m)$$

distance value — state — map

How much should I trust my readings?



Typical Sources of Stochasticity

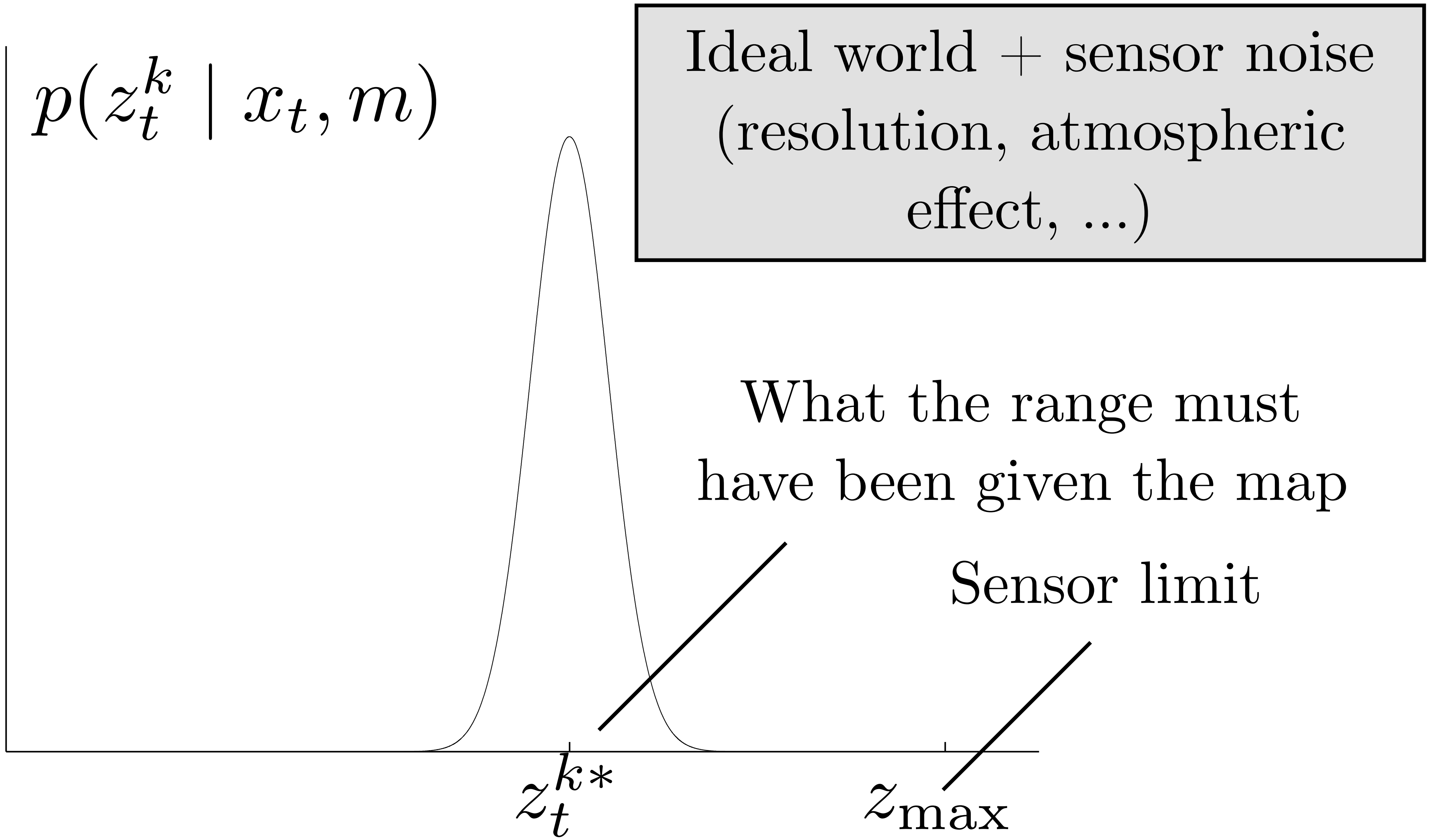
1. Simple measurement noise in distance value

2. Presence of unexpected objects

3. Sensor failures

4. Randomness

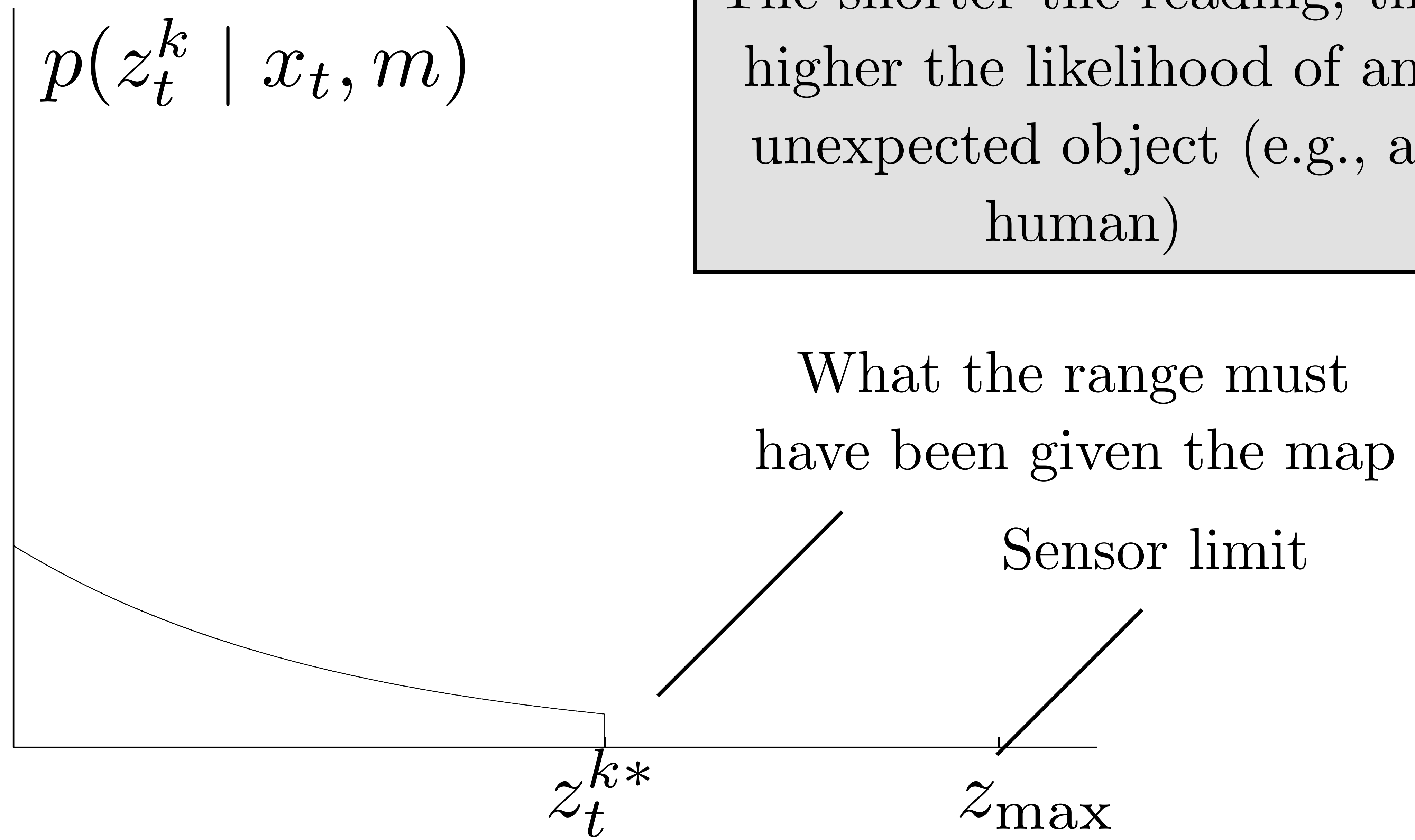
Factor 1: Simple Measurement Noise



$$p_{\text{hit}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \mathcal{N}(z_t^k; z_t^{k*}, \sigma_{\text{hit}}^2) & \text{if } 0 \leq z_t^k \leq z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

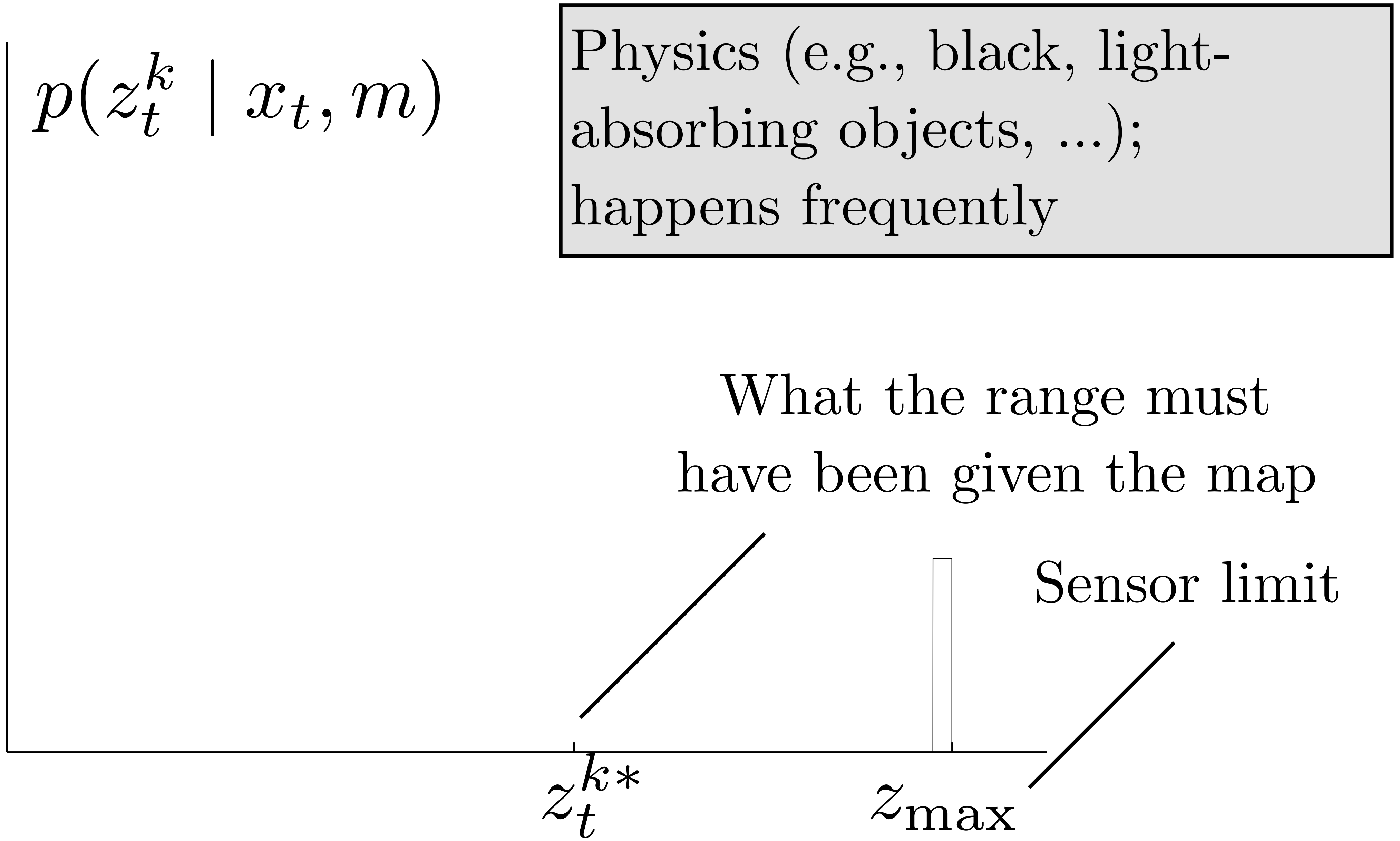
Factor 2: Unexpected Objects

The shorter the reading, the higher the likelihood of an unexpected object (e.g., a human)



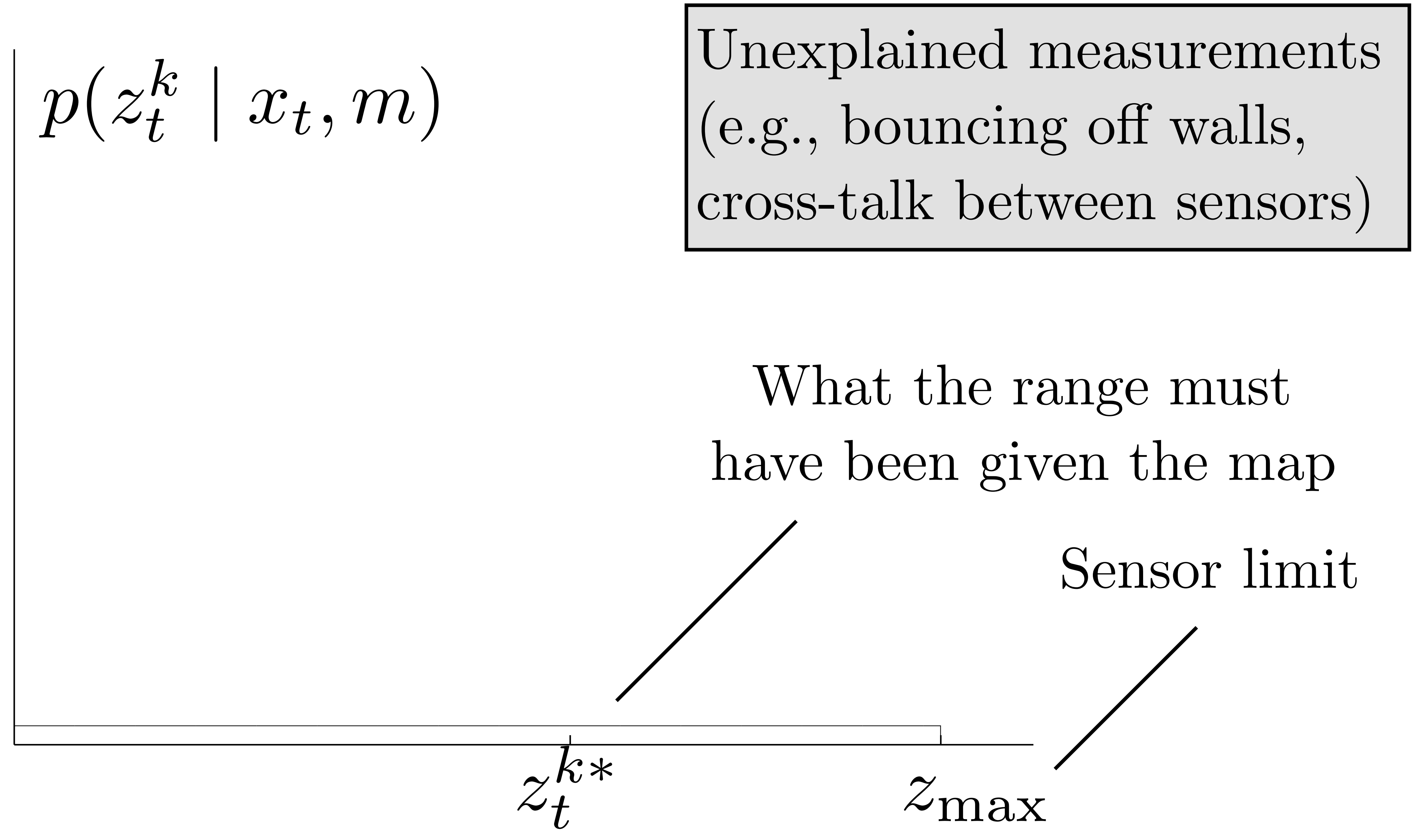
$$p_{\text{short}}(z_t^k \mid x_t, m) = \begin{cases} \eta\, \lambda_{\text{short}}\, e^{-\lambda_{\text{short}} z_t^k} & \text{if } 0 \le z_t^k \le z_t^{k*} \\ 0 & \text{otherwise} \end{cases}$$

Factor 3: Sensor Failures



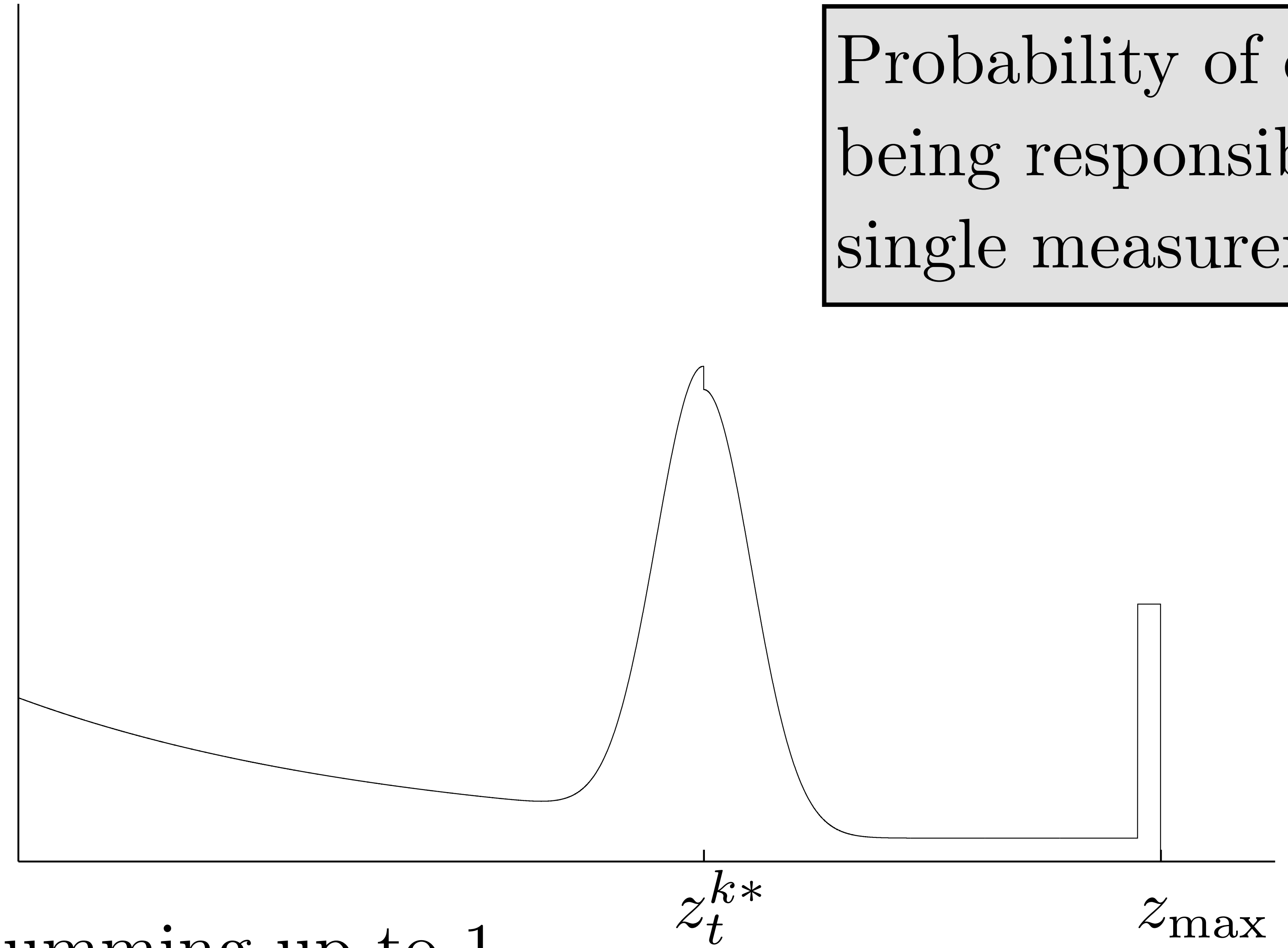
$$p_{\max}(z_t^k \mid x_t, m) \quad = \quad I(z = z_{\max}) \quad = \begin{cases} 1 & \text{if } z = z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Factor 4: Random Measurements



$$p_{\text{rand}}(z_t^k \mid x_t, m) = \begin{cases} \frac{1}{z_{\max}} & \text{if } 0 \le z_t^k < z_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Combined Measurement Model

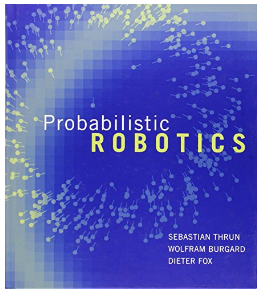


Probability of each mode being responsible for a single measurement.

Weights summing up to 1

$$p(z_t^k \mid x_t, m) = \begin{pmatrix} z_{\text{hit}} \\ z_{\text{short}} \\ z_{\text{max}} \\ z_{\text{rand}} \end{pmatrix}^T \cdot \begin{pmatrix} p_{\text{hit}}(z_t^k \mid x_t, m) \\ p_{\text{short}}(z_t^k \mid x_t, m) \\ p_{\text{max}}(z_t^k \mid x_t, m) \\ p_{\text{rand}}(z_t^k \mid x_t, m) \end{pmatrix}$$

Measurement Algorithm



Chapter 6

Input: State of the robot x, Map m, True laser scan z

Output: Probability p

1. Use x to figure out the pose of the sensor

2. Ray-cast (shoot out rays) from sensor to the map

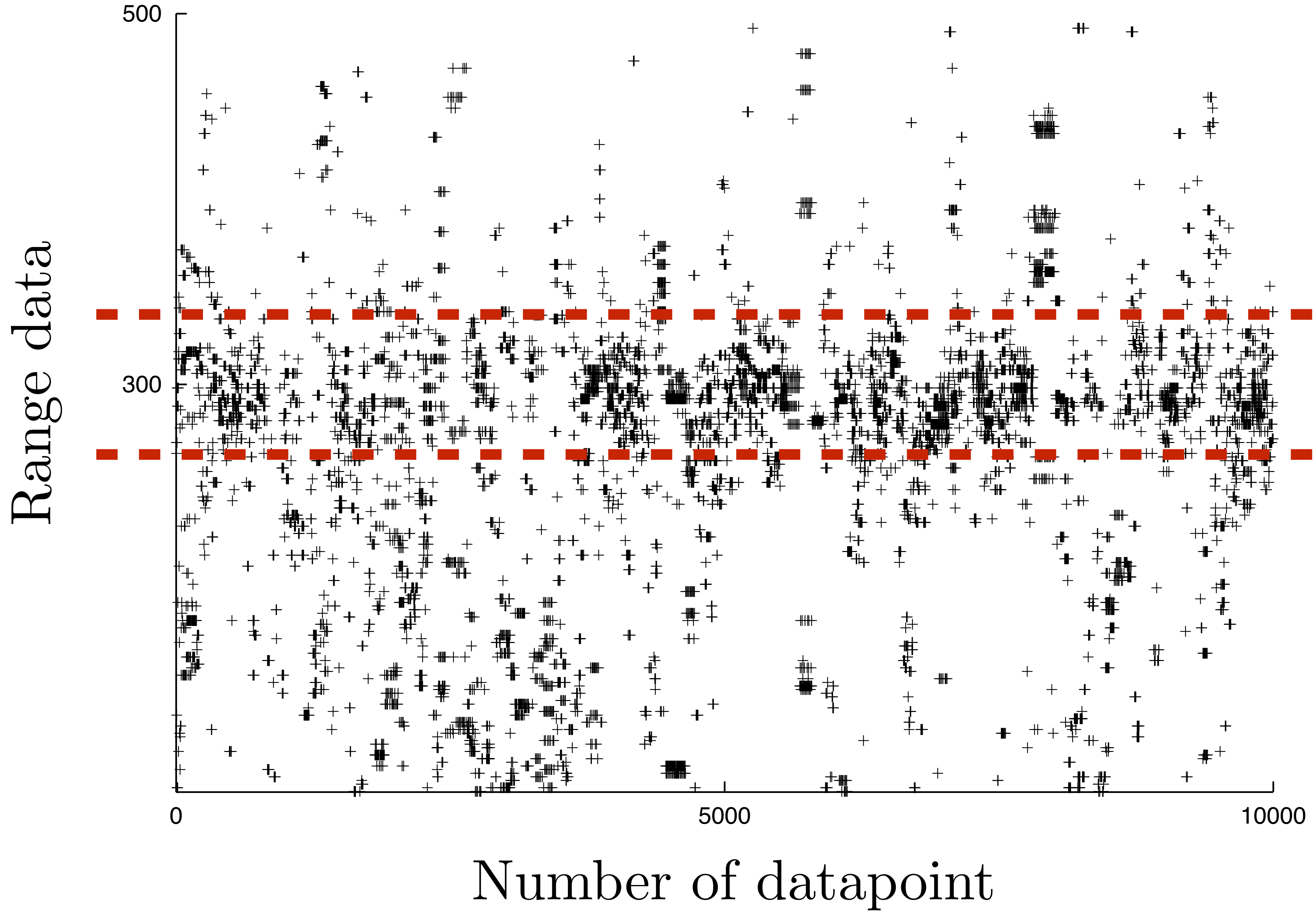
3. Get back a simulated laser-scan z*

4. Go over every ray in z* and compare with z. Compute combined likelihood based on how much they match / mismatch.

5. Multiply all probabilities to get p

Question: How do we Tune Parameters?

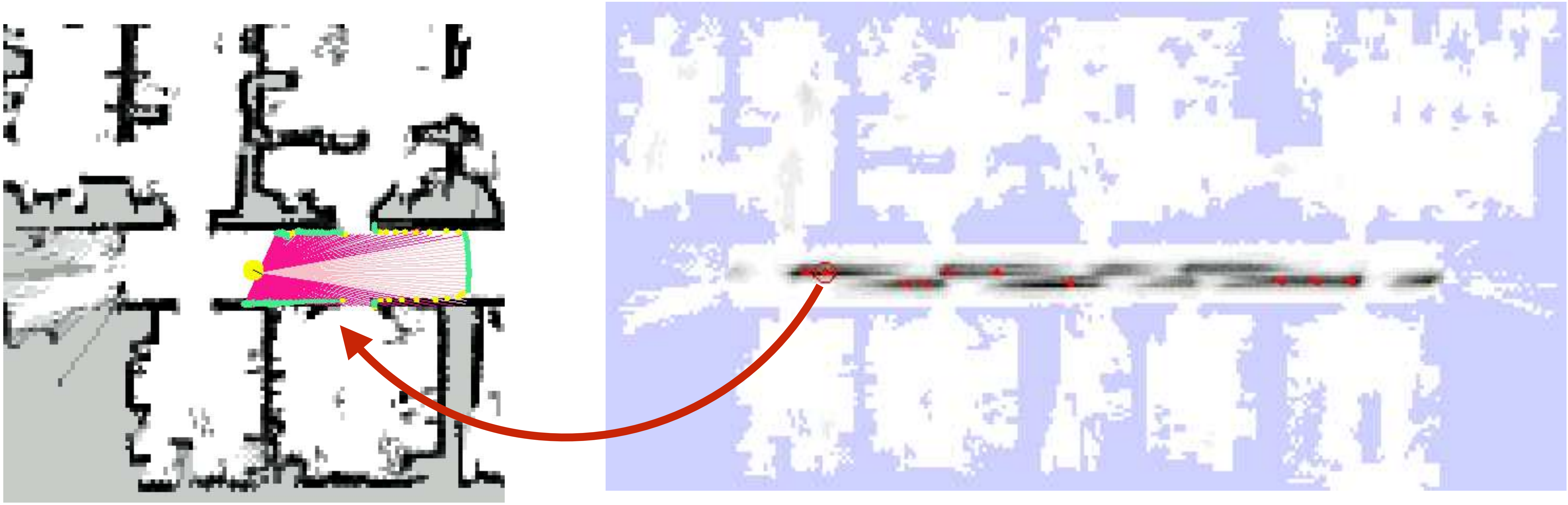
In theory: Collect lots of data and optimize parameters to maximize data likelihood



Example: Place a robot 300 cm from a wall and collect lots of data

Question: How do we Tune Parameters?

In practice: Simulate a scan and plot the likelihood from different positions



Actual scan

Likelihood at various locations

Problem: Overconfidence

$$P(z_t|x_t, m) = \prod_{i=1}^{K} P(z_t^k|x_t, m)$$

Independence assumption may result in repetition of mistakes